# Probability Sheet

## 1 Permitations

1. How many ways are there of arranging the letters of the word MATRIX:
   (a) If there are no restrictions
   (b) If they must begin with T
   (c) If they must begin with T and end with M
   (d) If they must begin with a vowel
   (e) If they must not begin with a vowel
2. How many four digit numbers can be made from the digits 1,3,4 and 5 (without repetition)?
   (a) How many of these are even?
   (b) How many are odd?
   (c) How many are over 4,000?
3. How many ways are there of arranging the letters of the word Monster:
   (a) If there are no restrictions
   (b) If they must begin with a vowel
   (c) If they must begin with a consonant
   (d) If the vowels must be together
   (e) if the vowels must be apart
4. How many natural numbers can be made using some or all of the digits 0, 1, 2 and 3, with no repetitions? (Note: Zero is not considered a natural number.)
5. A code for breaking into a computer system consists of two different letters followed by three different digits.
   (a) How many different codes are possible?
   (b) A user has been given a code, but cannot remember it fully. All that she can remember is that the first letter is B and the first digit is 8. How many different possible codes fit this description?
6. Two girls and three boys are to be lined up for a photograph. How many different arrangements are possible:

(a) If there are no restrictions

(b) If they must be boy-girl-boy-girl-boy

7. How many four digit numbers can be formed from the digits 0, 1, 2, 3,....,9, if no digit is repeated in a number?

   How many of these numbers

   (a) are greater than 8000?

   (b) are divisible by 10?

# 2 Combinations

1. An examination paper consists of 9 questions
   i. In how many ways can 5 questions be selected?
   ii. If question 1 is compulsory, in how many ways can 5 questions then be selected?

2. In how many different ways may 5 colours be selected from 10 different colours including red, blue, and green.
   i. if blue and green are always included
   ii. if red is always included
   iii. if red and blue are always included but green excluded?

3. A committee of six persons is to be chosen from five men and three women. In how many ways can this be done
   i. when there are 4 men on each committee
   ii. when there is a majority of men on each committee?

4. A student has to choose three subjects from the following range; biology, music, art, chemistry, economics, history, Greek. How many different choices can the student make:
   i. If there are no restrictions
   ii. If Greek must be included as one of the three subjects
   iii. If Greek must be excluded

5. There are eight people, including Mr. and Mrs. Jones, on a committee. How many subcommittees of four can be selected
   i. if all members are eligible for the subcommittee
   ii. if Mr. and Mrs. Jones are included on each subcommittee
   iii. if neither Mr. nor Mrs. Jones can be included on the subcommittee?

6. In how many ways can a committee of five be chosen from 11 people:
   i. If one particular person must be included
   ii. If one particular person must be excluded

iii. If two particular people wil not work on the same committee (that is, if one is included, the other must be excluded)

7. In how many ways can a committe of six be selected from five men and four women if each committee consists of
   i. An equal number of men and women
   ii. At least three men
   iii. At least three women

8. A committee of six is to be formed from eight students and five teachers. How many different committees can be formed if there are to be more teachers than students?

9. A team of five players is to be chosen from six boys and five girls. If there must be more boys than girls, how many different teams can be formed?

# 3 The Addition Rule

1. An unbiased six-sided die is thrown. Find the probability that the number obtaiined is:
   i. Even
   ii. Prime
   iii. Even or Prime.

2. A number is selected at random from the integers 1 to 30 inclusive. Find the probability that the number is:
   i. a multiple of 3
   ii. a multiple of 5
   iii. a multiple of 3 or a multiple of 5

3. If two fair dice are thrown, what is the probability of getting:
   i. the same number on both dice
   ii. a total of 8
   iii. the same number on both dice or a total of 8?

4. In a group of 40 adults, 8 out of the 14 women and 2 out of the 26 men wear glasses. What is the probability that a person chosen at random:
   i. is a woman
   ii. is a person who wears glasses
   iii. is a woman or a person who wears glasses?

5. A card is drawn at random from a pack of 52. Find the probability that the card drawn is:
   i. a spade

ii. an ace

iii. a spade or an ace

iv. a red card or an ace

6. A card is selected at random from a pack of 52. Find the probability that the card is:

   i. a spade or a club

   ii. a queen or a red card

   iii. a heart or a red ten

   iv. not a heart or a red ten.

7. Two dice are cast. What is the probability of getting:

   i. a total of two or a total of six

   ii. a total greater than nine or a total which is prime.

# 4 Tree Diagrams

1. When a certain player takes a penalty, the probability that he will score $\frac{5}{6}$ each time. In a match, he takes two penalties. What is the probability that he will score exactly once?

2. Three-quarters of all light bulbs in a country are made by Glare, and one-quarter are made by Brighteyes. Five per cent of all Glare lightbulbs last for a year (or more). Ten per cent of all Brighteyes lightbulbs last for a year (or more). If you buy a lightbulb at random in this country, what is the probability that it will last for at least a year?

3. Three teachers, A, B and C, enter a 10 km race. The probability that each will complete the race are 0.8, 0.6 and 0.5 respectively. Assuming that their performances are independent, find the probability that :

   i. They all complete the race.

   ii. At least two complete the race.

4. If a day is fine, the probability that Mary will be late for school is 0.4. If the day is wet, the probability goes up to 0.5. In Ireland, 20% of all days are wet.
   Find the probability that on a random day in Ireland, Mary will be late for school.

5. A student takes two tests, A and B. The tests are independent of each other, The probability of passing test A is 0.6 and the probability of passing test B is 0.5. Find the probability of passing:

   i. Both tests

   ii. At least one of the two tests

6. Bob catches a train to work every day from Monday to Friday. The probability that the train is late on Monday is 0.45. the probability that the train is late on any other day is 0.3.

i. A day is chosen at random. Find the probability of the following.
   (a) It is Monday and the train is late
   (b) The train is late

ii. Given that the train is late, what is the probability that it is Monday?

7. A university student walks, cycles or drives to college with probabilities 0.1, 0.3 and 0.6, respectively. if she walks, she has a probability of 0.35 of being late. If she cycles, the probability of being late is 0.1. If she drives, the probability of being late is 0.55. Find, correct to three decimal places, the probability that:
   i. She will be late on a particular day.
   ii. She walked, given that she was late.
   iii. She walked, given that she was not late.

# 5 Combinatorics

1. A bag contains five black marbles and two white ones. Two marbles are taken out, without replacement. What is the probability that one is white and the other is black?

2. A bag contains five red sweets and four green ones. Pete takes three sweets out in succession and eats them.
   What is the probability that:
   i. All three are red.
   ii. One is red and two are green

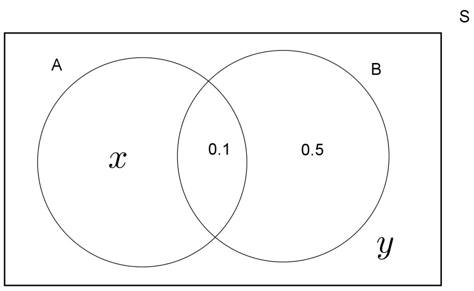
3. A bag contains five green, three blue and four red marbles. two marbles are withdrawn, without replacement.
   Find the probability that:
   i. Both are green
   ii. Both are green or both are blue
   iii. Both are the same colour
   iv. They are different colours
   v. One is red and one is not red.

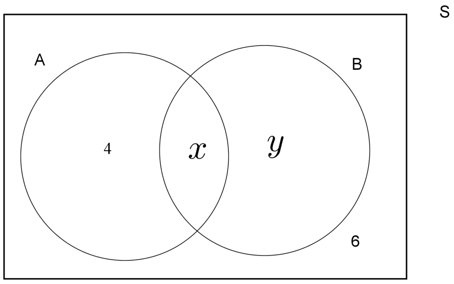
4. For a lottery, 35 cards numbered 1 to 35 are placed in a drum. Five cards will b chosen at random from the drum as the winning combination.
   i. How many different combinations are possible?
   ii. How many of all the possible combinations will match exactly four numbers with the winning combination?
   iii. How many of all the possible combinations will match exactly three numbers with the winning combination?
   iv. Show that the probability of matching at least three numbers with the winning combination is approximately 0.014.

5. A team of three students is to be chosen at random to take part in a chess competition. The team is to be chosen from seven maths students and three medical students. Find the probability that the team consists of:
   i. Only maths students
   ii. Only medical students
   iii. Two maths students and one medical student
6. A bag contains eight black, three white and five red beads. three beads are picked at random, without replacement.
   Find the probability that:
   i. All three have the same colour.
   ii. One is white and the others are not white.
   iii. All three are of different colours
   iv. At least two are the same colour.
7. Nine tickets, numbered 11,12,13,14,15,16,17,18 and 19 are placed in a hat. One ticket is taken out but not replaced. Another ticket is taken out. Find the probability that :
   i. Both are prime numbers
   ii. Both are even or both are odd
   iii. One is prime but the other is not.
8. Ten discs, each marked with a different whole number from 1 to 10, are placed in a box. Three of the discs are drawn at random (without replacement) from the box.
   i. What is the probability that the disc with the number 5 is drawn?
   ii. What is the probability that the three numbers on the discs are even¿
   iii. What is the probability that the product of the three numbers on the discs drawn is odd?
   iv. What is the probability that the largest number on the discs drawn is five?

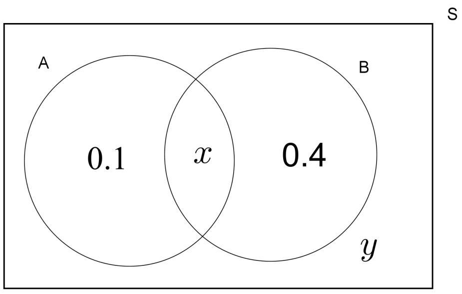
# 6 Independent Events

1. E and F are independent events, such that $P(E) = \frac{1}{5}$ and $P(F) = \frac{5}{8}$.
   Find:
   i. $P(E \cap F)$
   ii. $P(E \cup F)$
2. If A and B are independent events, find the value of :
   i. $x$
   ii. $y$
   iii. $P(A|B)$

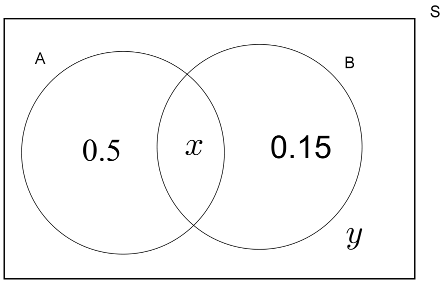


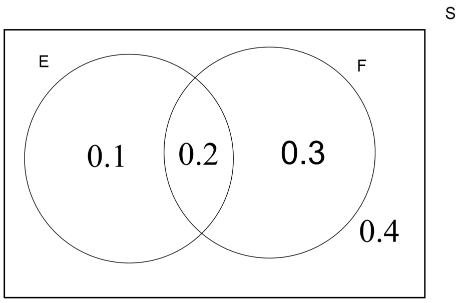

3. The Venn diagram shows the number of elements in each subset. An element is picked at random from S. If $P(A) = \frac{1}{3}$ and $P(B) = \frac{7}{9}$:
   i. Find the values of $x$ and $y$
   ii. Investigate if $P(A|B) = P(B|A)$.
   iii. Investigate if A and B are independent.
4. Given that A and B are independent events, find two possible values of $x$ and the corresponding values of $y$.



5. (a) Given $P(A|B) = \frac{5}{8}$, find the values of $x$ and $y$.
   (b) Investigate if A and B are independent.




6. E and F are two events of an outcome space, S. The probabilities of E and F are shown in the Venn diagram below:




   i. Write down $P(E), P(F)$ and $P(E \cap F)$
   ii. Find $P(E|F)$
   iii. What do we mean when we say that two events are 'independent'? Give a verbal answer and a mathematical answer.
   iv. Investigate if E and F are independent.

# 7 Conditional Probability

1. Two events, $A$ and $B$, are such that $P(A \cap B) = 0.44$ and $P(B) = 0.8$. Find $P(A|B)$.

2. $A$ and $B$ are two events such that $P(A) = \frac{3}{10}, P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$.
   Find $P(A|B)$ and $P(B|A)$

3. At a school, 24% of all students play soccer and Gaelic football and 30% of all students play soccer. What is the probability that a student plays Gaelic football given that they play soccer?

4. $P(A) = \frac{1}{2}$ $P(B) = \frac{1}{3}$ $P(A \cap B) = \frac{1}{4}$
   Find:
   i. $P(A|B)$
   ii. $P(B|A)$

iii. $P(A'|B)$

iv. $P(A|B')$

v. $P(A \cup B)$

vi. $P[A|(A \cup B)]$

5. $E$ and $F$ are two events such that $P(E) = \frac{2}{5}, P(F) = \frac{1}{2}$, and $P(E|F) = \frac{1}{9}$ Find:

   i. $P(E \cap F)$

   ii. $P(F|E)$

   iii. $P(E \cup F)$

6. $P(A) = 0.5$ $P(A \cup B) = 0.6$ $P(A|B) = 0.75$
   Find $P(B)$
   (Hint: Let $P(B) = x$ and let $P(A \cap B) = y$. You should be able to set up a pair of simultaneous equations.)

7. $A$ and $B$ are two events such that $P(A) = \frac{3}{10}, P(B) = \frac{1}{2}$ and $P(A \cup B) = \frac{3}{5}$.
   Find $P(A|B)$ and $P(B|A)$

8. A team wins 60% of home matches and 20% of away matches. Half of their matches are at home and half are away. They won their last match. Find the probability that:

   i. It was at home.

   ii. It was away

9. In a certain county, 70% of the population live in a townland, and 30% live in a rural area. Fifty per cent of townlanders listen to the local radio station. whereas only 20% of rural dwellers listen to the local radio station. A person from the county is picked at random. Find the probability that this person is a townlander, given that the person listens to the local radio station.

10. A couple have four children.

    i. Find the probability that all four are boys, given that there are three boys.

    ii. 'If your first three children are boys, your fourth child is more likely to be a girl' Is this statement true? Give a reason for your answer.

# 8 Expected Values

1. A card is chosen from the following poker hand

   - a ten of Spades
   - a ten of Hearts
   - a ten of diamonds
   - a five of clubs
   - and a five of spades

i. Copy and complete this probability distribution table:

| Number on card | 10 | 5 |
|---|---|---|
| Probability | | |

ii. Find the expected value of the number on the card.

iii. Is the expected value one of the possible outcomes?

2. A pair of fair six-sided dice is rolled and the total on the two dice is noted.

i. Copy and complete the following probability distribution table:

| Total $(x)$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 0 | 11 | 12 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| P$(x)$ | $\frac{1}{36}$ | | | | $\frac{5}{36}$ | | | | | | $\frac{1}{36}$ |

ii. Calculate the expected value.

iii. Is the expected value a possible outcome?

3. The table below gives motor insurance information for fully licensed, 17-20 year old drivers in Ireland in 2007. All drivers who had their own insurance policy are included.

| | Number of drivers | Number of claims | Average cost per claim |
|---|---|---|---|
| Male | 9634 | 977 | €6108 |
| Female | 6743 | 581 | €6051 |

Questions (i) to (v) refer to drivers in the table above only.

i. What is the probability that a randomly selected male driver made a claim during the year? Give your answer correct to three decimal places

ii. What is the probability that a randomly selected female driver made a claim during the year? Give your answer correct to three decimal places.

iii. What is the expected value of the cost of claims on a male driver's policy?

iv. What is the expected value of the cost of claims on a female driver's policy?

v. The male drivers were paying an average of €1688 for insurance in 2007, and the female drivers were paying an average of €1024. Calculate the average surplus for each group, and comment your answer.(Note: the surplus is the amount paid for the policy minus the expected cost of claims)

vi. A 40-year-old female driver with a full license has a probability of 0.07 of making a claim during the year. The average cost of such claims is €3900. How much should a company charge such drivers for insurance in order to show a surplus of €175 per policy?

4. The number of laptops per household in a survey of 100 houses in a town gave the following frequency distribution:

| Number of laptops | 0 | 1 | 2 | 3 |
|---|---|---|---|---|
| Frequency | 10 | 75 | 10 | 5 |

i. Draw up a probability distribution table for the variable $X$, where $X$ is the number of laptops in a house picked at random in the town.

ii. Find the expected value for $X$.

iii. Find the probability that a household has three laptops, given that there is at least one laptop in the house.

5. The random variable $X$ has the following probability distribution:

| $x$ | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|
| $P(X = x)$ | 0.1 | a | b | 0.2 | 0.1 |

   i. Given that the expected value $E(X) = 2.9$, find the value of a and the value of b.
   ii. Find $P(X = 5|X > 3)$.

# 9 Binomial Distribution

1. A coin is tossed three times. What is the probability of obtaining the following?
   i. Exactly one head
   ii. Exactly two heads
   iii. Exactly three heads
   iv. At least one head
2. In the first 56 days of school, Samantha was late 14 times. Take the relative frequency as the probability that she will be late. In five consecutive days, find the probability that she will be late:
   i. never
   ii. Only once
   iii. More than once
3. In a game of chess against a particular opponent, the probability that Boris wins is 85%. He plays five games against the opponent, where no draws are allowed. What is the probability, correct to three decimal places, that Boris will:
   i. Only win the second and fourth games
   ii. Win exactly two games
4. Whenever Anne's mobile phone rings, the probability that she answers the call is $\frac{3}{4}$. A friend phones Anne six times. What is the probability that:
   i. She misses all the calls
   ii. She misses the first two calls and answers the others
   iii. She answers exactly one of the calls
   iv. She answers at least two of the calls
5. A multiple choice test has seven questions. Each question has three choices. Tony takes the test and guesses the answer on each question. Find, correct to three decimal places, the probability that Tony gets:
   i. All seven correct
   ii. Exactly four questions correct
   iii. At least two questions correct

6. A fair coin is flipped $n$ times. Show that the probability of getting one head or less is $\frac{n+1}{2^n}$.
7. Whenever Jimmy fires an arrow he has $\frac{1}{4}$ chance of hitting a target. He fires 10 arrows. Find the probability (to the nearest percentage) that:
   i. He hits the target the first three times but misses the rest
   ii. He hits the target exactly three times
   iii. He hits the target at least once
8. Kenny is a very good dart player. Over a long period of time, it was found that he hits a bull's eye with 90% accuracy. What is the probability that Kenny hits his fourth bull's eye on his sixth shot?
9. A fair coin is flipped repeatedly until exactly four heads appear. Find the probability that this will take exactly six flips.
10. A couple decide to keep having children until they have exactly two boys. Find the probability that this will happen:
    i. At the birth of their second child
    ii. At the birth of their fifth child
11. People are stopped at random on a street and asked what day of the week they were born on. This process is repeated until two people who were born on Sunday are stopped. Find the probability that this will take exactly 12 people. (You may assume they all know the day of their birth).
12. A missile has a 25% chance of hitting a target. How many missiles must be fired to ensure that there is a greater than 90% chance that the target will be hit at least once.
13. How many times must a fair six-sided die be rolled so that the chances of getting at least one 6 is greater than 90%?

# 10 Applications of Normal Distribution

1. The average age of CEOs is 56 years. Assume the variable is normally distributed. If the standard deviation is 4 years, find the probability that the age of a randomly selected CEO will be in the following range.
   i. Between 53 and 59 years old
   ii. Between 58 and 63 years old
   iii. Between 50 and 55 years old.
2. If Leaving Certificate Higher Level maths marks are normally distributed with a mean 68 and standard deviation 10, what percentage of students will get an A?
3. The average life of a brand of automobile tires is 30,000 miles, with a standard deviation of 2,000 miles. If a tire is selected and tested, find the probability that it will have the following lifetime. Assume the variable is normally distributed.

i. Between 25,000 and 28,000 miles

ii. Between 27,000 and 32,000 miles

iii. Between 31,500 and 33,500 miles.

4. The average time for a courier to travel from Pittsburgh to Harrisburg is 200 minutes, and the standard deviation is 10 minutes. If one of these trips is selected at random, find the probability that the courier will have the following travel time. Assume the variable is normally distributed.

   i. At least 180 minutes

   ii. At least 205 minutes

5. The average amount of rain per year in South Summerville is 49 inches. The standard deviation is 5.6 inches. Find the probability that next year South Summerville will receive the following amount of rain. Assume the variable is normally distributed.

   i. At most 51 inches of rain

   ii. At least 58 inches of rain

6. The annual number of hours of sunshine in Ireland is normally distributed with a mean of 1,550 hours and a standard deviation of 150 hours. What is the probability that next year there will be:

   i. Less than 1,300 hours of sunshine in Ireland

   ii. More than 1,800 hours of sunshine in Ireland

   iii. Between 1,300 and 1,800 hours of sunshine in Ireland.

7. A brisk walk at 4 miles per hour burns an average of 300 calories per hour. If the standard deviation of the distribution is 8 calories, find the probability that a person who walks one hour at the rate of 4 miles per hour will burn the following calories. Assume the variable is normally distributed.

   i. More than 280 calories

   ii. Less than 293 calories

   iii. Between 285 and 320 calories

8. During September, the average temperature of Laurel Lake is 64.2° and the standard deviation is 3.2°. Assume the variable is normally distributed. For a randomly selected day, find the probability that the temperature will be as follows:

   i. Above 62°

   ii. Below 67°

   iii. Between 65° and 68°

9. In order to qualify for letter sorting, applicants are given a speed-reading test. The scores are normally distributed, with a mean of 80 and a standard deviation of 8. If only the top 15% of the applicants are selected, find the cutoff score.

10. The scores on a test have a mean of 100 and a standard deviation of 15. If a personnel manager wishes to select from the top 75% of applicants who take the test, find the cutoff score. Assume the variable is normally distributed.

11. For an educational study, a volunteer must place in the middle 50% on a test . If the mean for the population is 100 and the standard deviation is 15, find the two limits (upper and lower) for the scores that would enable a volunteer to participate in the study. Assume the variable is normally distributed.

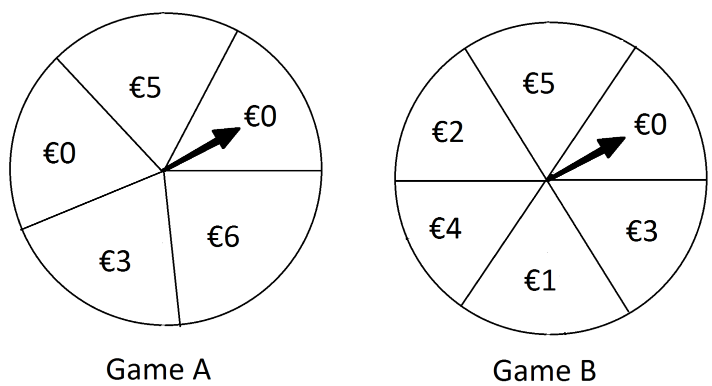
12. A contractor decided to build homes that will include the middle 80% of the market. If the average size (in square feet) of homes built is 1,810, find the maximum and minimum sizes of the homes the contractor should build. Assume that the standard deviation is 92 square feet and the variable is normally distributed.

13. If the average price of a new home if €145,000, find the maximum and minimum prices of the houses a contractor will build to include the middle 80% of the market. Assume that the standard deviation of prices is €1,500 and the variable is normally distributed.

14. An athletic association wants to sponsor a footrace. the average time it takes to run the course if 58.6 minutes, with a standard deviation of 4.3 minutes. If the association decides to award certificates to the fastest 20% of the racers, what should the cutoff time be? Assume the variable is normally distributed.

# 11 Exam Questions

1. **2014 Paper 2** Two different games of chance, shown below, can be played at a charity fund-raiser. In each game, the player spins an arrow on a wheel and wins the amount shown on the sector that the arrow stops in. Each game is fair in that the arrow is just as likely to stop in one sector as in any other sector on that wheel.



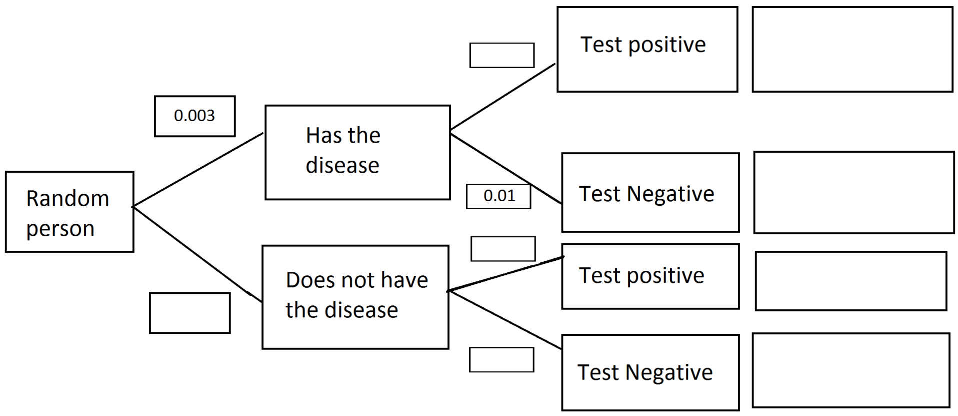
(a) John played Game A four times and tells us that he has won a total of €8. In how many different ways could he have done this?

(b) To spin either arrow once, the player pays €3. Which game of chance would you expect to be more successful in raising funds for charity? Give reason for your answer

(c) Mary plays Game B six times. Find the probability that the arrow stops in the €4 sector exactly twice.

2. **2014 Paper 2** Blood tests are sometimes used to indicate if a person has a particular disease. Sometimes such tests give an incorrect result, either indicating the person has the disease when they do not (called a false positive) or indicating that they do not have the disease when they do (called a false negative).
It is estimated that 0.3% of a large population have a particular disease. A test developed to detect the disease gives a false positive in 4% of tests and a false negative in 1% of tests. A person picked at random is tested for the disease.

(a) i. Write the probability associated with each branch of the tree diagram in the blank boxes provided.




ii. Hence, or otherwise, calculate the probability that a person selected at random from the population tests positive for the disease.

iii. A person tests positive for the disease. What is the probability that the person actually has the disease? Give your answer correct to three significant figures.

iv. The health authority is considering using a test on the general population with a view to treatment of the disease. Based on your results, do you think that the above test would be an effective way to do this? Give a reason for your answer.

(b) A generic drug used to treat a particular condition has a success rate of 51%. A company is developing two drugs, A and B, to treat the condition. They carried out clinical trials on two groups of 500 patients suffering from the condition. The results showed that Drug A was successful in the case of 296 patients. The company claims that Drug A is more successful in treating the condition than the generic drug.

i. Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to justify the company's claim. State the null hypothesis and state your conclusion clearly.

ii. The null hypothesis was accepted for Drug B. estimate the greatest number of patients in the trial who could have been successfully treated with Drug B.

3. **2013 Paper 2**

(a) Explain each of the following terms:

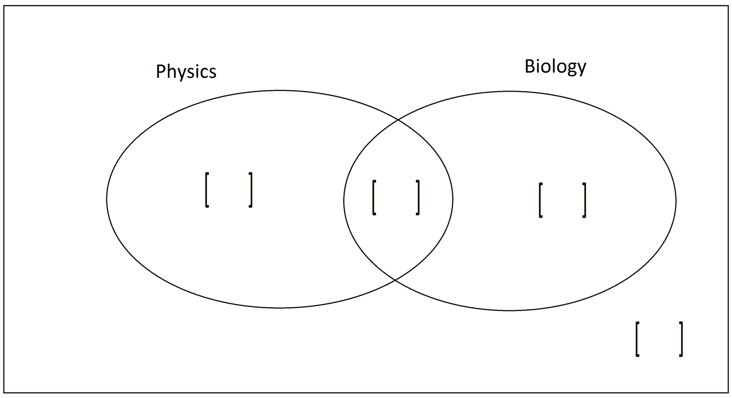
i. Sample space.

ii. Mutually exclusive events.

iii. Independent events.

(b) In a class of 30 students, 20 study physics, 6 study biology and 4 study both Physics and Biology.

i. represent the information on the Venn Diagram.




A student is selected at random from this class. The events E and F are:
E: The student studies Physics
F: The student studies Biology.

ii. By calculating probabilities, investigate if the events E and F are independent.

4. **2013 Paper 2**

(a) A random variable $X$ follows a normal distribution with mean 60 and standard deviation 5.

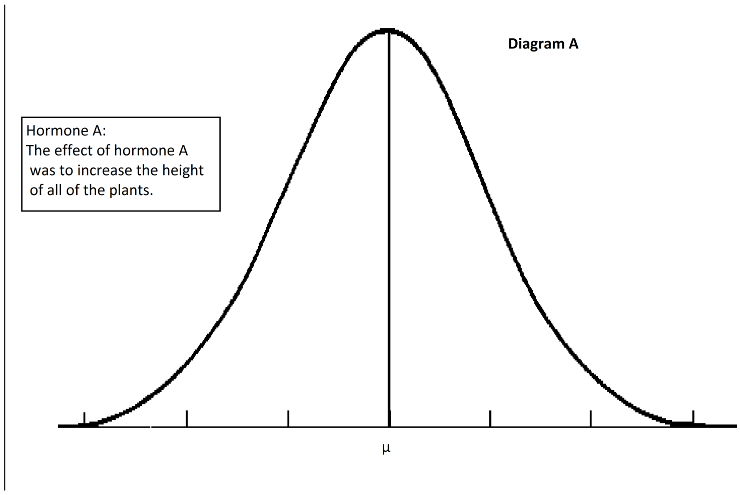
i. Find $P(X \leq 68)$

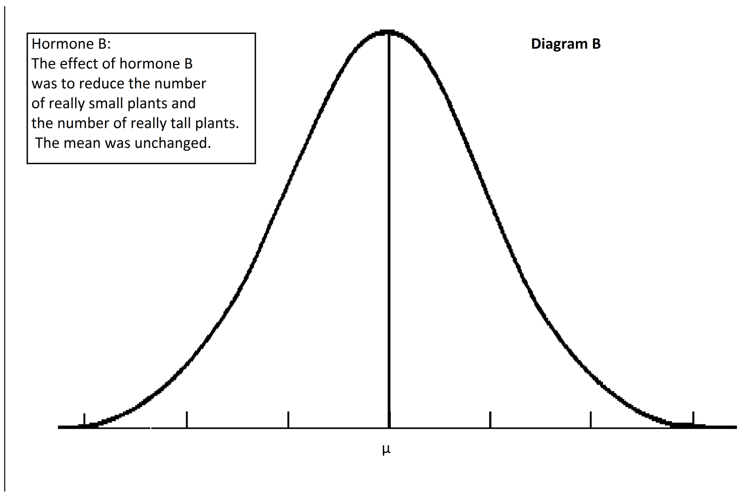
ii. Find $P(52 \leq X \leq 68)$.

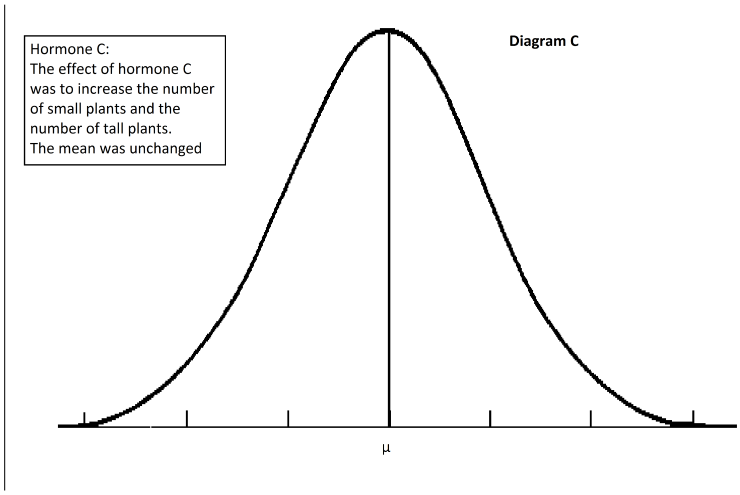
(b) The heights of a certain type of plant, when ready to harvest, are known to be normally distributed, with a mean of $\mu$ . A company tests the effects of three different growth hormones on this type of plant. The three hormones were used on a different large sample of the crop. After applying each hormone, it was found that the heights of the plants in the samples were still normally distributed at harvest time.

The diagrams A,B and C show the expected distribution of the heights of the plants, at harvest time, without the use of the hormones.

The effect, on plant growth, of each of the hormones is described beside each diagram. Sketch, on each diagram, a new distribution to show the effect of the hormones.






5. **2013 Paper 2**

*Go Fast Airlines* provides internal flights in Ireland, short haul flights to Europe and long haul flights to America and Asia. On ling haul flights the company sells economy class, business class and executive class tickets. All passengers have a baggage allowance of 20 kg and must pay a cost per kg for any weight over the 20 kg allowance. Each month the company carries out a survey among 1000 passengers. Some of the results of the survey for may are shown below.

| | | |
|---|---|---|
| Gender | Male: 479 | Female: 521 |

| | | |
|---|---|---|
| Previously flown with *Go Fast Airlines* | Yes: 682 | No: 318 |

| | | |
|---|---|---|
| Would fly again with *Go Fast Airlines* | Yes: 913 | No:87 |

| | | |
|---|---|---|
| Passenger Age | Mean age: 42 | Median age: 31 |
| Spend on in-flight facilities | Mean spend: €18.65 | Median spend: €32.18 |

| Was flight delayed? | Yes | No | Don't know |
|---|---|---|---|
| | 231 | 748 | 21 |

| Passenger satisfaction with overall service | Satisfied | Not satisfied | Don't know |
|---|---|---|---|
| | 664 | 238 | 98 |

(a) *Go Fast Airlines* used a **stratified random sample** to conduct the survey.

i. Explain what is meant by a **stratified random sample**.

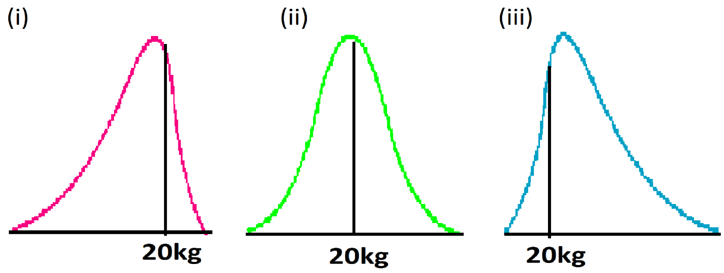
ii. Write down 4 different passenger groups that the company might have included in their sample.

(b) i. What is the probability that a passenger selected at random from this sample

- had his/her flight delayed Answer:
- was not satisfied with the overall service Answer:

ii. An employee suggest that the probability of selecting a passenger whose flight was delayed and who was not satisfied with the overall service should be equal to the product of the two probabilities in **(i)** above. Do you agree with the employee?

(c) Which of the graphs below do you think is most likely to represent the distribution of the weight of passengers baggage?




Give a reason for your answer.

(d) i. Draw a sketch of the possible distribution of the ages of the passengers based on the data in the survey.

ii. Explain your answer

(e) i. The company repeatedly asserts that 70% of their customers are satisfied with their overall service. Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to conclude that their claim is valid in May. Write the null hypothesis and state your conclusion clearly.

ii. A manager of the airline says: "If we survey 2000 passengers from June on, we will halve the margin of error in our surveys." Is the manager correct? Explain your answer.

(f) The response of ten individual passengers to the question on age and in-flight spend are given below.

| | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Age(years) | 46 | 29 | 37 | 18 | 25 | 75 | 52 | 35 | 40 | 31 |
| In-flight spend (euro) | 30 | 15 | 20 | 0 | 10 | 45 | 25 | 20 | 20 | 30 |

i. Draw a scatter plot of the data.

ii. Calculate the correlation coefficient between passengers age and in-flight spend.

iii. What can you conclude from the completed scatter plot and the correlation coefficient?

iv. Sketch the line of best fit in the completed scatter plot above.

6. **2012 Paper 2** A certain basketball player scores 60% of the free-throw shots she attempts. During a particular game, she gets six free throws.

(a) What assumption(s) must be made in order to regard this as a sequence of Bernoulli trials?

(b) Based on such assumption(s), find, correct to three decimal places, the probability that:

i. she scores on exactly four of the six shots.

ii. she scores for the second time on the fifth shot.

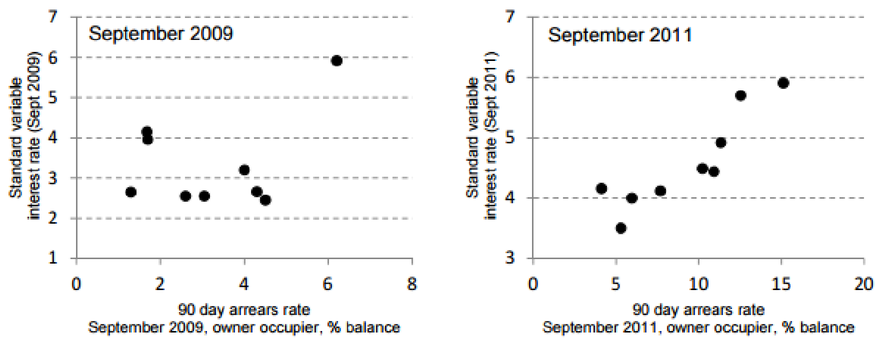
7. **2012 Paper 2** A company produces calculator batteries. The diameter of the batteries is supposed to be 20 mm. The tolerance is 0.25 mm. Any batteries outside this tolerance are rejected. You may assume that this is the only reason for rejecting the batteries.

   (a) The company has a machine that produces batteries with diameters that are normally distributed with mean 20 mm and standard deviation 0.1 mm. Out of every $10,000$ batteries produced by this machine, how many, on average, are rejected?

   (b) A setting on the machine slips, so that the mean diameter of the batteries increases to 20.05 mm, while the standard deviation remains unchanged. Find the percentage increase in the rejection rate for batteries from this machine.

8. **2012 Paper 2** To buy a home, people usually take out loans called *mortgages*. If one of the repayments is not made in time, the mortgage is said to be *in arrears*. One way of considering how much difficulty the borrowers in a country are having with their mortgages is to look at the percentage of all mortgages that are in arrears for 90 days or more. For the rest of this question, the term *in arrears* means in arrears for 90 days or more.

   The two charts below are from a report about mortgages in Ireland. The charts are intended to illustrate the connection, if any, between the percentages of mortgages that are in arrears and the interest rates being charged for mortgages. Each dot on the chart represents a group of people paying a particular interest rate to a particular lender. The arrears rate is the percentage in arrears.




(Source: Goggin et al. *Variable Mortgage Rate Pricing in Ireland*, Central Bank of Ireland, 2012)

   (a) Paying close attention to the scales on the charts, what can you say about the change from September 2009 to September 2011 with regard to:

      i. the arrears rates?

      ii. the rates of interest being paid?

      iii. the relationship between the arrears rate and the interest rate?

   (b) What additional information would you need before you could estimate the median interest rate being paid by mortgage holders in September 2011?

(c) Regarding the relationship between the arrears rate and the interest rate for September 2011, the authors of the report state: "The direction of causality ... is important" and they go on to discuss this.
Explain what is meant by the "direction of causality" in this context.

(d) A property is said to be in " negative equity" if the person owes more mortgage than the property is worth. A report about mortgaged properties in Ireland in December 2010 has the following information:

- Of the 475136 properties examined, 145414 of them were in negative equity.
- Of the ones in negative equity, 11644 were in arrears.
- There were 317355 properties that were neither in arrears or negative equity.

i. What is the probability that a property selected at random (from all those examined) will be in negative equity? Give your answer correct to two decimal places.

ii. What is the probability that a property selected at random from all those in negative equity will also be in arrears? Give your answer correct to two decimal places.

iii. Find the probability that a property selected at random from all those in arrears will also be in negative equity. Give your answer correct to two decimal places.

(e) The study in part **(d)** was so large that it can be assumed to represent the population. Suppose that, in early 2012, researchers want to know whether the proportion of properties in negative equity has changed. They analyse 2000 randomly selected properties with mortgages. They discover that 522 of them are in negative equity. Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to conclude that the situation has changed since December 2010.
Be sure to state the null hypothesis clearly, and to state the conclusion clearly.

9. **2014 Sample Paper 2** The random variable $X$ has a discrete distribution. The probability that it takes a value other that $13, 14, 15$ or $16$ is negligible.

(a) Complete the probability distribution table below and hence calculate $E(X)$, the expected value of $X$.

| $x$ | 13 | 14 | 15 | 16 |
|---|---|---|---|---|
| $P(X = x)$ | 0.383 | 0.575 | | 0.004 |

(b) If $X$ is the age, in complete years, on 1 January 2013 of a student selected at random from among all second-year students in Irish schools, explain what $E(X)$ represents.

(c) If ten students are selected at random from this population, find the probability that exactly six of them were 14 years old on 1 January 2013. Give your answers correct to three significant figures.