

Functions and Calculus

1 Rates of Change

1. The population, P , of a new housing estate t years after a certain date is given by the formula $P = 100(5 + t - 0.25t^2)$.
Find the rate of change of the population after 3 years.

2. The distance, in metres, travelled by an object in t seconds is given by the formula
$$s = t^3 - 2t^2 + 3t - 4.$$

Find (i) the speed and (ii) the acceleration when $t = 4$.

3. An object is projected upwards and its height in metres above its point of projection after t seconds is

$$h = 600t - 5t^2.$$

- i. After how many seconds is the object momentarily at rest?
ii. Find the greatest height, in km, the object reaches.
4. A particle P moves in a straight line. After t seconds, the displacement, in metres, of P from a fixed point O on the line is given by $s = t^3 - 2t^2 + 4t$. Calculate
i. the distance between P and O at the time $t = 2$
ii. the times at which the velocity of p equals 4 metres per second.
5. A particle moves in a straight line so that its position, s cm, relative to O at time t seconds ($t \geq 0$) is given by

$$s = 2t^3 - 5t^2 + 4t - 5$$

- i. When its velocity zero, and what is its acceleration at these points?
ii. When its acceleration zero, and what is its velocity at that time?
6. A particle moving in a straight line is x cm from the point O at time t seconds ($t \geq 0$), where $x = t^3 - 11t^2 + 24t - 3$.
i. Find its initial position and velocity.
ii. Find its velocity at any time.
iii. At what times is the particle stationary?
iv. What is the position of the particle when it is stationary?
v. For how long is the particle's velocity negative?
vi. Find its acceleration at any time.

- vii. When is the particle's acceleration zero, and what is its velocity and position at that time?
7. A car accelerates in a straight line so that its distance s (in metres) from its starting point p after t seconds is given by the function $s(t) = t^2$. Find:
- The distance of the car from p after 4 seconds
 - The distance of the car from p after 5 seconds
 - The speed of the car in terms of t
 - The speed of the car after 4 seconds
8. A ball is thrown upward from the ground level with an initial speed of 9.8 m/s so that its height in metres after t s is given by $y = 9.8t - 4.9t^2$
- What is the acceleration of the ball at any time t ?
 - How high does the ball go?
 - How fast is it moving when it strikes the ground?
9. A ball is thrown downward from the top of a 100-metre high tower with an initial speed of 2m/s. Its height in metres above the ground t s later is $y = 100 - 2t - 4.9t^2$.
- How long does it take to reach the ground?
 - What is its average velocity during the fall?
 - At what instant is its velocity equals to its average velocity?
10. The distance an aircraft travels along a runway before takeoff is given by $D = t^2$, where D is measured in metres from the starting point, and t is measured in seconds from the time the brake is released. If the aircraft will become airborne when its speed reaches 200 km/h, how long will it take to become airborne, and what distance will it travel in that time?
11. A ball is thrown upward with initial velocity v m/s and reaches a maximum height of h m.
- How high would it have gone if its initial velocity was $2v$?
 - How fast must it be thrown upward to achieve a maximum height of $2h$ m ?
12. A rock falls from the top of a cliff and hits the ground at the base of the cliff at a speed of 160 ft/s. How high is the cliff?
13. With full brakes applied, a freight train can decelerate at a constant rate of $1/6m/s^2$. How far will the train travel while braking to a full stop from an initial speed of 60 km/h?
14. The position of an object moving along the s -axis is given at time t by

$$s = \begin{cases} t^2 & \text{if } 0 \leq t \leq 2 \\ 4t - 4 & \text{if } 2 < t < 8 \\ -68 + 20t - t^2 & \text{if } 8 \leq t \leq 10 \end{cases}$$

- i. Determine the velocity and acceleration at any time t .
- ii. Is the velocity continuous?
- iii. Is the acceleration continuous?
- iv. What is the maximum velocity and when is it attained?