Completing the Square

1. Represent each of the following expressions as a perfect square:

    i. $x^2 + 6x + 9$

    ii. $k^2 + 18k + 81$

    iii. $x^2 + 10x + 25$

    iv. $y^2 - 8y + 16$

    v. $x^2 + 20x + 100$

    vi. $a^2 - 12a + 36$

    vii. $x^2 + 2xy + y^2$

    viii. $4x^2 + 4x + 1$

    ix. $9x^2 + 6x + 1$

    x. $4x^2 + 12x + 9$

2. Complete the square for the following expressions:

    (a) $x^2 + 10x + 30$

    (b) $y^2 + 6y + 10$

    (c) $x^2 - 8x + 13$

    (d) $x^2 + 12x + 20$

    (e) $x^2 - 4x + 1$

    (f) $x^2 + 7x + 12$

    (g) $y^2 - 9y + 10$

    (h) $y^2 + 3y - 5$

    (i) $3x^2 + 12x + 18$

    (j) $2x^2 + 10x + 11$

    (k) $2x^2 - 11x + 10$

    (l) $35 + 10x - x^2$

    (m) $26 - 12x - 2x^2$

3. Prove that $x^2 - 10x + 30 \geq 0$ for all values of $x \in R$.

4. Prove that $3x^2 + 24x + 54 \geq 0$ for all values of $x \in R$.

5. Prove that $18x - 3x^2 - 12 \leq 0$ for all values of $x \in R$.

6. The graph of $y = a(x - p)^2 + q$ has a turning point (p,q). By **completing the square**, find the turning point of each of the following quadratic functions.

   (a) $f(x) = x^2 + 6x + 10$

   (b) $g(x) = x^2 + 8x + 4$

   (c) $h(x) = 5 + 8x - x^2$

   (d) $f(x) = x^2 + 11x + 10$

   (e) $g(x) = 2x^2 + 4x + 12$

   (f) $h(x) = 1 + 6x - 3x^2$

7. Solve each of the following quadratic equations by completing the square, representing each answer in the form $a \pm \sqrt{b}$:

   (a) $x^2 - 8x + 2 = 0$

   (b) $x^2 - 6x + 7 = 0$

   (c) $x^2 + 10x - 20 = 0$

   (d) $x^2 - 4x + 2 = 0$

   (e) $3x^2 + 12x - 3 = 0$