

## Measures of Variation

1. The stem and leaf plot gives the ages of 31 people attending a meeting about a government proposal not to grant medical cards to all people over the age of 70.

5	1, 4, 4, 4
5	5, 9, 9, 9, 9
6	3, 3, 3, 4, 5
6	6, 6, 7, 7, 8, 8, 9
7	1, 1, 2, 3, 3, 3, 3, 3, 4
7	5

- i. Find  $Q_1$  , the lower quartile.
  - ii. Find  $Q_3$  , the upper quartile.
  - iii. Calculate the interquartile range.
  - iv. Describe the shape of the distribution.
  - v. Explain the shape of the distribution in the context of this question.
2. The following data indicates the electricity consumption (in kilowatt-hours) for 20 typical two-bedroom apartments in Dublin.

9	11	16	10	11	9	8	13	11	9
13	11	7	9	7	12	14	13	11	7

- i. Draw a stem and leaf diagram to illustrate the data.
  - ii. Describe the distribution.
  - iii. Find the range.
  - iv. Find the median.
  - v. Find the interquartile range.
3. The percentage of silica was calculated in each of 22 meteorites. The data is displayed below.

20.77	22.56	22.71	22.99	26.39	27.08	27.32	27.33	27.57	27.81
28.69	29.36	30.25	31.89	32.88	33.23	33.28	33.40	33.52	33.83
33.95	34.82								

- (a) Provide the following summary statistics

- i. The mean.
  - ii. The median
  - iii. The minimum value
  - iv. The maximum value
  - v. The range
  - vi. The interquartile range.
- (b)
- i. What is the difference between the mean and the median for the data set given above?
  - ii. What does this difference tell us about the shape of the distribution?
4. Examine the data sets X and Y. Without doing any calculations, decide which set has the larger standard deviation. Then check by doing the standard deviation by hand.
- i.  $X = (15, 17, 19, 21, 23)$ ,  $Y = (14, 16, 18, 20, 22)$
  - ii.  $X = (9, 13, 14, 15, 19)$ ,  $Y = (14, 18, 19, 20, 24)$
  - iii.  $X = (x, x + 1, x + 2, x + 3)$ ,  $Y = (x + 10, x + 11, x + 12, x + 13)$
  - iv.  $X = (x + 2, x + 5, x + 8, x + 9)$ ,  $Y = (x + 22, x + 25, x + 28, x + 29)$
5. The gross national product (GNP) is the value of all goods and services produced by an economy in a year. The table gives Ireland's GNP (in €m ) for the period 2006-2009.

Year	2006	2007	2008	2009
GNP	154,078	162,853	154,672	131,241

- i. Calculate the mean and the standard deviation for the data
  - ii. Why would an economist be interested in the standard deviation from the mean?
  - iii. In what year was the maximum value of GNP?
  - iv. In what year was the minimum value of GNP?
  - v. In your opinion, is the economy in recession? Explain your answer.
6. The frequency distribution shows the ages of 100 people. Note: 0-10 means 0 is

Age	0-10	10-20	20-30	30-50	50-80
Frequency	10	19	25	30	16

included but 10 is not , etc.

- i. Estimate the mean age.
  - ii. Find to the nearest integer the standard deviation from the mean.
7. Weights (in grams) of samples of the contents in cans of regular Coke and Diet Coke are given below.

Regular Coke				
370	371	369	371	363
362	369	367	367	365

Diet Coke				
352	352	349	354	353
352	350	354	357	351

Regular Coke			
Weight (g)	360-365	365-370	370-375
Frequency			

Diet Coke			
Weight (g)	345-350	350-355	355-360
Frequency			

- i. Complete the grouped frequencies for the data.  
 Note: 360-365 means 360 is included but 365 is not etc.  
 Note: 345-350 means 345 is included, but 350 is not etc.
  - ii. Display the grouped frequency distributions on two histograms.
  - iii. Using only evidence from your histograms, state which set has the greater standard deviation.
  - iv. Confirm your answer to part (iii) by calculating the standard deviation for both distributions.
8. The array of numbers 1, 2, 4, 5, 8, 16 has mean  $\mu$  and standard deviation  $\sigma$  . Verify that  $\mu - \sigma = 1$
  9. The array of numbers 1.8, 2.6, 4.8, 7.2 has mean  $\mu$  and standard deviation  $\sigma$  . Verify that  $\mu - \sigma = 2$

10. The two machines, X and Y, are used to pack biscuits. A random sample of 10 packets was taken from each machine and the mass of each packet was measured to the nearest gram.
- Find the standard deviation of the masses of the packets taken in the sample from each machine.

Machine X (mass in g)	195	197	197	198	199	199	200	200	201	204
Machine Y (mass in g)	191	193	194	197	199	200	202	203	205	206

- By comparing the results for the standard deviations, comment on which machine is more reliable.
11. In a school there are two fifth year Mathematics teachers called Sir and Miss. Over the last 10 years the mean mark achieved by Sir's students on the Leaving Certificate Mathematics exam was 60%, with a standard deviation of 2%. During the same period, Miss' students also have a mean mark of 60%, but the standard deviation of these students' marks is 10%. Students in this school have a choice of teacher when entering fifth year. Which teacher should a student who is good at Mathematics choose? Which teacher should a student who is weak at Mathematics choose? Justify your answers.
12. On 1 September 2010 the mean age of the first year students in a school was 12.4 years and the standard deviation was 0.6 years. One year later, all of these students had moved into second year and no other students had joined them.
- State the mean and the standard deviation of the ages of these students on 1 September 2011. Give a reason for each answer. A new group of first year students began on 1 September 2011. This group had a similar age distribution and was of a similar size to the first year group of September 2010.
  - State the mean age of the combined group of the first year and second year students on 1 September 2011.
  - State whether the standard deviation of the ages of this combined group is less than, equal to or greater than 0.6 years. Give a reason for your answer.
13. The owner of a small clothes shop records the number of customers she has during a typical six-day week in June. The number of customers each day were: Monday 15, Tuesday 12, Wednesday 15, Thursday 14, Friday 21, Saturday 31.
- Calculate  $\mu$ , the mean number of customers per day.
  - Calculate  $\sigma$ , the standard deviation, correct to one decimal place.
  - Calculate  $\mu + \sigma$  and  $\mu - \sigma$ .
  - On how many days is the number of customers within one standard deviation of the mean?
  - On what percentage of the days is the number of customers within one  $\pm$  one standard deviation of the mean?
- Later during a week in July the number of customers was:  
Monday 17, Tuesday 14, Wednesday 17, Thursday 16, Friday 23, Saturday 33  
Notice there are  $\pm 2$  customers on each day when compared to the previous week.

- vi. Write down the mean number of customers for the July week.
  - vii. Write down the standard deviation for the July week.
14. Do men or women have more friends? To answer this question, a researcher asked 15 men and 15 women to list the names of friends they had spent time with during the past year. The researcher then counted the number of friends for each man and each woman. The results are given below.  
 Men: 5, 9, 10, 4, 22, 13, 19, 21, 28, 17, 11, 13, 17, 16, 20.  
 Women: 35, 21, 10, 18, 9, 23, 39, 33, 29, 18, 20, 17, 15, 14, 29.
- i. Verify that  $\mu_{men} = 15$  and  $\mu_{women} = 22$  .
  - ii. Using your calculator or otherwise, find the standard deviations  $\sigma_{men}$  and  $\sigma_{women}$  , correct to one decimal place.
15. In a school with 125 girls, each student is tested to see how many sit-up exercises she can do in one minute. The results are given in the table below:

Number of sit-ups	Number of students	Cumulative number of students
15	11	11
16	21	32
17	33	$p$
18	$q$	99
19	18	117
20	8	125

- i. (a) Write down the value of  $q$   
 (b) Find the value of  $p$
- ii. Find the median number of sit-ups.
- iii. Find the mean number of sit-ups, correct to the nearest integer.