Logarithms and Exponential Functions Revision Series 2020

LEAMY MATHS COMMUNITY

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Log Rules

$$\log_a y = x \quad \rightarrow \quad a^x = y$$

Example: What is the value of $\log_2 16$?

$$\log_2 16 = x$$

Log Rules

$\log_a y = x \quad \rightarrow \quad a^x = y$

Example: What is the value of $\log_2 16$?

$$\log_2 16 = x$$

$$2^x = 16$$

Log Rules

$\log_a y = x \quad \rightarrow \quad a^x = y$

Example: What is the value of $\log_2 16$?

$$\log_2 16 = x$$
$$2^x = 16$$
$$2^x = 2^4$$

Log Rules

$\log_a y = x \quad \rightarrow \quad a^x = y$

Example: What is the value of $\log_2 16$?

$$\log_2 16 = x$$
$$2^x = 16$$
$$2^x = 2^4$$
$$x = 4$$

So the value of $\log_2 16$ is 4.

Log Rules

$\log_a(xy) = \log_a x + \log_a y$

Example: What is the value of $\log_4 2 + \log_4 32$?

$$\log_4 2 + \log_4 32$$

Log Rules

$\log_a(xy) = \log_a x + \log_a y$

Example: What is the value of $\log_4 2 + \log_4 32$?

$$\log_4 2 + \log_4 32$$

$$= \log_4((2)(32))$$

Log Rules

$\log_a(xy) = \log_a x + \log_a y$

Example: What is the value of $\log_4 2 + \log_4 32$?

$$\log_4 2 + \log_4 32$$
$$= \log_4((2)(32))$$
$$= \log_4 64$$

Log Rules

$\log_a(xy) = \log_a x + \log_a y$

Example: What is the value of $\log_4 2 + \log_4 32$?

$$\log_4 2 + \log_4 32$$
$$= \log_4((2)(32))$$
$$= \log_4 64$$

$$\log_4 64 = x$$

Log Rules

$\log_a(xy) = \log_a x + \log_a y$

Example: What is the value of $\log_4 2 + \log_4 32$?

$$\log_4 2 + \log_4 32$$
$$= \log_4((2)(32))$$
$$= \log_4 64$$

$$\log_4 64 = x$$
$$4^x = 64$$

Log Rules

$\log_a(xy) = \log_a x + \log_a y$

Example: What is the value of $\log_4 2 + \log_4 32$?

$$\log_4 2 + \log_4 32$$
$$= \log_4((2)(32))$$
$$= \log_4 64$$

$$\log_4 64 = x$$
$$4^x = 64$$
$$x = 3$$

So the value of $\log_4 2 + \log_4 32$ is 3.

Log Rules

$\log_a(\frac{x}{y}) = \log_a x - \log_a y$

Example: What is the value of $\log_3 54 - \log_3 2$?

$$\log_3 54 - \log_3 2$$

Log Rules

$\log_a(\frac{x}{y}) = \log_a x - \log_a y$

Example: What is the value of $\log_3 54 - \log_3 2$?

$$\log_3 54 - \log_3 2$$

$$= \log_3(\frac{54}{2})$$

Log Rules

$\log_a(\frac{x}{y}) = \log_a x - \log_a y$
Example: What is the value of $\log_3 54 - \log_3 2$?

$$\log_3 54 - \log_3 2$$
$$= \log_3(\frac{54}{2})$$
$$= \log_3 27$$

Log Rules

$\log_a(\frac{x}{y}) = \log_a x - \log_a y$
Example: What is the value of $\log_3 54 - \log_3 2$?

$$\log_3 54 - \log_3 2$$
$$= \log_3(\frac{54}{2})$$
$$= \log_3 27$$

$$\log_3 27 = x$$

Log Rules

$\log_a(\frac{x}{y}) = \log_a x - \log_a y$
Example: What is the value of $\log_3 54 - \log_3 2$?

$$\log_3 54 - \log_3 2$$
$$= \log_3(\frac{54}{2})$$
$$= \log_3 27$$

$$\log_3 27 = x$$
$$3^x = 27$$

Log Rules

$\log_a(\frac{x}{y}) = \log_a x - \log_a y$
Example: What is the value of $\log_3 54 - \log_3 2$?

$$\log_3 54 - \log_3 2$$
$$= \log_3(\tfrac{54}{2})$$
$$= \log_3 27$$

$$\log_3 27 = x$$
$$3^x = 27$$
$$x = 3$$

So the value of $\log_3 54 - \log_3 2$ is 3

Log Rules

$\log_a x^q = q \log_a x$

Example: What is the value of $2\log_3 6 - \log_3 4$?

$$2\log_3 6 - \log_3 4$$

Log Rules

$\log_a x^q = q \log_a x$

Example: What is the value of $2\log_3 6 - \log_3 4$?

$$2\log_3 6 - \log_3 4$$
$$= \log_3 6^2 - \log_3 4$$

Log Rules

$\log_a x^q = q \log_a x$

Example: What is the value of $2\log_3 6 - \log_3 4$?

$$2\log_3 6 - \log_3 4$$
$$= \log_3 6^2 - \log_3 4$$
$$= \log_3 36 - \log_3 4 = \log_3 9$$

Log Rules

$\log_a x^q = q \log_a x$

Example: What is the value of $2\log_3 6 - \log_3 4$?

$$2\log_3 6 - \log_3 4$$

$$= \log_3 6^2 - \log_3 4$$

$$= \log_3 36 - \log_3 4 = \log_3 9$$

$$\log_3 9 = x$$

Log Rules

$\log_a x^q = q \log_a x$

Example: What is the value of $2\log_3 6 - \log_3 4$?

$$2\log_3 6 - \log_3 4$$
$$= \log_3 6^2 - \log_3 4$$
$$= \log_3 36 - \log_3 4 = \log_3 9$$

$$\log_3 9 = x$$
$$3^x = 9$$

Log Rules

$\log_a x^q = q \log_a x$

Example: What is the value of $2\log_3 6 - \log_3 4$?

$$2\log_3 6 - \log_3 4$$
$$= \log_3 6^2 - \log_3 4$$
$$= \log_3 36 - \log_3 4 = \log_3 9$$

$$\log_3 9 = x$$
$$3^x = 9$$
$$x = 2$$

So the value of $2\log_3 6 - \log_3 4$ is 2

Log Equations

Question 1

$$\log_3(2x+5) = 2$$

Log Equations

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$$\log_3(2x+5) = 2$$
$$3^2 = 2x+5$$

Log Equations

Question 1

$$\log_3(2x+5) = 2$$
$$3^2 = 2x + 5$$
$$9 = 2x + 5$$

Log Equations

Question 1

$$\log_3(2x+5) = 2$$
$$3^2 = 2x+5$$
$$9 = 2x+5$$
$$4 = 2x$$

Log Equations

Question 1

$$\log_3(2x+5) = 2$$
$$3^2 = 2x+5$$
$$9 = 2x+5$$
$$4 = 2x$$
$$2 = x$$

Question 3

$$\log_2 x + \log_2(x+2) = 3$$

Question 3

$$\log_2 x + \log_2(x+2) = 3$$
$$\log_2(x^2 + 2x) = 3$$

Question 3

$$\log_2 x + \log_2(x+2) = 3$$
$$\log_2(x^2 + 2x) = 3$$
$$2^3 = x^2 + 2x$$

Question 3

$$\begin{aligned}
\log_2 x + \log_2(x+2) &= 3 \\
\log_2(x^2 + 2x) &= 3 \\
2^3 &= x^2 + 2x \\
8 &= x^2 + 2x
\end{aligned}$$

Question 3

$$\log_2 x + \log_2(x+2) = 3$$
$$\log_2(x^2 + 2x) = 3$$
$$2^3 = x^2 + 2x$$
$$8 = x^2 + 2x$$
$$x^2 + 2x - 8 = 0$$

Question 3

$$
\begin{aligned}
\log_2 x + \log_2(x+2) &= 3 \\
\log_2(x^2+2x) &= 3 \\
2^3 &= x^2 + 2x \\
8 &= x^2 + 2x \\
x^2 + 2x - 8 &= 0 \\
(x+4)(x-2) &= 0
\end{aligned}
$$

Question 3

$$\log_2 x + \log_2(x+2) = 3$$
$$\log_2(x^2+2x) = 3$$
$$2^3 = x^2 + 2x$$
$$8 = x^2 + 2x$$
$$x^2 + 2x - 8 = 0$$
$$(x+4)(x-2) = 0$$
$$x = -4 \quad x = 2$$

We cannot get the log of a negative, so therefore we disallow $x = -4$, so our only solution is $x = 2$

Question 5

$$2\log_5 x - \log_5(x-1) = \log_5 4$$

Question 5

$$2\log_5 x - \log_5(x-1) = \log_5 4$$
$$\log_5 x^2 - \log_5(x-1) = \log_5 4$$

Question 5

$$2\log_5 x - \log_5(x-1) = \log_5 4$$
$$\log_5 x^2 - \log_5(x-1) = \log_5 4$$
$$\log_5\left(\frac{x^2}{x-1}\right) = \log_5 4$$

Question 5

$$2\log_5 x - \log_5(x-1) = \log_5 4$$
$$\log_5 x^2 - \log_5(x-1) = \log_5 4$$
$$\log_5\left(\frac{x^2}{x-1}\right) = \log_5 4$$
$$\frac{x^2}{x-1} = 4$$

Question 5

$$
\begin{aligned}
2\log_5 x - \log_5(x-1) &= \log_5 4 \\
\log_5 x^2 - \log_5(x-1) &= \log_5 4 \\
\log_5\left(\frac{x^2}{x-1}\right) &= \log_5 4 \\
\frac{x^2}{x-1} &= 4 \\
x^2 &= 4(x-1)
\end{aligned}
$$

Question 5

$$
\begin{aligned}
2\log_5 x - \log_5(x-1) &= \log_5 4 \\
\log_5 x^2 - \log_5(x-1) &= \log_5 4 \\
\log_5\left(\frac{x^2}{x-1}\right) &= \log_5 4 \\
\frac{x^2}{x-1} &= 4 \\
x^2 &= 4(x-1) \\
x^2 &= 4x-4
\end{aligned}
$$

Question 5

$$
\begin{aligned}
2\log_5 x - \log_5(x-1) &= \log_5 4 \\
\log_5 x^2 - \log_5(x-1) &= \log_5 4 \\
\log_5\left(\frac{x^2}{x-1}\right) &= \log_5 4 \\
\frac{x^2}{x-1} &= 4 \\
x^2 &= 4(x-1) \\
x^2 &= 4x-4 \\
x^2-4x+4 &= 0
\end{aligned}
$$

Question 5

$$
\begin{aligned}
2\log_5 x - \log_5(x-1) &= \log_5 4 \\
\log_5 x^2 - \log_5(x-1) &= \log_5 4 \\
\log_5\left(\frac{x^2}{x-1}\right) &= \log_5 4 \\
\frac{x^2}{x-1} &= 4 \\
x^2 &= 4(x-1) \\
x^2 &= 4x-4 \\
x^2-4x+4 &= 0 \\
(x-2)(x-2) &= 0
\end{aligned}
$$

Question 5

$$2\log_5 x - \log_5(x-1) = \log_5 4$$
$$\log_5 x^2 - \log_5(x-1) = \log_5 4$$
$$\log_5\left(\frac{x^2}{x-1}\right) = \log_5 4$$
$$\frac{x^2}{x-1} = 4$$
$$x^2 = 4(x-1)$$
$$x^2 = 4x-4$$
$$x^2 - 4x + 4 = 0$$
$$(x-2)(x-2) = 0$$
$$x = 2 \quad x = 2$$

Change of Base

Question 16(b):

$$\log_3 x = \log_9(5x - 4)$$

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$$\log_3 x = \log_9(5x - 4)$$

$$\log_3 x = \frac{\log_3(5x - 4)}{\log_3 9}$$

Change of Base

Question 16(b):

$$\log_3 x = \log_9(5x - 4)$$

$$\log_3 x = \frac{\log_3(5x - 4)}{\log_3 9}$$

$$\log_3 x = \frac{\log_3(5x - 4)}{2}$$

Change of Base

Question 16(b):

$$\log_3 x = \log_9(5x-4)$$
$$\log_3 x = \frac{\log_3(5x-4)}{\log_3 9}$$
$$\log_3 x = \frac{\log_3(5x-4)}{2}$$
$$2\log_3 x = \log_3(5x-4)$$

Change of Base

Question 16(b):

$$\log_3 x = \log_9(5x-4)$$

$$\log_3 x = \frac{\log_3(5x-4)}{\log_3 9}$$

$$\log_3 x = \frac{\log_3(5x-4)}{2}$$

$$2\log_3 x = \log_3(5x-4)$$

$$\log_3 x^2 = \log_3(5x-4)$$

Change of Base

Question 16(b):

$$\log_3 x = \log_9(5x-4)$$
$$\log_3 x = \frac{\log_3(5x-4)}{\log_3 9}$$
$$\log_3 x = \frac{\log_3(5x-4)}{2}$$
$$2\log_3 x = \log_3(5x-4)$$
$$\log_3 x^2 = \log_3(5x-4)$$
$$x^2 = 5x-4$$

Change of Base

Question 16(b):

$$\log_3 x = \log_9(5x - 4)$$
$$\log_3 x = \frac{\log_3(5x-4)}{\log_3 9}$$
$$\log_3 x = \frac{\log_3(5x-4)}{2}$$
$$2\log_3 x = \log_3(5x-4)$$
$$\log_3 x^2 = \log_3(5x-4)$$
$$x^2 = 5x - 4$$
$$x^2 - 5x + 4 = 0$$

Change of Base

Question 16(b):

$$\log_3 x = \log_9(5x-4)$$
$$\log_3 x = \frac{\log_3(5x-4)}{\log_3 9}$$
$$\log_3 x = \frac{\log_3(5x-4)}{2}$$
$$2\log_3 x = \log_3(5x-4)$$
$$\log_3 x^2 = \log_3(5x-4)$$
$$x^2 = 5x-4$$
$$x^2 - 5x + 4 = 0$$
$$(x-4)(x-1) = 0$$
$$x = 4 \quad x = 1$$

Introduction to the Exponential Function

The exponential function is used when a quantity grows or decays at a rate proportional to its current value.

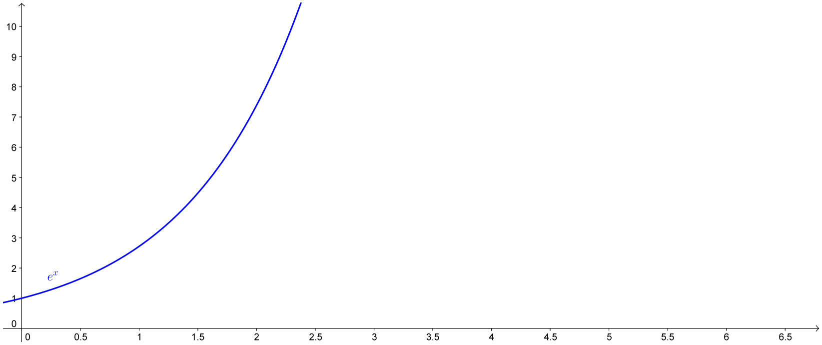
The exponential function is used to model many situations and processes. Some of these include the growth of bacteria, radio carbon dating, depreciation and continuously compounding interest.

The basic exponential function is written as $y = e^x$.

Properties

- Note that $e^x > 0$ for every value of x.
- As $x \to \infty$, $e^x \to \infty$ very fast!
- As $x \to -\infty$, $e^x \to 0$ very fast!
- $e^0 = 1$

$y = e^x$



Modelling with the Exponential Function

A certain fish population is believed to be growing exponentially. When first studied the fish population was 50,000. Now, five years later, it has increased to 75,000. If the hypothesis of exponential growth is valid, what will the population be three years from now?

The model $y = 50,000e^{.08019t}$ models this situation. We can use this model to make further predictions about the fish population.

$y = Ae^{rt}$

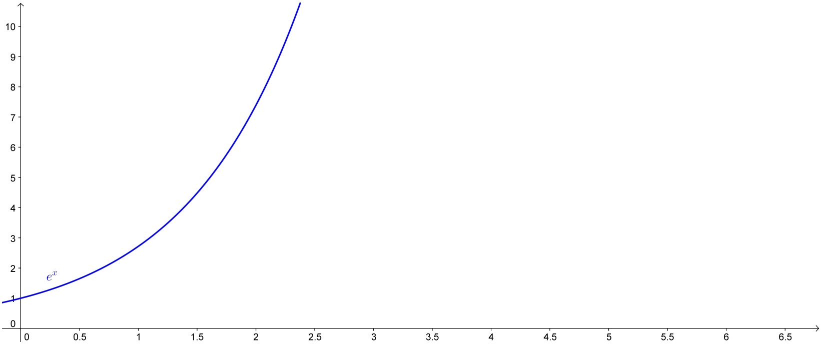
We use the basic model $y = Ae^{rt}$ to describe many situations where a quantity grows or decays at a rate proportional to its current value.

$$y = Ae^{rt}$$

- A is the initial value.
- r is the growth/decay rate.
- t is time, the independent variable.

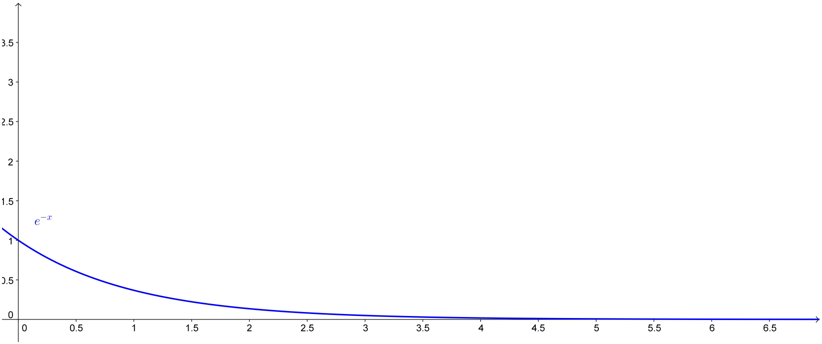
$y = e^{rt}, \ r > 0$

Note that $y = e^{rt}, \ r > 0$ is the exponential growth curve.



$y = e^{rt}, \ r < 0$

Note that $y = e^{rt}, \ r < 0$ is the exponential decay curve.



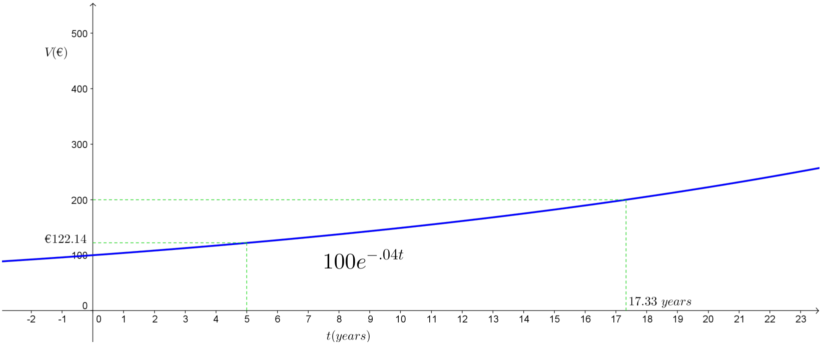
$y = Ae^{rt}$

A €100 investment gains value by continuously compounding at a rate of 4% annually. Create a model in the form $V = Ae^{rt}$, where V is the value of the investment (€), and t is time in years.

$$V = 100e^{.04t}$$

1. How much will the investment be after 5 years?
2. How long will it take the investment to double?

$V = 100e^{.04t}$


500
400
300
200
100
0
$V(€)$
€122.14
$100e^{-.04t}$
17.33 *years*
-2 -1 0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23
$t(years)$

$V = 100e^{.04t}$

How much will the investment be after 5 years?

$V = 100e^{.04t}$

How much will the investment be after 5 years?

$$V = 100e^{.04t}$$

$V = 100e^{.04t}$

How much will the investment be after 5 years?

$$V = 100e^{.04t}$$
$$V = 100e^{(.04(5))}$$

$V = 100e^{.04t}$

How much will the investment be after 5 years?

$$V = 100e^{.04t}$$

$$V = 100e^{(.04(5))}$$

$$V = 100(1.2214)$$

$V = 100e^{.04t}$

How much will the investment be after 5 years?

$$V = 100e^{.04t}$$
$$V = 100e^{(.04(5))}$$
$$V = 100(1.2214)$$
$$V = 122.14$$

So, after 5 years the investment will be worth €122.14.

$V = 100e^{.04t}$

How long will it take the investment to double?

$V = 100e^{.04t}$

How long will it take the investment to double?

$$200 = 100e^{.04t}$$

$V = 100e^{.04t}$

How long will it take the investment to double?

$$200 = 100e^{.04t}$$
$$2 = e^{.04t}$$

$V = 100e^{.04t}$

How long will it take the investment to double?

$$200 = 100e^{.04t}$$
$$2 = e^{.04t}$$
$$Ln(2) = Ln(e^{.04t})$$

$V = 100e^{.04t}$

How long will it take the investment to double?

$$
\begin{aligned}
200 &= 100e^{.04t} \\
2 &= e^{.04t} \\
Ln(2) &= Ln(e^{.04t}) \\
Ln(2) &= .04t
\end{aligned}
$$

$V = 100e^{.04t}$

How long will it take the investment to double?

$$\begin{aligned}
200 &= 100e^{.04t} \\
2 &= e^{.04t} \\
Ln(2) &= Ln(e^{.04t}) \\
Ln(2) &= .04t \\
t &= \frac{Ln(2)}{.04}
\end{aligned}$$

$V = 100e^{.04t}$

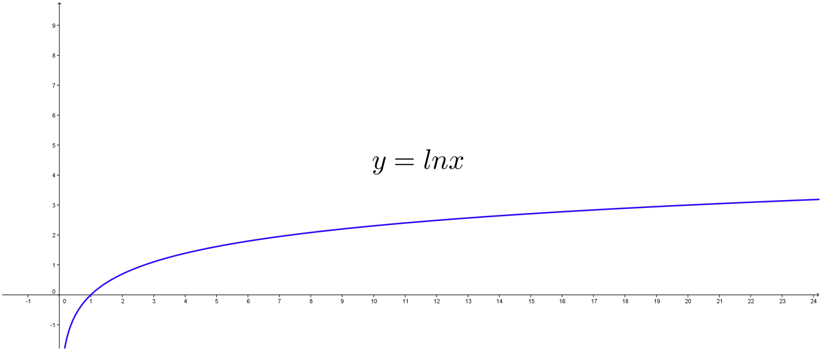
How long will it take the investment to double?

$$
\begin{aligned}
200 &= 100e^{.04t} \\
2 &= e^{.04t} \\
Ln(2) &= Ln(e^{.04t}) \\
Ln(2) &= .04t \\
t &= \frac{Ln(2)}{.04} \\
t &= 17.33
\end{aligned}
$$

Properties

- The function $y = Ln(x)$ (also written as $Log_e(x)$) is the inverse function of e^x , i.e.,
$$y = Ln(x) \text{ means } x = e^y$$
by definition.
- Since $e^y > 0$, $Ln(x)$ only makes sense for $x > 0$, i.e., $Ln(x)$ does not exist for $x < 0$.
- Since $1 = e^0$ we have $Ln(1) = 0$
- $Ln(e^x) = x$ **and** $e^{Ln(x)} = x$

$y = lnx$



Exponential Equations

Since the Exponential function and the Natural Log function are the inverse of each other we can use the natural log function ($Ln(x)$) when trying to solve equations with exponentials.

Example:

Exponential Equations

Since the Exponential function and the Natural Log function are the inverse of each other we can use the natural log function ($Ln(x)$) when trying to solve equations with exponentials.
Example:

$$e^x = 10$$

Exponential Equations

Since the Exponential function and the Natural Log function are the inverse of each other we can use the natural log function ($Ln(x)$) when trying to solve equations with exponentials.
Example:

$$e^x = 10$$
$$Ln(e^x) = Ln(10)$$

Exponential Equations

Since the Exponential function and the Natural Log function are the inverse of each other we can use the natural log function ($Ln(x)$) when trying to solve equations with exponentials.
Example:

$$e^x = 10$$
$$Ln(e^x) = Ln(10)$$
$$x = Ln(10)$$

Exponential Equations

Since the Exponential function and the Natural Log function are the inverse of each other we can use the natural log function ($Ln(x)$) when trying to solve equations with exponentials.
Example:

$$\begin{aligned} e^x &= 10 \\ Ln(e^x) &= Ln(10) \\ x &= Ln(10) \\ x &= 2.302585 \end{aligned}$$

Example

Solve for x:

Example

Solve for x:

$$50e^{.1x} = 3000$$

Example

Solve for x:

$$50e^{.1x} = 3000$$
$$e^{.1x} = 60$$

Example

Solve for x:

$$50e^{.1x} = 3000$$
$$e^{.1x} = 60$$
$$Ln(e^{.1x}) = Ln(60)$$

Example

Solve for x:

$$50e^{.1x} = 3000$$
$$e^{.1x} = 60$$
$$Ln(e^{.1x}) = Ln(60)$$
$$.1x = Ln(60)$$

Example

Solve for x:

$$\begin{aligned} 50e^{.1x} &= 3000 \\ e^{.1x} &= 60 \\ Ln(e^{.1x}) &= Ln(60) \\ .1x &= Ln(60) \\ x &= \frac{Ln(60)}{.1} \end{aligned}$$

Example

Solve for x:

$$50e^{.1x} = 3000$$
$$e^{.1x} = 60$$
$$Ln(e^{.1x}) = Ln(60)$$
$$.1x = Ln(60)$$
$$x = \frac{Ln(60)}{.1}$$
$$x = 40.9434456$$

Practice

Solve the following equations:

1. $e^x = 20$
2. $e^x = -1$
3. $e^{3x} = 12$
4. $25e^{.3x} = 200$
5. $150e^{.0325x} = 600$

Solutions

1. $x = 2.996$
2. No Solution because $e^x > 0$
3. $x = 0.8283$
4. $x = 6.931$
5. $x = 42.655$

$Y = Ae^{rt}$

A laptop bought at the end of 2016 for €1, 100 depreciates continuously in value at a rate of 25% a year. Assuming this can be modeled by the exponetial function;

$Y = Ae^{rt}$

A laptop bought at the end of 2016 for €1,100 depreciates continuously in value at a rate of 25% a year. Assuming this can be modeled by the exponetial function;

1. What will the laptop be worth at the end of 2018?

$Y = Ae^{rt}$

A laptop bought at the end of 2016 for €1,100 depreciates continuously in value at a rate of 25% a year. Assuming this can be modeled by the exponetial function;

1. What will the laptop be worth at the end of 2018?
2. During what year will the price of the laptop drop below €400?

$Y = Ae^{rt}$

A laptop bought at the end of 2016 for €1, 100 depreciates continuously in value at a rate of 25% a year. Assuming this can be modeled by the exponetial function;

1. What will the laptop be worth at the end of 2018?
2. During what year will the price of the laptop drop below €400?

We can model this situation with the function $V = 1100e^{-.25t}$.

$V = 1100e^{-.25t}$

What will the laptop be worth at the end of 2018?

$V = 1100e^{-.25t}$

What will the laptop be worth at the end of 2018?

$$V = 1100e^{-.25(2)}$$

$V = 1100e^{-.25t}$

What will the laptop be worth at the end of 2018?

$$V = 1100e^{-.25(2)}$$
$$V = 1100(.606530659)$$

$V = 1100e^{-.25t}$

What will the laptop be worth at the end of 2018?

$$V = 1100e^{-.25(2)}$$
$$V = 1100(.606530659)$$
$$V = 667.1837$$

$V = 1100e^{-.25t}$

What will the laptop be worth at the end of 2018?

$$V = 1100e^{-.25(2)}$$
$$V = 1100(.606530659)$$
$$V = 667.1837$$

So, at the end of 2018, the laptop will be worth €667.18

$V = 1100e^{-.25t}$

During what year will the price of the laptop drop below €400?

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t &= 4.04
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$$r = .08109$$

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So in 3 years,(8 years since time = 0), the population of fish will be around 95657

Practice

1. An investment of €505 gains value from continuously compounding interest at a rate of $3\frac{1}{2}\%$. What will the investment be worth after 5 years? When will it be worth over €1000?
2. The population of the world in 1950 was 2.518 billion and in 1960 it was 3.021 billion. Assuming exponential growth, derive a formula for the population. What is the expected population of the world in 2012 using this data.
3. A car was bought three years ago for €25,000. It is now valued at €15,000. Assuming that the value is depreciating exponentially, estimate the value one year from now to the nearest euro.

Solutions

1. - $V = 505e^{.035t}$
   - €601.58
   - $t = 19.52$ years
2. - $V = 2.518e^{0.01821t}$
   - 7.787 Billion
3. - $V = 25000e^{-0.1703t}$
   - €$12,650.24$