Chain Rule SOLUTIONS

1. (a) $\frac{dy}{dx} = 3\cos(3x)$
   (b) $\frac{dy}{dx} = -2x\sin(x^2)$
   (c) $\frac{dy}{dx} = \frac{3}{3x+5}$
   (d) $f'(x) = 3x^2e^{x^3}$
   (e) $f'(x) = (24x+20)(3x^2+5x)^3$
   (f) $\frac{dy}{dx} = 3x^2\cos(x^3+7)$
   (g) $f'(x) = \cos x e^{\sin x}$
   (h) $f'(x) = \frac{2x+2}{x^2+2x}$
   (i) $\frac{dy}{dx} = \frac{3x^2+1}{2\sqrt{x^3+x}}$ (S*)
   (j) $\frac{dy}{dx} = 2\sin x\cos x$ (S*)
   (k) $\frac{dy}{dx} = -6\sin(6x-7)$
   (l) $\frac{dy}{dx} = -\frac{3x^2}{(2x^3-3)^{\frac{3}{2}}}$ (S*)
   (m) $f'(x) = 21(3x+2)^6$
   (n) $f'(x) = 3\cos^2 x\sin x$
   (o) $\frac{dy}{dx} = (4x^3+3x^2)e^{x^4+x^3}$
   (p) $\frac{dy}{dx} = -\tan x$ (S*)
   (q) $\frac{dy}{dx} = \frac{1}{2\sqrt{x}}\cos\sqrt{x}$ (S*)

2. (a) $\frac{dy}{dx} = 3x^3\cos(3x) + 3x^2\sin(3x)$ (S*)
   (b) $\frac{dy}{dx} = -4x^2\sin(x^2) + 2\cos(x^2)$
   (c) $\frac{dy}{dx} = (36x+24)(4x-7)^2 + 3(4x-7)^3$
   (d) $\frac{dy}{dx} = (2x^4+x^3)e^{x^2}$ (S*)
   (e) $\frac{dy}{dx} = \frac{5x}{x+2} + 5\ln(x+2)$
   (f) $\frac{dy}{dx} = \frac{(x^2+2x)e^x}{x^4}$
   (g) $f'(x) = \frac{-1}{\sqrt{(x+1)(x-1)^3}}$ (S*)
   (h) $f'(x) = 10\sin(5x)\cos(5x)$
   (i) $f'(x) = 27\cos^2(9x+4)\sin(9x+4)$
   (j) $f'(x) = 2\sin x\cos x e^{sin^2 x}$

(k) $\frac{dy}{dx} = \frac{2}{x^2-1}$

(l) $\frac{dy}{dx} = 15\sin^2(5x)\cos(5x)e^{\sin^3(5x)}$

Part (i)

$$\begin{aligned} y &= \sqrt{x^3+x} \\ y &= (x^3+x)^{\frac{1}{2}} \\ \frac{dy}{dx} &= \frac{1}{2}(x^3+x)^{-\frac{1}{2}}(3x^2+1) \\ \frac{dy}{dx} &= \frac{1}{2}\frac{1}{(x^3+x)^{\frac{1}{2}}}(3x^2+1) \\ \frac{dy}{dx} &= \frac{3x^2+1}{2\sqrt{x^3+x}} \end{aligned}$$

Part (j)

$$\begin{aligned} y &= \sin^2 x \\ y &= (\sin x)^2 \\ \frac{dy}{dx} &= 2(\sin x)^1(\cos x) \\ \frac{dy}{dx} &= 2\sin x\cos x \end{aligned}$$

Part (l)

$$\begin{aligned} y &= \frac{1}{\sqrt{2x^3-3}} \\ y &= (2x^3-3)^{-\frac{1}{2}} \\ \frac{dy}{dx} &= -\frac{1}{2}(2x^3-3)^{-\frac{3}{2}}(6x^2) \\ \frac{dy}{dx} &= -3x^2\frac{1}{(2x^3-3)^{\frac{3}{2}}} \\ \frac{dy}{dx} &= -\frac{3x^2}{(2x^3-3)^{\frac{3}{2}}} \end{aligned}$$

Part (p)

$$\begin{aligned} y &= \ln(\cos x) \\ \frac{dy}{dx} &= \frac{1}{\cos x}(-\sin x) \\ \frac{dy}{dx} &= \frac{-\sin x}{\cos x} \\ \frac{dy}{dx} &= -\tan x \end{aligned}$$

Part (q)

$$\begin{aligned} y &= \sin\sqrt{x} \\ \frac{dy}{dx} &= \cos\sqrt{x}.\frac{1}{2\sqrt{x}} \\ \frac{dy}{dx} &= \frac{\cos\sqrt{x}}{2\sqrt{x}} \end{aligned}$$