Probability

- 1. A bag contains five black marbles and two white ones. Two marbles are taken out, without replacement. What is the probability that one is white and the other is black?
- 2. In a class of 56 students, each studies at least one of the following subjects: Biology, Chemistry, Physics. The Venn diagram shows the number of students studying the various combinations of subjects.



- i. A student is picked at random from the whole class. Find the probability that the student does not study Biology.
- ii. A student is picked at random from those who study at least two of the subjects. Find the probability that the student does not study Biology.
- iii. Two students are picked at random from the whole class. Find the probability that they both study Physics. V Two students are picked at random from those who study Chemistry. Find the probability that exactly one of them studies Biology.
- 3. Two cards are taken from a standard pack, without replacement. What is the probability that one will be a Club and the other a Spade?
- 4. The following data gave the age and gender of 25 pupils in a class on a given day.

	Boys	Girls
Number of pupils aged 16 years	5	7
Number of pupils aged 17 years	7	6

- i. One of the pupils is picked at random. What is the probability that a boy aged 16 years or a girl aged 17 years is picked?
- ii. Each pupil in the class is given his/her examination results. Only three pupils scored full marks. Determine the probability that these three pupils are of the same age and the same gender.
- 5. When a certain player takes a penalty, the probability that he will score $\frac{5}{6}$ each time. In a match, he takes two penalties. What is the probability that he will score exactly once?
- 6. i. Two cards are drawn at random from a normal pack of 52 cards, where R = the probability of a red card and B = the probability of a black card. Complete the probability tree diagram.



ii. Hence, find the probability that both cards are black.

- 7. A fair six-sided die is rolled twice. Find the probability that one result is odd and the other even.
- 8. Three teachers, A, B and C, enter a 10 km race. The probability that each will complete the race are 0.8, 0.6 and 0.5 respectively. Assuming that their performances are independent, find the probability that :
 - i. They all complete the race.
 - ii. At least two complete the race.
- 9. A student sits a multiple choice test. In each question, four possible answers are offered, but only one is correct. The student guesses the answers to two of the questions. What is the probability that she gets exactly one of these right?
- 10. A team of three students is to be chosen at random to take part in a chess competition. The team is to be chosen from seven maths students and three medical students. Find the probability that the team consists of:
 - i. Only maths students
 - ii. Only medical students
 - iii. Two maths students and one medical student

- 11. Three-quarters of all light bulbs in a country are made by Glare, and one-quarter are made by Brighteyes. Five per cent of all Glare lightbulbs last for a year (or more). Ten per cent of all Brighteyes lightbulbs last for a year (or more). If you buy a lightbulb at random in this country, what is the probability that it will last for at least a year?
- 12. The probability that coffee machine will break down in its first year of operation is 0.05.
 - i. What is the probability that the machine will not break down in its first year of operation?
 - ii. A restaurant has just bought two of these machines, which are installed at the same time. Find the probability that at the end of the first year of operation just one machine has broken down. Assume that the performance of each machine is independent.
- 13. If a day is fine, the probability that Mary will be late for school is 0.4. If the day is wet, the probability goes up to 0.5. In Ireland, 20% of all days are wet.Find the probability that on a random day in Ireland, Mary will be late for school.
- 14. A student takes two tests, A and B. The tests are independent of each other, The probability of passing test A is 0.6 and the probability of passing test B is 0.5. Find the probability of passing:
 - i. Both tests
 - ii. At least one of the two tests
- A bag contains five red sweets and four green ones. Pete takes three sweets out in succession and eats them.

What is the probability that:

- i. All three are red.
- ii. One is red and two are green
- 16. A woman visits her local supermarket twice in one week. The probability that she uses a bank card is 0.6 and the probability that she uses cash is 0.4.
 - i. Find the probability that
 - (a) She uses cash on both visits
 - (b) She uses her bank card once and cash once
 - (c) she uses cash
 - ii. Given that she uses cash, what is the probability that she uses her bank card on the first visit?
- 17. A bag contains five green, three blue and four red marbles. two marbles are withdrawn, without replacement. Find the probability that:

find the probability that

- i. Both are green
- ii. Both are green or both are blue

- iii. Both are the same colour
- iv. They are different colours
- v. One is red and one is not red.
- 18. Bob catches a train to work every day from Monday to Friday. The probability that the train is late on Monday is 0.45. the probability that the train is late on any other day is 0.3.
 - i. A day is chosen at random. Find the probability of the following.
 - (a) It is Monday and the train is late
 - (b) The train is late
 - ii. Given that the train is late, what is the probability that it is Monday?
- 19. A bag contains eight black, three white and five red beads. three beads are picked at random, without replacement. Find the probability that:
 - i. All three have the same colour.
 - ii. One is white and the others are not white.
 - iii. All three are of different colours
 - iv. At least two are the same colour.
- 20. For a lottery, 35 cards numbered 1 to 35 are placed in a drum. Five cards will b chosen at random from the drum as the winning combination.
 - i. How many different combinations are possible?
 - ii. How many of all the possible combinations will match exactly four numbers with the winning combination?
 - iii. How many of all the possible combinations will match exactly three numbers with the winning combination?
 - iv. Show that the probability of matching at least three numbers with the winning combination is approximately 0.014.
- 21. Nine tickets, numbered 11,12,13,14,15,16,17,18 and 19 are placed in a hat. One ticket is taken out but not replaced. Another ticket is taken out. Find the probability that :
 - i. Both are prime numbers
 - ii. Both are even or both are odd
 - iii. One is prime but the other is not.
- 22. Two fair six-sided dice, A and B, are cast. What is the probability of getting:
 - i. A total of two or a total of six
 - ii. A total greater than nine or a total that is prime
 - iii. A total that is three times as great as other possible totals.

- 23. Twenty tickets are numbered 1-20. Numbers 1-10 are red, 11-16 are yellow, and 17-20 are green. A ticket is picked at random and replaced. A second ticket is then picked at random. Find the probability that:
 - i. Both are red
 - ii. Both are yellow and even-numbered
 - iii. Both are yellow or even numbered
 - iv. The sum of the numbers on the two tickets is 36.
- 24. There are two boxes on a table labelled A and B. Inside box A are four red spheres and two green spheres. Inside box B are five red cubes and three green cubes. Julie is invited to select one object at random. She may take an object from either box. Find the probability that the object she selects is:
 - i. A sphere
 - ii. Green.
- 25. i. A tennis player gets 50% of his first serves 'in' and 80% of his second serves 'in'. Find the probability that this player will get a 'double fault' (where both and second serves go out).
 - ii. Another player gets 70% of both her first and second serves 'in'. What is the probability that this player will get a double fault?
- 26. Karen is about to sit an examination at the end of an English course. The course has 20 prescribed texts. Six of these are novels, four are plays, and 10 are poems. The examination consists of a question on one of the novels, a question on one of the plays and a question on one of the poems. Karen has studied four of the novels, three of the plays and seven of the poems. Find the probability of the following:
 - i. Karen has studied all three of the texts on the examination
 - ii. Karen has studied none of the texts on the examination
 - iii. Karen has studied at least two of the texts on the examination
- 27. Three people are picked at random. What is the probability that:
 - i. All three have birthdays in different months.
 - ii. At least two have their birthdays in the same month. (N.B Take all months to be equally likely.)
- 28. Ten discs, each marked with a different whole number from 1 to 10, are placed in a box. Three of the discs are drawn at random (without replacement) from the box.
 - i. What is the probability that the disc with the number 5 is drawn?
 - ii. What is the probability that the three numbers on the discs are even;
 - iii. What is the probability that the product of the three numbers on the discs drawn is odd?
 - iv. What is the probability that the largest number on the discs drawn is five?

- 29. i. A poker hand of five cards is dealt randomly from a pack of 52. Find, correct to four decimal places, the probability of being dealt a flush of spades. (where all 5 cards are spades)
 - ii. Find the probability of being dealt a flush in any suit.
- 30. A fishing team has four members, Anthony, Bernard, Catherine and Denise. Of the fish caught, 50% are ray, 20% are salmon and the rest are trout, while 20% of the fish caught are made by Anthony, 45% by Bernard, 10% by Catherine and 25% by Denise. All catches are independent. A fish is selected at random.
 - i. Calculate the probability that the fish is
 - (a) A ray caught by Anthony
 - (b) A trout not caught by Anthony
 - ii. The probability of the selected fish being a ray caught by Catherine is 0.05. What is the probability that a catch made by Catherine is a ray?
 - iii. What is the probability that the fish selected is a ray caught by Bernard or Denise?
 - iv. Given that the fish selected is a ray, find the probability it is caught by Denise.
- 31. Five people are chosen at random. Find the probability that two or more were born:
 - i. On the same day of the week
 - ii. In the same month (taking all months to be equally likely)
- 32. A factory has three machines, P, Q and R producing large numbers of a certain item. Of the total production, 40% is produced on P, 50% on Q and 10% on R. The records show that 1% of items produced on P are defective, 2% of items produced on Q are defective, and 6% of items produced on R are defective. The occurrence of a defective item is independent of each machine and all other items.
 - i. Using D for defective and D' for not defective, write out all possible outcomes and show that the sum of the probabilities add up to 1. One item is chosen at random from a certain day's output.
 - ii. Calculate the probability that the item chosen is defective.
 - iii. Given that the item chosen is defective, find the probability that it was produced on machine ${\cal Q}$
- 33. i. Four people are asked their birthday (e.g. 10th Nov, 25th Jan etc.) Show that the probability that two or more of them have the same birthday (correct to four decimal places) is 0.0164. (Ignore 29th Feb)
 - ii. The Famous Birthday Problem: There are 23 people in a room. Show that the probability that two or more have the same birthday s greater than 0.5.
- 34. i. The probability that a person has a disease is 0.02. What is the probability that they do not have the disease?
 - ii. there is a test for the disease. However, the test has some errors. If you have the disease, there is a probability of 0.95 that the test is positive for the disease. If

you do not have the disease, there is a probability of 0.01 that the test is positive for the disease. A person takes the test. Using a tree diagram or otherwise, calculate the probability that the person has:

- (a) the disease and a positive test result
- (b) A positive test result
- iii. Calculate the probability that the person has the disease, given that the person had a positive result. Give your answer correct to three decimal places.
- iv. Five thousand people have a positive result for the test. How many of these people would you expect to have the disease?
- v. Is this test worth carrying out? Justify your answer.
- 35. Three girls, and one boy meet and discuss their birthdays. they discover that three of them have their birthdays in the same week (i.e. a aseven day week). Find the probability that:
 - i. The three whose birthdays lie in that week are all girls
 - ii. All four were born on different days of the week.
- 36. x and y are randomly selected integers with $1 \le x \le 10$ and $1 \le y \le 10$. p is the point with coordinates (x, 0) and q is the point with coordinates (0, y). Find the probability of the following:



- i. The slope of pq is equal to -1
- ii. The slope of pq is greater than -1
- iii. The length of [pq] is less than or equal to 5.
- 37. Four people are asked (independently) to think of one letter in the word TRIAN-GLES. Find the probability that:
 - i. They all think of the letter T
 - ii. They all think of vowels

- iii. They all think of different letters.
- 38. There are 30 days in June. Seven students have their birthdays in June. The birthdays are independent of each other and all dates are equally likely.
 - i. What is the probability that all seven students have the same birthday?
 - ii. What is the probability that all seven students have different birthdays?
 - iii. Show that the probability that at least two have the same birthday is greater than 0.5.
- 39. Three people are asked to choose at random one of the numbers between 1 and 10 (inclusive). Find the probability that:
 - i. They all choose the same number
 - ii. They all choose different numbers
 - iii. At least two choose the same number.
- 40. A classroom contains 15 desks, which are arranged in rows. The front roe contains three desks. Fifteen students are seated at random in the classroom, eight of whom are boys and seven of whom are girls. Each desk seats only one student. What is the probability of the following?
 - i. Three girls occupy the front row of desks
 - ii. There are more boys than girls seated in the front row
 - iii. There are two girls and one boy seated in the front row, with the two girls seated next to each other.
- 41. A wheel of fortune is divided into eight sections, each of which is equally likely to end up the 'winning' sector (where the arrow points). Sections 1,2 and 3 are grey. Sections 4,5,6, and 7 are black. Section 8 is white.

The wheel is turned twice. Find the probability that:

- i. Black wins twice
- ii. The same colour wins twice
- iii. The total of the two winning numbers is 13
- 42. A bag contains discs of three different colours. There are five red discs, one white disc and x black discs. Three discs are picked together at random.
 - i. Write down an expression in x for the probability that the three discs are all different colours.
 - ii. If the probability that the three discs are all different colours is equal to the probability that they are all black, find x
- 43. Two people are asked (independently of one another) to choose two letters (each) from the letters of the word FRACTION. Find the probability that :
 - i. They both choose two consonants
 - ii. One chooses two consonants and the other chooses two vowels
 - iii. They have no letters in common

iv. They both choose the same pair of letters

- 44. w white discs and r red discs are placed in a box. Two of the discs are drawn at random from the box. The probability that both discs are red is p.
 - i. Find p in terms of w and r
 - ii. When w = 1, find the value of r for which $p = \frac{1}{2}$
 - iii. There are other values of w and r that also give $p = \frac{1}{2}$. The next smallest such value of w is even. By investigating the even numbers in turn, find this value of w and the corresponding value of r.
- 45. There are 11 white socks and n black socks in a drawer. A person draws out two socks, at random. The probability that both are black is $\frac{1}{12}$. Find n
- 46. Eight cards are numbered 1 to 8. The cards numbered 1 and 2 are red, the cards numbered 3 and 4 are blue, the cards numbered 5 and 6 are yellow and the cards numbered 7 and 8 are black. Four cards are selected at random from the eight cards. Find the probability that the four cards selected are :
 - i. All different colours
 - ii. Two odd-numbered cards and two even-numbered cards
 - iii. All differnt colours, two odd-numbered and two even-numbered.
- 47. There are four black balls, 10 white balls and n red balls in a bag. Two balls are drawn without replacement. The probability that one is red and one is not red is found to $\frac{7}{15}$. Find two possible values for n.
- 48. Seven people are picked at random. Find the probability (correct to two significant figures) that their birthdays in a certain year (ignore leap years):
 - i. Fall on different days of the week
 - ii. Fall in different months (taking all months as equally likely)
 - iii. Fall on different dates of the year
- 49. A fair cubic die is relabelled so that it has three 1's, two 2's and one 6. The die is rolled.

T=[The number showing is 2] E=[The number showing is even] O=[The number showing is 1]

(a) i.
$$P(T)$$

ii. $P(E)$
iii. $P(O)$
iv. $P(T')$
v. $P(E')$
vi. $P(T \cap E)$
vii. $P(E' \cap T)$
viii. $P(O' \cap T)$

- (b) Are these pairs of sets mutually exclusive?
 - i. T and E
 - ii. T and O
 - iii. E and O
 - iv. T' and O'
- 50. Eva has a bag into which she is placing copper coins and silver coins. She places six copper coins and x silver coins in the bag. She then draws a coin at random from the bag.
 - (a) Find the value of x:
 - i. If P (the coin is copper) = 0.5
 - ii. If P (the coin is silver) = 0.25
 - iii. If P (the coin is copper) = 0.2
 - (b) Why is it impossible for : P (the coin is silver) = 0.2 ?
- 51. In a certain city, 60% of the cars are from country A, 30% from country B and 10% from country C. Thirty per cent of cars from country A, 25% of cars from country B and 20% of cars from country C last for 10 years or more. Find the probability that:
 - i. A car bought at random in this city, will last for ten years or more.
 - ii. A car will last for 10 years or more, given that it is not from country A.
 - iii. A car is from country C, given that it is 10 years old.
- 52. Three fair six-sided dice are rolled. Find the probability that the total on all three dice is 5 or less.
- 53. A soccer team have five penalty takers (A, B, C, D and E) for a penalty shoot out. Each of the five has to take one penalty. The probabilities for each player's success is given in the following table:

Player	A	B	C	D	E
Probability of scoring	0.9	0.8	0.75	0.5	0.4

Find the probability that:

- i. All five score their penalties
- ii. At least one misses
- iii. They all score, except for E
- iv. They all score, except for D
- v. Exactly four of the five score
- 54. A university student walks, cycles or drives to college with probabilities 0.1, 0.3 and 0.6, respectively. if she walks, she has a probability of 0.35 of being late. If she cycles, the probability of being late is 0.1. If she drives, the probability of being late is 0.55. Find, correct to three decimal places, the probability that:

- i. She will be late on a particular day.
- ii. She walked, given that she was late.
- iii. She walked, given that she was not late.
- 55. During the season, a hurling team (called The Random Variables) played 32 matches. Twenty of these matches were in fine weather and they won 16 of them. The other 12 were in rainy weather and they won only four of these.

Give your answers to the following questions as fractions in their lowest terms.

- i. What is the relative frequency of the Random Variables winning in fine weather?
- ii. What is the relative frequency of the Random Variables winning in rainy weather?
- iii. The Random Variables are playing in the Cup Final next Sunday. The weather forecasters say that the probability of fine weather is $\frac{3}{4}$ and the probability of rainy weather is $\frac{1}{4}$. Using the relative frequency as probabilities, find the probability that they will win.
- 56. A teacher has one euro coin and three 50c coins in his back pocket. He has two euro coins and one 50c coin in his left pocket. He has three euro coins only in his right pocket. He rolls a die. If it comes up as 1,2 or 3 he will give a coin randomly from his back pocket as a prize to the best pupil. If a 4 or a 5 comes up, he will give a coin from his left pocket. If a 6 comes up, he will give a coin from his right pocket. What is the probability that the coin given will be a euro?