



## Logs SOLUTIONS



1. (a) 2  
(b) 3  
(c) 5  
(d) 3  
(e) 4  
(f)  $\frac{3}{2}$   
(g)  $\frac{4}{3}$   
(h)  $\frac{3}{2}$   
(i)  $\frac{1}{3}$   
(j)  $\frac{4}{5}$
2. (a)  $x = \frac{2}{3}$   
(b)  $x = \frac{4}{3}$   
(c)  $x = -2$   
(d)  $x = -1$   
(e)  $x = -3$   
(f)  $x = \frac{1}{6}$   
(g)  $x = -4$   
(h)  $x = 4$   
(i)  $x = 81$
3. (a) 2  
(b) 3  
(c) 1  
(d) 3  
(e) 3  
(f) 2  
(g) 1  
(h) 2





4. i.  $a + b$   
ii.  $2a + b$  (S\*)  
iii.  $b - a$  (S\*)  
iv.  $a + 2b$   
v.  $2a + 2b$  (S\*)  
vi.  $2b - 3a$   
vii.  $b - \frac{1}{2}a$  (S\*)  
viii.  $\frac{1}{2}b - 3a$   
ix.  $b + 1$  (S\*)  
x.  $2a + b + 1$  (S\*)  
xi.  $1 + a - b$  (S\*)  
xii.  $\frac{1}{3}(a + b + 1)$  (S\*)
5.  $x = 2$
6.  $x = \frac{-13}{2}$
7.  $x = 2$
8.  $x = 9$
9.  $x = 2$
10.  $x = 8$  (S\*)
11.  $x = \frac{-1}{2}$  (S\*)
12.  $x = 8$  (S\*)
13.  $x = 3$
14.  $x = 2$   $x = \frac{3}{2}$  (S\*)
15.  $x = 2$   $y = 1$  (S\*)
16.  $x = 3$   $y = -2$
17.  $x = 1$   $y = 2$  or  $x = 2$   $y = 1$  (S\*)
18.  $x = 1$   $y = 3$  (S\*)
19. i.  $x = 1.86$   
ii.  $x = 4.19$   
iii.  $x = 1.7$  (S\*)  
iv.  $x = 0.55$   
v.  $x = 0.9$  (S\*)  
vi.  $x = 6.23$  (S\*)





- vii.  $x = 6.62$   
viii.  $x = -0.22$   
ix.  $x = 4.21$  (S\*)
20. i.  $x = 0$   $x = 1.26$  (S\*)  
ii.  $x = 0.58$   $x = 0$   
iii.  $x = -1$   $x = 0.63$   
iv.  $x = -2$   $x = 1.58$
21. (a)  $x = 5$   
(b)  $x = 4$   $x = 1$   
(c)  $x = \sqrt{2}$   
(d)  $x = 1$   $x = 3$  (S\*)  
(e)  $x = 7$  (S\*)  
(f)  $x = \frac{3}{2}$  (S\*)  
(g)  $x = 0.47397$  (S\*)  
(h)  $x = 1396.5$  (S\*)
22. (a)  $x = 4$   $x = 8$   
(b)  $x = 9$   $x = \frac{1}{3}$  (S\*)  
(c)  $x = 5$   $x = \frac{1}{125}$   
(d)  $x = \frac{1}{64}$   $x = \frac{1}{16}$   
(e)  $x = 6$   $x = 36$  (S\*)

**Question 4(ii)**

$$\begin{aligned}\log_3 20 &\rightarrow \\ &= \log_3 (4)(5) \\ &= \log_3 4 + \log_3 5 \\ &= \log_3 2^2 + \log_3 5 \\ &= 2 \log_3 2 + \log_3 5 \\ &= 2a + b\end{aligned}$$

**Question 4(iii)**

$$\begin{aligned}\log_3\left(\frac{5}{2}\right) &\rightarrow \\ &= \log_3 5 - \log_3 2 \\ &= b - a\end{aligned}$$



**Question 4(v)**

$$\begin{aligned}\log_3 100 &\rightarrow \\ &= \log_3 4 + \log_3 25 \\ &= \log_3 2^2 + \log_3 5^2 \\ &= 2 \log_3 2 + 2 \log_3 5 \\ &= 2a + 2b\end{aligned}$$

**Question 4(vii)**

$$\begin{aligned}\log_3 \frac{5}{\sqrt{2}} &\rightarrow \\ &= \log_3 5 - \log_3 \sqrt{2} \\ &= \log_3 5 - \log_3 2^{\frac{1}{2}} \\ &= \log_3 5 - \frac{1}{2} \log_3 2 \\ &= b - \frac{1}{2}a\end{aligned}$$

**Question 4(ix)**

$$\begin{aligned}\log_3 15 &\rightarrow \\ &= \log_3 5 + \log_3 3 \\ &= b + 1\end{aligned}$$

**Question 4(x)**

$$\begin{aligned}\log_3 60 &\rightarrow \\ &= \log_3 4 + \log_3 5 + \log_3 3 \\ &= \log_3 2^2 + \log_3 5 + \log_3 3 \\ &= 2 \log_3 2 + \log_3 5 + \log_3 3 \\ &= 2a + b + 1\end{aligned}$$

**Question 4(xi)**

$$\begin{aligned}\log_3 \frac{6}{5} &\rightarrow \\ &= \log_3 6 - \log_3 5 \\ &= \log_3 2 + \log_3 3 - \log_3 5 \\ &= a + 1 - b\end{aligned}$$

**Question 4(xii)**

$$\begin{aligned}\log_3 \sqrt[3]{30} &\rightarrow \\ &= \log_3 (30)^{\frac{1}{3}} \\ &= \frac{1}{3} \log_3 30 \\ &= \frac{1}{3} (\log_3 2 + \log_3 5 + \log_3 3) \\ &= \frac{1}{3} (a + b + 1)\end{aligned}$$



**Question 10**

$$2 \log_7 x - \log_7 2 = \log_7 32$$

$$\log_7 x^2 - \log_7 2 = \log_7 32$$

$$\log_7 \frac{x^2}{2} = \log_7 32$$

$$\frac{x^2}{2} = 32$$

$$x^2 = 64$$

$$x = \sqrt{64}$$

$$x = 8 \quad x = -8$$

**Disallow**  $x = -8$

**Solution:**  $x = 8$

**Question 11**

$$\log_2(x+1) = 2 \log_2(x+2) - \log_2(x+5)$$

$$\log_2(x+1) = \log_2(x+2)^2 - \log_2(x+5)$$

$$\log_2(x+1) = \log_2 \left( \frac{(x+2)^2}{(x+5)} \right)$$

$$x+1 = \frac{(x+2)^2}{(x+5)}$$

$$(x+1)(x+5) = (x+2)^2$$

$$x^2 + 6x + 5 = x^2 + 4x + 4$$

$$2x = -1$$

$$x = -\frac{1}{2}$$

**Question 12**

$$2 \log_6(x-2) = 2$$

$$\log_6(x-2)^2 = 2$$

$$6^2 = (x-2)^2$$

$$36 = x^2 - 4x + 4$$

$$x^2 - 4x - 32 = 0$$

$$(x-8)(x+4) = 0$$

$$x = 8 \quad x = -4$$

**Disallow**  $x = -4$

**Solution:**  $x = 8$





### Question 14

$$\log(7x - 6) - 2 \log x = \log 2$$

$$\log(7x - 6) - \log x^2 = \log 2$$

$$\log \frac{7x - 6}{x^2} = \log 2$$

$$\frac{7x - 6}{x^2} = 2$$

$$7x - 6 = 2x^2$$

$$2x^2 - 7x + 6 = 0$$

$$(2x - 3)(x - 2) = 0$$

$$x = \frac{3}{2} \quad x = 2$$

### Question 15

$$\log_2(3x - 2y) = 2$$

$$2^2 = 3x - 2y$$

$$4 = 3x - 2y$$

$$\log_3(x + 2y) = \log_3 4$$

$$x + 2y = 4$$

$$3x - 2y = 4$$

$$x + 2y = 4$$

Solve simultaneous equations:

$$x = 2 \quad y = 1$$

### Question 17

$$\log_4 x + \log_4 y = \frac{1}{2}$$

$$\log_4 xy = \frac{1}{2}$$

$$4^{\frac{1}{2}} = xy$$

$$2 = xy$$

$$\log_5(x + y) = \log_5 3$$

$$x + y = 3$$

$$2 = xy$$

$$x + y = 3$$

$$x = 3 - y$$

$$2 = (3 - y)(y)$$

$$2 = 3y - y^2$$

$$y^2 - 3y + 2 = 0$$

$$(y - 2)(y - 1) = 0$$

$$y = 2 \quad y = 1$$

$$x = 3 - y$$

$$x = 1 \quad x = 2$$





## Question 18

$$\log_2 4 - \log_2 x = \log_2(x + y)$$

$$\log_2 \frac{4}{x} = \log_2(x + y)$$

$$\frac{4}{x} = x + y$$

$$4 = x^2 + xy$$

$$\log_{16} 2 + \log_{16}(x + y) = \frac{3}{4}$$

$$\log_{16}(2x + 2y) = \frac{3}{4}$$

$$16^{\frac{3}{4}} = 2x + 2y$$

$$8 = 2x + 2y \quad (\div 2)$$

$$4 = x + y$$

$$x^2 + xy = 4$$

$$x + y = 4$$

$$x = 4 - y$$

$$(4 - y)^2 + (4 - y)y = 4$$

$$16 - 8y + y^2 + 4y - y^2 = 4$$

$$12 - 4y = 0$$

$$4y = 12$$

$$y = 3$$

$$x = 4 - y$$

$$x = 1$$

## Question 19(iii)

$$10^x = 50$$

$$\log 10^x = \log 50$$

$$x \log 10 = \log 50$$

$$x = \frac{\log 50}{\log 10}$$

$$x = 1.7$$

## Question 19(v)

$$4^{5x} = 500$$

$$\log 4^{5x} = \log 500$$

$$5x \log 4 = \log 500$$

$$5x = \frac{\log 500}{\log 4}$$

$$5x = 0.45$$

$$x = 0.9$$



**Question 19(vi)**

$$\begin{aligned}2^{x+1} &= 150 \\ \log 2^{x+1} &= \log 150 \\ (x+1) \log 2 &= \log 150 \\ x+1 &= \frac{\log 150}{\log 2} \\ x+1 &= 7.23 \\ x &= 6.23\end{aligned}$$

**Question 19(ix)**

$$\begin{aligned}2^{3x-8} &= 25 \\ \log 2^{3x-8} &= \log 25 \\ (3x-8) \log 2 &= \log 25 \\ 3x-8 &= \frac{\log 25}{\log 2} \\ 3x-8 &= 4.63 \\ 3x &= 12.63 \\ x &= 4.21\end{aligned}$$

**Question 20(i)**

$$\begin{aligned}3^{2x} - 5(3^x) + 4 &= 0 \\ (3^x)^2 - 5(3^x) + 4 &= 0 \\ \text{Let } y &= 3^x \\ y^2 - 5y + 4 &= 0 \\ (y-4)(y-1) &= 0 \\ y = 4 \quad y = 1 \\ 3^x = 4 \quad 3^x = 1 \\ \log 3^x = \log 4 \quad \log 3^x = \log 1 \\ x \log 3 = \log 4 \quad x \log 3 = \log 1 \\ x = \frac{\log 4}{\log 3} \quad x = \frac{\log 1}{\log 3} \\ x = 1.26 \quad x = 0\end{aligned}$$





**Question 21(d)**

$$\log_3(x+3) = \log_9(10x+6)$$

$$\log_3(x+3) = \frac{\log_3(10x+6)}{\log_3 9}$$

$$\log_3(x+3) = \frac{\log_3(10x+6)}{2} \quad (\times 2)$$

$$2 \log_3(x+3) = \log_3(10x+6)$$

$$\log_3(x+3)^2 = \log_3(10x+6)$$

$$(x+3)^2 = 10x+6$$

$$x^2 + 6x + 9 - 10x - 6 = 0$$

$$x^2 - 4x + 3 = 0$$

$$(x-3)(x-1) = 0$$

$$x = 3 \quad x = 1$$

**Question 21(e)**

$$\log_2(x+1) + \log_8(x+1) = 4$$

$$\log_2(x+1) + \frac{\log_2(x+1)}{\log_2 8} = 4$$

$$\log_2(x+1) + \frac{\log_2(x+1)}{3} = 4 \quad (\times 3)$$

$$3 \log_2(x+1) + \log_2(x+1) = 12$$

$$4 \log_2(x+1) = 12 \quad (\div 4)$$

$$\log_2(x+1) = 3$$

$$2^3 = x+1$$

$$8 = x+1$$

$$7 = x$$





### Question 21(f)

$$\begin{aligned} \log_{25} 2 + \log_{25}(x+1) + \log_{125}(2x+2) &= \frac{5}{6} \\ \log_{25} 2 + \log_{25}(x+1) + \frac{\log_{25}(2x+2)}{\log_{25} 125} &= \frac{5}{6} \\ \log_{25}(2)(x+1) + \frac{\log_{25}(2x+2)}{\frac{3}{2}} &= \frac{5}{6} \quad (\times \frac{3}{2}) \\ \frac{3}{2} \log_{25}(2x+2) + \log_{25}(2x+2) &= \frac{5}{4} \quad (\times 4) \\ 6 \log_{25}(2x+2) + 4 \log_{25}(2x+2) &= 5 \\ 10 \log_{25}(2x+2) &= 5 \quad (\div 10) \\ \log_{25}(2x+2) &= \frac{1}{2} \\ 25^{\frac{1}{2}} &= 2x+2 \\ 5 &= 2x+2 \\ 3 &= 2x \\ \frac{3}{2} &= x \end{aligned}$$

### Question 21(g)

$$\begin{aligned} \log_3(x+5) + \log_2(x+5) &= 4 \\ \frac{\log_2(x+5)}{\log_2 3} + \log_2(x+5) &= 4 \\ \frac{\log_2(x+5)}{1.611} + \log_2(x+5) &= 4 \quad (\times 1.611) \end{aligned}$$

### Question 21(h)

$$\begin{aligned} \log_5(2x-1) - \log_5 5(2x-1) &= 2 \\ \log_5(2x-1) - \frac{\log_5(2x-1)}{\log_5 15} &= 2 \end{aligned}$$





### Question 22(b)

$$2 \log_x 3 - \log_3 x + 1 = 0$$

$$2 \frac{\log_3 3}{\log_3 x} - \log_3 x + 1 = 0$$

$$\frac{2}{\log_3 x} - \log_3 x + 1 = 0 \quad (\times \log_3 x)$$

$$2 - (\log_3 x)^2 + \log_3 x = 0$$

$$(\log_3 x)^2 - \log_3 x - 2 = 0$$

$$\text{Let } y = \log_3 x$$

$$y^2 - y - 2 = 0$$

$$(y - 2)(y + 1) = 0$$

$$y = 2 \quad y = -1$$

$$\log_3 x = 2 \quad \log_3 x = -1$$

$$3^2 = x \quad 3^{-1} = x$$

$$9 = x \quad \frac{1}{3} = x$$

### Question 22(e)

$$\log_6 x + 2 \log_x 6 = 3$$

$$\log_6 x + 2 \frac{\log_6 6}{\log_6 x} = 3$$

$$\log_6 x + \frac{2}{\log_6 x} = 3 \quad (\times \log_6 x)$$

$$(\log_6 x)^2 + 2 = 3 \log_6 x$$

$$(\log_6 x)^2 - 3 \log_6 x + 2 = 0$$

$$\text{Let } y = \log_6 x$$

$$y^2 - 3y + 2 = 0$$

$$(y - 2)(y - 1) = 0$$

$$y = 2 \quad y = 1$$

$$\log_6 x = 2 \quad \log_6 x = 1$$

$$6^2 = x \quad 6^1 = x$$

$$36 = x \quad 6 = x$$

