Financial Series

1. Calculate the future value of 36 monthly instalments of €20.00 at an interest rate of 0.5% per month. What is the total interest earned on these savings?

2. Marie has saved €30.00 per month since her 18th birthday. If her bank has guaranteed her an interest rate of 4% per annum, find
   (a) the equivalent monthly rate of interest, correct to two decimal places
   (b) the value of her savings on her 21st birthday.

3. A special savings account offers AER of 4% per annum. If I invest €2000 per year in this account, how much will my investment be worth in 5 years time?

4. Show that the future value of a series of n payments of €P , earning an interest rate of i% per annum, can be written as:
$$\text{Future Value} = P(1+i)\frac{(1+i)^n - 1}{i}$$

5. Show that the present value of a series of n payments of €P, earning an interest rate of i% per annum, can be written as:
$$\text{Present value} = \frac{P}{(1+i)^n}\frac{(1+i)^n - 1}{i}$$

6. Anne received a cheque in the post for €6523.33 after saving for 5 years with her bank in a scheme offering 9% per annum. If she invested €A per annum,
   (a) write down a geometric series representing the value of her investment over the 5 years
   (b) find the value of A

7. Use the future value formula to find the final value if €200 is invested every month for 2 years. The interest rate is 9% per annum, compounded monthly.

8. George wants to make regular payments into an account that pays 8.5% per annum compound interest in order to have €10 000 after 7 years. Find the amount of each annual payment.

9. Ella wants to have €5000 in 3 years time. She invests in an annuity that pays 7.2% per annum, compounded quarterly. How much does she need to deposit each quarter to achieve her target of €5000?

10. A company wishes to raise capital to expand. It offers a 10-year €2,000 bond that will pay €100 every year for ten years. Given that the expected market interest rate over the lifetime of the bond is 5%, is €2,000 a fair price to pay for this bond? Be careful to state clearly any assumptions that have been made.

11. ABC Finance issues a bond offer as detailed below:
    10-year bond
    Pays €30 at the end of every six months for 10 years
    AER=6% interest compounded bi-annually.
    What is a fair price for this bond?

12. An investment bond quotes an annual equivalent rate of 13.5%. If interest on the investment fund is compounded bi-annually, what is the rate of interest for each compounding period? Answer correct to four significant figures.

13. Verify that the figures given in this advertisement are accurate.

Investments	Description
Savings certificates	Interest 21% after 5.5 years, AER 3.53% tax free
Savings bonds	Interest 10% after 3 years, AER 3.23% tax free

14. O'Reilly-Elwood Finance Company issues a bond offer as detailed below:
    10 year bond
    Pays €100 at the end of every six months for 10 years
    AER=5% interest compounded bi-annually.
    What is a fair price for this bond?

15. A building society offers a savings account with an AER of 4%. If a customer saves €2,000 per annum starting now, how much will the customer have in five years' time?

16. A bank offers a savings account with an effective annual rate of 3%. If a customer saves €1,000 per annum starting now, how much will the customer have in six years' time?

17. A building society offers a savings account with an AER of 4.5%. If a customer saves €2,000 per annum starting now, how much will the customer have in 4 years' time?

18. A building society offers a savings account with an AER of 4% compounded monthly. If a customer saves €150 each month starting now, how much will the customer have in five years' time?

19. A bank offers a savings account with an AER of 10% compounded monthly. If a customer saves €110 per month starting now, how much will the customer have in five years' time?

20. Nicki deposits €200 at the end of each quarter in her savings account. the money earns 5.5% (AER). How much will the investment be worth at the end of four years? State clearly any assumptions that you make.

21. Eoin wants to have €5,000 in three years' time to travel to the USA on a J1 Visa. How much will he need to deposit at the end of each month into an account that pays 8% (EAR)? (Remember : EAR and AER are used interchangeably.)

22. Mercedes took part in a TV game show and won the top prize. She is given two options:

(a) Receive €1,000 at the end of every month for the next 20 years.

(b) Take a lump sum now.

If the AER IS 8%, what is the minimum amount Mercedes should accept as a lump sum?

23. Suppose you expect to receive a payment of €200 at the end of each year for an indefinite period of time. What is the present value of this annuity?

24. Chelsea's parents wish to set up a regular savings account from the day she is born so that on her 21st birthday she will have €21,000. How much should they plan to deposit each month if they choose a regular savings plan with an AER of 3.5%?

25. Julie contributed €200 at the end of each week for 20 years to a pension fund earning 4.5% AER.

(a) Find the rate of interest per week that, if compounded weekly, would be equivalent to an AER of 4.5% (assume a 52-week year)

(b) What was her lump sum payment when she retired?

(c) Julie used her lump sum to purchase an annuity at 3.8% AER, giving her a regular payment at the start of each month for the next 20 years. What was her monthly payment ?

26. Most lottery games in the USA allow winners of the jackpot prize to choose between two forms of the prize: an annual- payments option or a cash- value option. In the case of the New York Lotto, there are 26 annual payments in the annual-payments option, with the first payment immediately, and the last payment in 25 years' time. The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The cash-value option pays a smaller amount than this.

(a) If the amount of the first annual payment is A, write down, in terms of A, the amount of the second, third, fourth and 26th payments.

(b) The 26 payments form a geometric series. Use this fact to express the advertised jackpot prize in terms of A.

(c) Find, correct to the nearest dollar, the value of A that corresponds to an advertised jackpot prize of $21.5 million.

(d) A winner who chooses the cash-value option receives, immediately the total of the present values of the 26 annual payments. The interest rate used for the present-value calculations is 4.78%. We want to find the cash value of the prize referred to in part (c).

　i. Complete the table below to show the actual amount and the present value of each of the first three annual payments.

Payment Number	Time to payment (years)	Actual amount $	Present Value $
1	0		
2	1		
3	2		

ii. Write down, in terms of n, an expression for the present value of the nth annual payment.

iii. Find the amount of prize money payable under the cash-value option. That is, find the total of the present values of the 26 annual payments. Give your answer in millions, correct to one decimal place.

(e) The jackpot described in parts (c) and (d) above was won by an Irish woman earlier this year. She chose the cash-value option. After tax, she received $7.9 million. What percentage of tax was charged on her winnings? Give your answer to one place of decimals.

27. Padraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. The he calculates how much he needs to invest in order to achieve this. He asumes that in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.

(a) Write down the present value of a future payment of €20,000 in one years' time.

(b) Write down, in terms of t, the present value of a future payment of €20,000 in t years' time.

(c) Padraig wants to have a fund that could, from the date of his retirement, give him a payment of €20,000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value on the date of retirement of the fund required.

(d) Padraig plans to invest a fixed amount of money every month in order to generate the fund calculated in (c). His retirement is $40 \times 12 = 480$ months away.

i. Find, correct to four significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of 3%.

ii. Write down, in terms of n and p, the value on the retirement date of a payment of €P made n months before the retirement date.

iii. If Padraig makes 480 equal monthly payments of €P from now until his retirement, what value of P will provide him with the funds he wants?

iv. If Padraig waits for 10 years before starting his pension investments, how much will he then have to pay each month in order to generate the same pension fund?