Factor Theorem 2 - SOLUTIONS

1. $a = -3, b = 2$
2. $p = -48, q = 128$ (S*)
3. $a = -3, b = 4$
4. $a = 2, b = -9$

Question 2

$$
\begin{array}{r|llll}
 & & & x & +8 \\
\hline
x^2 - 8x + 16 & x^3 & & +px & +q \\
 & -x^3 + 8x^2 & & -16x & \\
\hline
 & & 8x^2 + (-16 + 1p)\,x & & +q \\
 & & -8x^2 & +64x & -128 \\
\hline
 & & & (48 + 1p)\,x + (-128 + 1q) &
\end{array}
$$

x column :

$$48 + 1p = 0$$
$$p = -48$$

constants column :

$$-128 + 1q = 0$$
$$q = 128$$

Question 5

$$
\begin{array}{r|llll}
 & & & x & +p \\
\hline
x^2 + px + q & x^3 + 2px^2 & & +2qx & +r \\
 & -x^3 - px^2 & & -qx & \\
\hline
 & & px^2 & +qx & +r \\
 & & -px^2 & -p^2x & -pq \\
\hline
 & & & (-1p^2 + 1q)\,x + (-1pq + 1r) &
\end{array}
$$

x column :

$$-1p^2 + 1q = 0$$
$$q = p^2$$

constants column :

$$-1pq + 1r = 0$$
$$r = pq$$
$$r = p(p^2)$$
$$r = p^3$$

Question 6

$$
\begin{array}{r|llll}
 & & & x & +6a \\ \hline
x^2+ax+b & x^3+7ax^2 & & +4bx & +c \\
 & -x^3-ax^2 & & -bx & \\ \hline
 & & 6ax^2 & +3bx & +c \\
 & & -6ax^2 & -6a^2x & -6ab \\ \hline
 & & & (-6a^2+3b)\,x & +(-6ab+1c)
\end{array}
$$

x column :

$$
\begin{aligned}
-6a^2+3b &= 0 \\
3b &= 6a^2 \\
b &= 2a^2
\end{aligned}
$$

constants column :

$$
\begin{aligned}
-6ab+1c &= 0 \\
c &= 6ab \\
c &= 6a(2a^2) \\
c &= 12a^3
\end{aligned}
$$

Question 7

$$
\begin{array}{r|llll}
 & & & x & -p \\ \hline
x^2+px+q & x^3 & & & -r \\
 & -x^3-px^2 & & -qx & \\ \hline
 & & -px^2 & -qx & -r \\
 & & px^2 & +p^2x & +pq \\ \hline
 & & & (1p^2+-1q)\,x & +(1pq+-1r)
\end{array}
$$

x column :

(i)
$$
\begin{aligned}
1p^2-1q &= 0 \\
p^2 &= q
\end{aligned}
$$

constants column :

$$
\begin{aligned}
1pq+1r &= 0 \\
pq &= r \\
p(p^2) &= r \\
p^3 &= r
\end{aligned}
$$

(ii)
$$
\begin{aligned}
(p^2)^3 &= (q)^3 \\
p^6 &= q^3
\end{aligned}
$$

$$
\begin{aligned}
(p^3)^2 &= (r)^2 \\
p^6 &= r^2
\end{aligned}
$$

So $q^3 = r^2$