Infinite Series

1. Calculate the sum to infinity of the following series. (If S_∞ exists)
   i. $1 + \frac{1}{2} + \frac{1}{4} + ...$
   ii. $1 + 2 + 4 + 8 + ...$
   iii. $9 + 3 + 1 + ...$
   iv. $8 - 4 + 2 - 1...$
   v. $\frac{1}{8} + \frac{1}{4} + \frac{1}{2} + ...$
2. Express $0.\dot{6}$ as an infinite geometric series and hence write $0.\dot{6}$ as a rational number.
3. Express $0.\dot{5}\dot{8}$ as a rational number.
4. Express $4.\dot{7}$ as a rational number.
5. $x + \frac{x}{1-x} + \frac{x}{(1-x)^2} + ...$, where $x > 2$, is a geometric series.
   i. Find S_∞ of the series in terms of x.
   ii. If $S_\infty = 3$, solve for x.
6. $x + \frac{x}{x+2} + \frac{x}{(x+2)^2} + ...$, where $x > 0$, is a geometric series.
   i. Find S_∞ of the series in terms of x.
   ii. If $S_\infty = \frac{15}{4}$, solve for x,
7. For each of the following geometric series, find the range of values of x, for which S_∞ exists.
   i. $1 + \frac{3}{x+1} + \frac{9}{(x+1)^2} + ...$
   ii. $\frac{1}{2} + \frac{1}{3x-1} + \frac{2}{(3x-1)^2} + ...$
   iii. $3 + x + \frac{x^2}{3} + ...$
8. Consider the geometric series: $1 + \cos^2(3\theta) + \cos^4(3\theta) + ...$
   i. If S_∞ of the series is 2, find all possible values of θ, where $0° < \theta < 90°$.
   ii. For what values of θ, $(0° < \theta < 90°)$, would S_∞ be undefined?
9. Consider the geometric series: $\log_4 a + \log_8 a + \log_{\sqrt{512}} a + ...$
   S_∞ of the series is $p \log_4 a$. Find the value of $p \in N$.
   (Hint: Use the change of base rule to express the series in terms of $\log_4 a$)