Quadratic/Rational/Modulus Inequalities SOLUTIONS

1 Quadratic Inequalities

1. $x < -5 \quad x > 3 \quad (-5 > x > 3)$
2. $-4 < x < 3$
3. $x \leq -3 \quad x \geq 6 \quad (-3 \geq x \geq 6)$
4. $-3 \leq x \leq 4 \quad$ (S*)
5. $x \leq -\frac{3}{2} \quad x \geq -1 \quad (-\frac{3}{2} \geq x \geq -1) \quad$ (S*)
6. $-6 < x < -\frac{1}{3}$
7. $x < -2 \quad x > \frac{1}{2} \quad (-2 > x > \frac{1}{2})$
8. $-\frac{3}{2} \leq x \leq 4$
9. $-9 \leq x \leq \frac{2}{3} \quad$ (S*)
10. $-3 < x < 3$
11. $x \leq -\frac{5}{2} \quad x \geq \frac{5}{2} \quad (-\frac{5}{2} \geq x \geq \frac{5}{2}) \quad$ (S*)
12. $-12 \leq x \leq 0$
13. $x < 0 \quad x > 5 \quad (0 \geq x \geq 5) \quad$ (S*)

2 Rational Inequalties

1. $x < 2 \quad x > 5$
2. $x < -\frac{9}{2} \quad x > -2$
3. $-2 \leq x < -1 \quad$ (S*)
4. $4 < x < 11$
5. $x < \frac{4}{3} \quad x > 5$

6. $-\frac{6}{5} < x < -\frac{1}{2}$ (S*)
7. $\frac{4}{3} < x \leq \frac{13}{4}$
8. $x < -8 \quad x > 2$
9. $-\frac{1}{2} < x < \frac{6}{5}$
10. $x \leq -\frac{1}{3} \quad x > \frac{1}{3}$ (S*)
11. $-2 < x \leq 1$
12. $x \leq -\frac{8}{5} \quad x > 4$ (S*)

3 Modulus Inequalities

1. $-2 < x < 1$
2. $x < -3 \quad x > 5$
3. $-8 \leq x \leq 2$
4. $x \leq -\frac{1}{2} \quad x \geq \frac{3}{2}$
5. $-\frac{7}{2} < x < \frac{1}{2}$
6. $-1 < x < 1$
7. $x \leq -2 \quad x \geq -\frac{1}{3}$(S*)
8. $-5 \leq x \leq -1$
9. $x < \frac{5}{6} \quad x > 12$
10. $-\frac{9}{4} < x < -\frac{3}{4}$(S*)
11. $x \leq \frac{8}{5} \quad x \geq \frac{12}{5}$
12. $\frac{7}{2} \leq x \leq \frac{9}{2}$
13. $x < -\frac{3}{2} \quad x > \frac{1}{2}$

Question 1.4

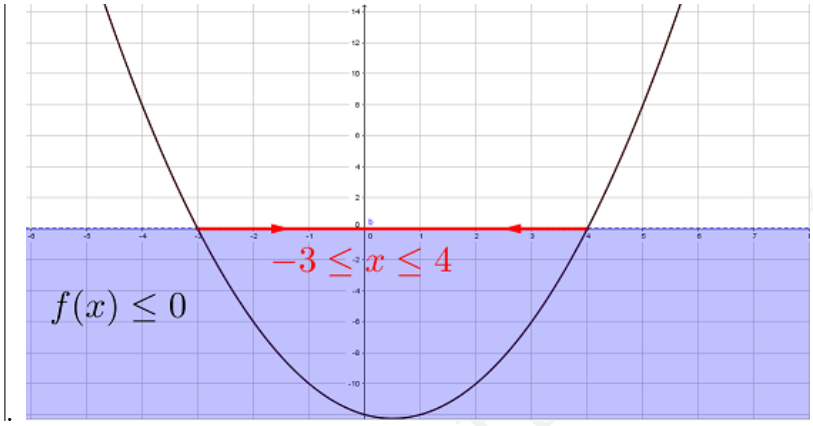
$$x^2 - x - 12 \leq 0$$

Roots: $f(x) = 0$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4 \quad x = -3$



Solution: $-3 \leq x \leq 4$

Question 1.5

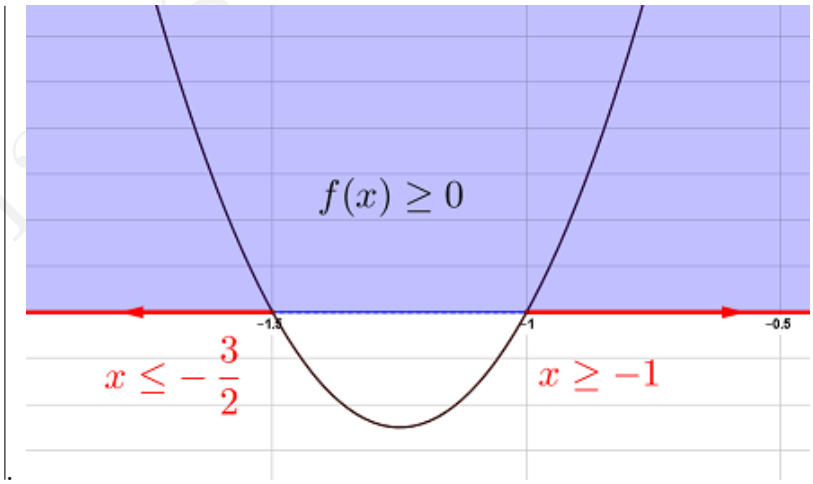
$$2x^2 + 5x + 3 \geq 0$$

Roots: $f(x) = 0$

$2x^2 + 5x + 3 = 0$

$(2x+3)(x+1) = 0$

$x = -\frac{3}{2} \quad x = -1$



Solution: $x \leq -\frac{3}{2} \qquad x \geq -1$

Question 1.9

$$18 - 25x \geq 3x^2$$

$$-3x^2 - 25x + 18 \geq 0 \quad (\times -1)$$

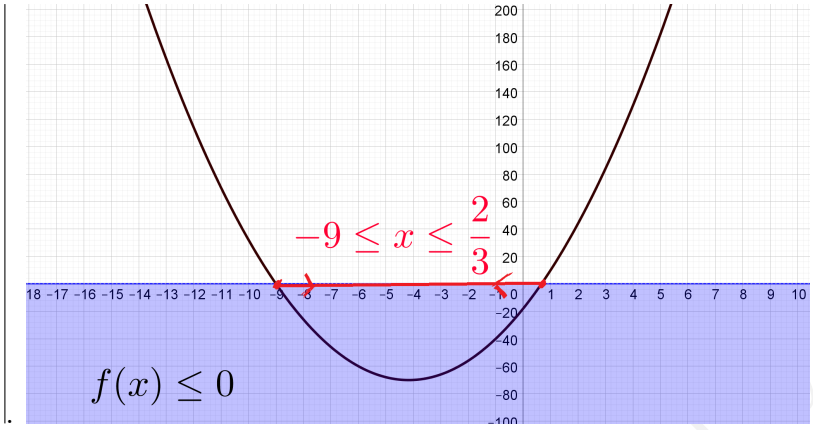
$$3x^2 + 25x - 18 \leq 0$$

Roots: $f(x) = 0$

$3x^2 + 25x - 18 = 0$

$(3x - 2)(x + 9) = 0$

$x = \frac{2}{3} \quad x = -9$



Solution: $-9 \leq x \leq \frac{2}{3}$

Question 1.11

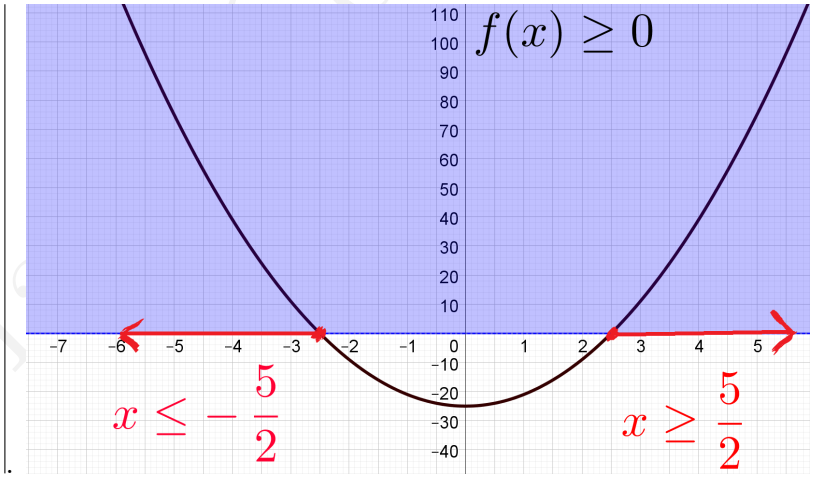
$$4x^2 \geq 25$$
$$4x^2 - 25 \geq 0$$

Roots: $f(x) = 0$

$4x^2 - 25 = 0$

$(2x - 5)(2x + 5) = 0$

$x = \frac{5}{2} \quad x = -\frac{5}{2}$



Solution: $x \leq -\frac{5}{2} \qquad x \geq \frac{5}{2}$

Question 1.13

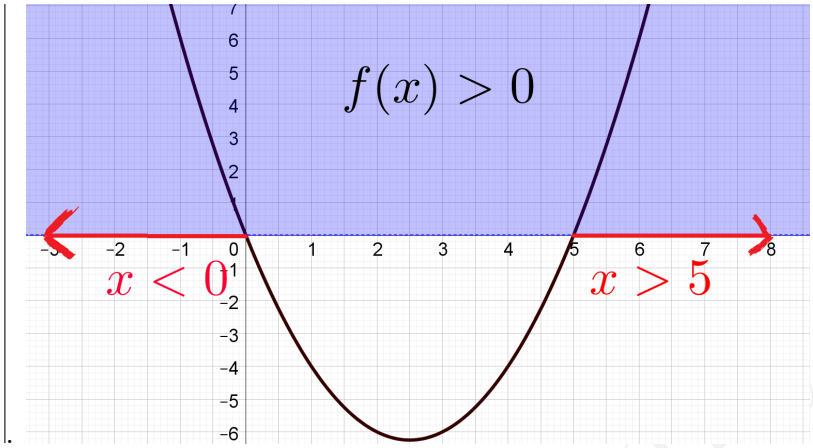
$$x^2 > 5x$$
$$x^2 - 5x > 0$$

Roots: $f(x) = 0$

$x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0 \quad x = 5$



Solution: $x < 0 \qquad x > 5$

Question 2.3

$$\frac{x-2}{x+1} \geq 4 \qquad (\times (x+1)^2)$$

$$(x+1)^2 \frac{x-2}{x+1} \geq 4(x+1)^2$$

$$(x+1)(x-2) \geq 4(x^2+2x+1)$$

$$x^2 - x - 2 \geq 4x^2 + 8x + 4$$

$$x^2 - 4x^2 - x - 8x - 2 - 4 \geq 0$$

$$-3x^2 - 9x - 6 \geq 0 \qquad (\times -1)$$

$$3x^2 + 9x + 6 \leq 0$$

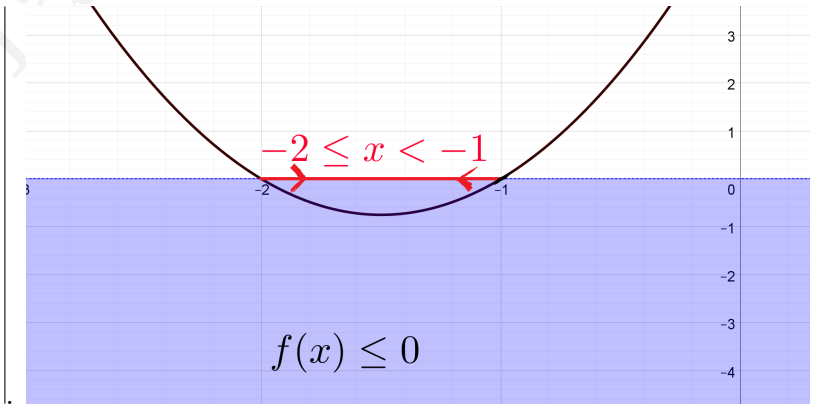
Roots: $f(x) = 0$

$3x^2 + 9x + 6 = 0 \quad (\div 3)$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x = -2 \quad x = -1$



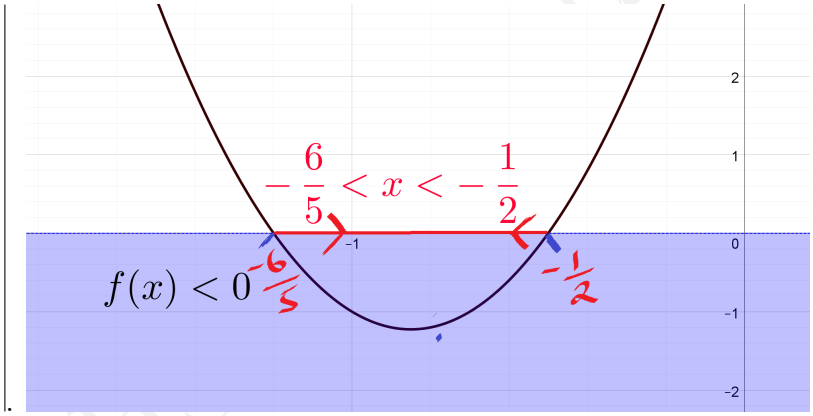
Solution: $-2 \leq x < -1 \qquad$ Note: $x \neq -1$

Question 2.6

$$\frac{3x-2}{2x+1} > 4 \qquad (\times(2x+1)^2)$$
$$(2x+1)^2\frac{3x-2}{x+1} > 4(2x+1)^2$$
$$(2x+1)(3x-2) > 4(4x^2+4x+1)$$
$$6x^2 - x - 2 > 16x^2 + 16x + 4$$
$$6x^2 - 16x^2 - x - 16x - 2 - 4 > 0$$
$$-10x^2 - 17x - 6 > 0 \qquad (\times -1)$$
$$10x^2 + 17x + 6 > 0$$

Roots: $f(x) = 0$

$$10x^2 + 17x + 6 = 0 \quad (\div 3)$$
$$(5x+6)(2x+1) = 0$$
$$x = -\frac{6}{5} \quad x = -\frac{1}{2}$$



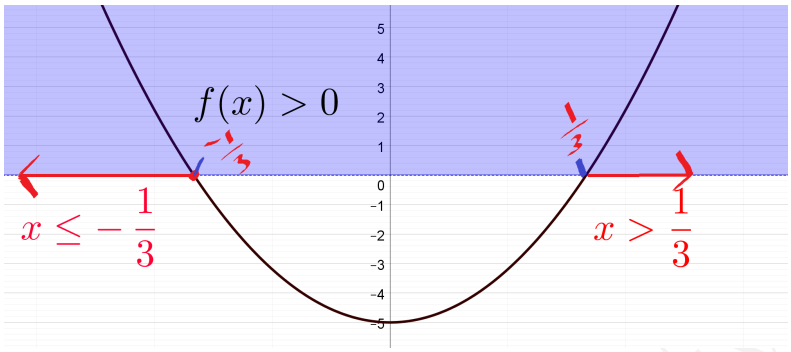
Solution: $-\frac{6}{5} < x < -\frac{1}{2}$

Question 2.10

$$\frac{x+3}{3x-1} \geq -\frac{4}{3} \qquad (\times 3(3x-1)^2)$$
$$3(3x-1)^2\frac{x+3}{3x-1} \geq -\frac{4}{3}3(3x-1)^2$$
$$3(3x-1)(x+3) \geq -4(3x-1)^2$$
$$3(3x^2+8x-3) \geq -4(9x^2-6x+1)$$
$$9x^2 + 24x - 9 \geq -36x^2 + 24x - 4$$
$$9x^2 + 36x^2 + 24x - 24x - 9 + 4 \geq 0$$
$$45x^2 - 5 \geq 0$$

Roots: $f(x) = 0$

$$45x^2 - 5 = 0 \qquad (\div 5)$$
$$9x^2 - 1 = 0$$
$$(3x-1)(3x+1) = 0$$
$$x = \frac{1}{3} \quad x = -\frac{1}{3}$$



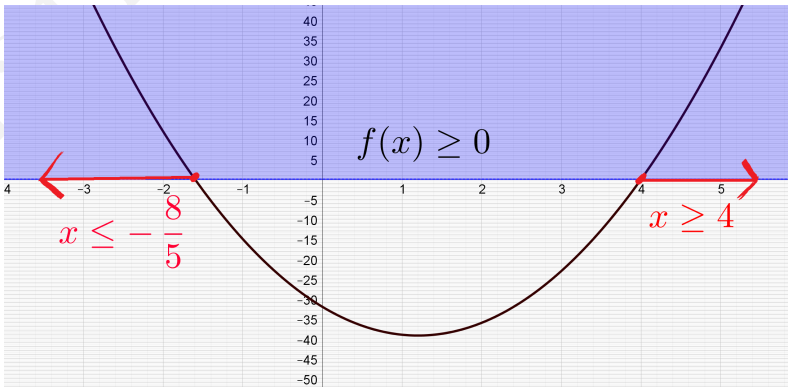
Solution: $x \leq -\frac{1}{3} \qquad x > \frac{1}{3}$ Note: $x \neq \frac{1}{3}$

Question 2.12

$$\frac{1-2x}{x-4} \leq -\frac{3}{4} \qquad (\times 4(x-4)^2)$$
$$4(x-4)^2\frac{1-2x}{x-4} \leq -\frac{3}{4}4(x-4)^2$$
$$4(x-4)(1-2x) \leq -3(x-4)^2$$
$$4(-2x^2+9x-4) \leq -3(x^2-8x+16)$$
$$-8x^2+36x-16 \leq -3x^2+24x-48$$
$$-8x^2+3x^2+36x-24x-16+48 \leq 0$$
$$-5x^2+12x+32 \leq 0 \qquad (\times -1)$$
$$5x^2-12x-32 \geq 0$$

Roots: $f(x) = 0$

$$5x^2 - 12x - 32 = 0$$
$$(5x+8)(x-4) = 0$$
$$5x+8 = 0 \quad x-4 = 0$$
$$x = -\frac{8}{5} \quad x = 4$$



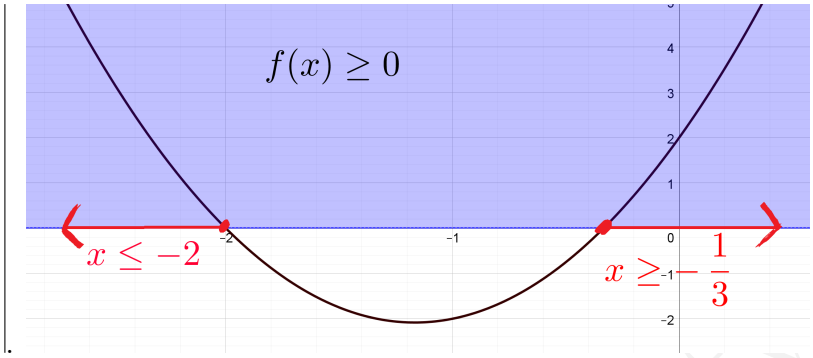
Solution: $x \leq -\frac{8}{5} \qquad x \geq 4$

Question 3.7

$$|2x-1| \leq |4x+3| \qquad \text{Square both sides} (|2x-1|)^2 \leq (|4x+3|)^2$$
$$4x^2-4x+1 \leq 16x^2+24x+9$$
$$4x^2-16x^2-4x-24x+1-9 \leq 0$$
$$-12x^2-28x-8 \leq 0 \qquad (\div -4)$$
$$3x^2+7x+2 \geq 0$$

Roots: $f(x) = 0$

$$3x^2 + 7x + 2 = 0$$
$$(3x + 1)(x + 2) = 0$$
$$3x + 1 = 0 \quad x + 2 = 0$$
$$x = -\frac{1}{3} \quad x = -2$$



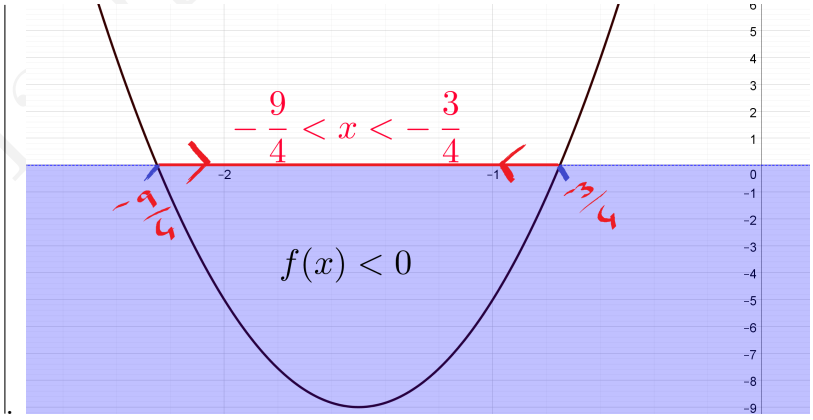
Solution: $x \leq -2 \quad x \geq -\frac{1}{3}$

Question 3.10

$$|2x + 3| < \frac{3}{2} \qquad \text{Square both sides} (|2x + 3|)^2 < \left(\frac{3}{2}\right)^2$$
$$4x^2 + 12x + 9 < \frac{9}{4} \qquad (\times 4)$$
$$16x^2 + 48x + 36 < 9$$
$$16x^2 + 48x + 36 - 9 < 0$$
$$16x^2 + 48x + 27 < 0$$

Roots: $f(x) = 0$

$$16x^2 + 48x + 27 = 0$$
$$(4x + 3)(4x + 9) = 0$$
$$4x + 3 = 0 \quad 4x + 9 = 0$$
$$x = -\frac{4}{3} \quad x = -\frac{9}{4}$$



Solution: $-\frac{9}{4} < x < -\frac{3}{4}$