Solutions

## 1.1 Logarithms

1. $x = 2$
2. $x = \frac{-13}{2}$
3. $x = 2$
4. $x = 9$
5. $x = 2$
6. $x = 8$
7. $x = \frac{-1}{2}$
8. $x = 8$
9. $x = 3$
10. $x = 2 \quad x = \frac{3}{2}$
11. $x = 2 \quad y = 1$
12. $x = 3 \quad y = -2$
13. $x = 1 \quad y = 2 \quad$ or $\quad x = 2 \quad y = 1$
14. $x = 1 \quad y = 3$

15. i. $a + b$
    ii. $2a + b$
    iii. $b - a$
    iv. $a + 2b$
    v. $2a + 2b$
    vi. $2b - 3a$
    vii. $b - \frac{1}{2}a$
    viii. $\frac{1}{2}b - 3a$
    ix. $b + 1$
    x. $2a + b + 1$
    xi. $1 + a - b$
    xii. $\frac{1}{3}(a + b + 1)$

16. (a) $x = 5$
    (b) $x = 4 \quad x = 1$
    (c) $x = \sqrt{2}$
    (d) $x = 1 \quad x = 3$
    (e) $x = 7$
    (f) $x = \frac{3}{2}$
    (g) $x = 0.47397$
    (h) $x = 1396.5$

17. (a) $x = 4 \quad x = 8$
    (b) $x = 9 \quad x = \frac{1}{3}$
    (c) $x = 5 \quad x = \frac{1}{125}$
    (d) $x = \frac{1}{64} \quad x = \frac{1}{16}$
    (e) $x = 6 \quad x = 36$

## 1.2 Introduction to Exponential Functions

1. (a) 7.76 Billion
   (b) 11.23 Billion
   (c) 2040
2. (a) 3.97 kg
   (b) 10 weeks
3. €45, 961.94
4. (a) $Y = 2.5e^{0.3185t}$
   (b) 8.94 million
   (c) 2020
5. (a) China: $r = .0052117$ India: $r = .01349$
   Therefore India has a higher growth rate.
   (b) 2021
6. (a) $N = 500e^{.231049t}$
   (b) 1587
   (c) 13 hours

## 1.3 Half Life and Carbon Dating

7. (a) 96%
   (b) 66%
   (c) 1.65%
8. 3435 years

## 1.4 Newtons Law of Cooling

9. (a) $T = 25 + 195e^{-0.041895t}$
   (b) 61 mins
   (c) $129°C$
10. (a) $53°C$
    (b) $44°C$
11. Another 13 mins.

## 1.5 Exam Questions

1. (a) $(3x + 4)(y - 3)$
   (b) $x = -\frac{4}{3} \quad x = e^3$
2. (a) $k = -62.5$
   (b) $x = 64$

| $x$(wpm) | 0 | 10 | 20 | 30 | 40 | 50 | 60 | 70 |
|---|---|---|---|---|---|---|---|---|
| $t(x)$(days) | 0 | 8 | 18 | 29 | 43 | 61 | 87 | 130 |

   (c)
   (d) Graph
   (e) i. 62 wpm
   ii. 17 days
3. $12^{th}$ day.
4. (a) $S = 1.1$
   (b) 1813593

(c) 190737

(d) $k = -0.05$

(e) $t = 8.44$ years, 2018

(f) 2743694

(g) -130712

5. (a) i. .

| $x$ | 0 | 0.5 | 1 | ln(4) |
|---|---|---|---|---|
| $f(x) = \dfrac{2}{e^x}$ | 2 | 1.21 | 0.74 | 0.5 |
| $g(x) = e^x - 1$ | 0 | 0.65 | 1.72 | 3 |

ii. graph

iii. $x = 0.7$

(b) $x = 0.693$

6. i. $3p - q$

ii. $2q + 2 - 4p$

7. (a) $y = 77°C$, $A = 77$

(b) $k = -0.0339$

(c) 31 minutes.

(d) .

(e) i. .

ii. Suggested value: $m = -0.05$ *Reason*: This value of $m$ gives a larger average rate of change for the cooling time required.

(f) i. $-2.52°C$ per min, $-1.86°C$ per min.

8. (a) 0.7851 or 78.51%

(b) 8900 years.

9. (a) $A = 2.920$, $b = 0.100$

(b) From the table $Q(2) = 2.391$, thus verifying the values for $A$ and $b$.

(c) .

(d) $k = 6.93$