Logs

1. Calculate the value of the following logs, without using a calculator:
   (a) $\log_3 9$
   (b) $\log_2 8$
   (c) $\log_2 32$
   (d) $\log_3 27$
   (e) $\log_{10} 10000$
   (f) $\log_4 8$
   (g) $\log_{125} 625$
   (h) $\log_9 27$
   (i) $\log_8 2$
   (j) $\log_{32} 16$
2. Solve each of the following equations for x, using indices:
   (a) $\log_{27} 9 = x$
   (b) $\log_8 16 = x$
   (c) $\log_{\frac{1}{2}} 4 = x$
   (d) $\log_{\frac{1}{5}} 5 = x$
   (e) $\log_{\frac{1}{3}} 27 = x$
   (f) $\log_8 \sqrt{2} = x$
   (g) $\log_{\sqrt{3}} \frac{1}{9} = x$
   (h) $\log_x 16 = 2$
   (i) $\log_3 x = 4$
3. Simplify the following expressions, representing them without logs:
   (a) $\log_6 4 + \log_6 9$
   (b) $\log_4 2 + \log_4 8 + \log_4 4$
   (c) $\log_6 72 - \log_6 12$
   (d) $\log_5 250 - \log_5 2$
   (e) $\log_2 48 - \log_2 2 - \log_2 3$

(f) $2\log_3 6 - \log_3 4$

(g) $\log_2 54 - 3\log_2 3$

(h) $2\log_5 10 - 2\log_5 2$

4. Given that $\log_3 2 = a$ and $\log_3 5 = b$, express the followingf in terms of a and b:

i. $\log_3 10$

ii. $\log_3 20$

iii. $\log_3 \frac{5}{2}$

iv. $\log_3 50$

v. $\log_3 100$

vi. $\log_3 \frac{25}{8}$

vii. $\log_3 \frac{5}{\sqrt{2}}$

viii. $\log_3 \frac{\sqrt{5}}{8}$

ix. $\log_3 15$

x. $\log_3 60$

xi. $\log_3 \frac{6}{5}$

xii. $\log_3 \sqrt[3]{30}$

Solve the following equations for x:

5. $\log_3(2x+5) = 2$

6. $\log_2(x+7) = -1$

7. $\log_2 x + \log_2(x+2) = 3$

8. $\log_3(10x) - \log_3(x+1) = 2$

9. $2\log_5 x - \log_5(x-1) = \log_5 4$

10. $2\log_7 x - \log_7 2 = \log_7 32$

11. $\log_2(x+1) = 2\log_2(x+2) - \log_2(x+5)$

12. $2\log_6(x-2) = 2$

13. $\log_9 x + \log_9(x-2) = \frac{1}{2}$

14. $\log(7x-6) - 2\log x = \log 2$

Solve the following pairs of equations for x and y:

15. $\log_2(3x-2y) = 2$ and $\log_3(x+2y) = \log_3 4$

16. $\log_2(x+y) = 0$ and $\log_2(2x+y) = 2$

17. $\log_4 x + \log_4 y = \frac{1}{2}$ and $\log_5(x+y) = \log_5 3$

18. $\log_2 4 - \log_2 x = \log_2(x+y)$ and $\log_{16} 2 + \log_{16}(x+y) = \frac{3}{4}$

Solve the following equations for x:

19. i. $5^x = 20$
    ii. $3^x = 100$
    iii. $10^x = 50$
    iv. $7^{3x} = 25$
    v. $4^{5x} = 500$
    vi. $2^{x+1} = 150$
    vii. $6^{x-3} = 660$
    viii. $5^{2x+5} = 1554$
    ix. $2^{3x-8} = 25$

20. i. $3^{2x} - 5(3^x) + 4 = 0$
    ii. $2^{2x+1} - 5(2^x) + 3 = 0$
    iii. $3^{2x+1} - 7(3^x) + 2 = 0$
    iv. $2^{2x+2} - 13(2^x) + 3 = 0$

To solve the following equations you should use the change of base formula:

21. (a) $\dfrac{\log_5(7x+1)}{3} = \log_{125}(5x+11)$
    (b) $\log_3 x = \log_9(5x-4)$
    (c) $\log_4 x + \log_2 x = \frac{3}{4}$
    (d) $\log_3(x+3) = \log_9(10x+6)$
    (e) $\log_2(x+1) + \log_8(x+1) = 4$
    (f) $\log_{25} 2 + \log_{25}(x+1) + \log_{125}(2x+2) = \frac{5}{6}$
    (g) $\log_3(x+5) + \log_2(x+5) = 4$
    (h) $\log_5(2x-1) - \log_{15}(2x-1) = 2$

22. (a) $6\log_x 2 + \log_2 x - 5 = 0$
    (b) $2\log_x 3 - \log_3 x + 1 = 0$
    (c) $\log_5 x + 2 = 3\log_x 5$
    (d) $\log_4 x + 6\log_x 4 + 5 = 0$
    (e) $\log_6 x + 2\log_x 6 = 3$