Functions
Revision Series 2017

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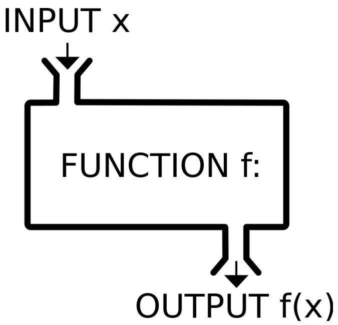
Introduction to Functions

A function is a rule which operates on one number(input) to give another number(output).

It is sometimes helpful to think of a function as a machine which changes an input, x, into an output, $f(x)$.



Example

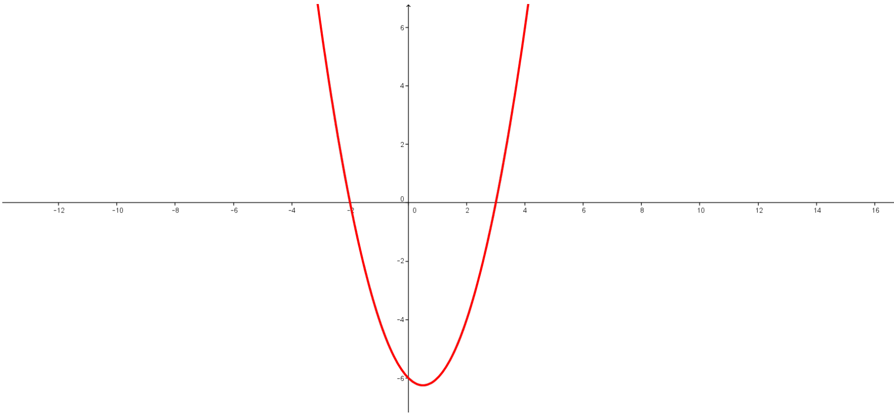
Take the function $f(x) = 2x + 6$. This may also be written as $f : x \to 2x + 6$ or $y = 2x + 6$

Entering $x = 1$ as the input;

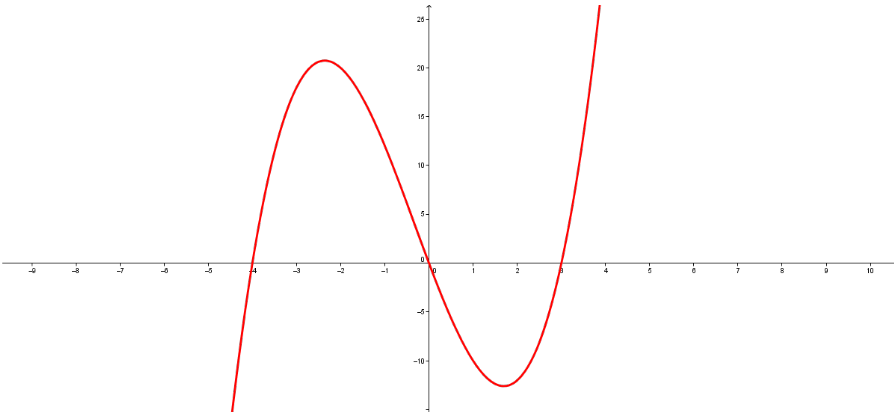
$$f(1) = 2(1) + 6 = 8$$
$$\Rightarrow f(1) = 8$$

This means for the input of $x = 1$, the function $f(x) = 2x + 6$ gave the output $y = 8$ We call the set of all inputs of a particular function the **Domain**, the set of possible outputs the **Co-Domain**, and the set of actual outputs the **Range**.

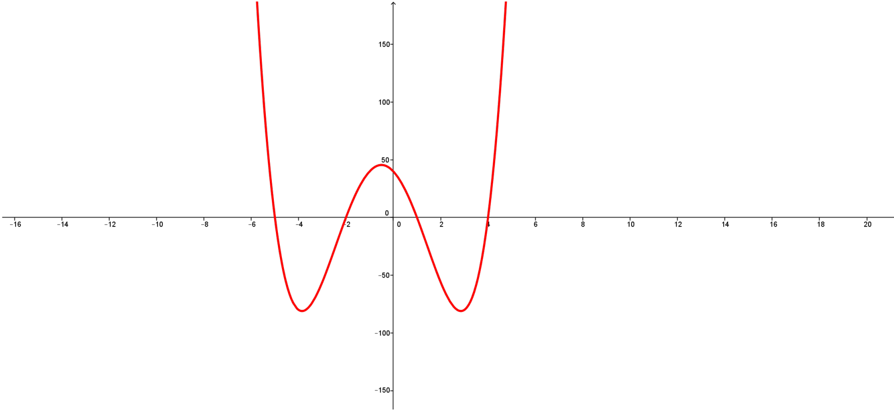
Quadratic



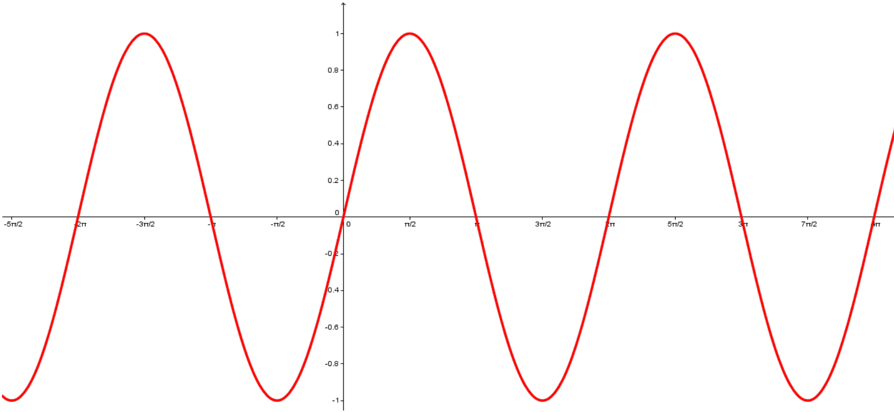
Cubic



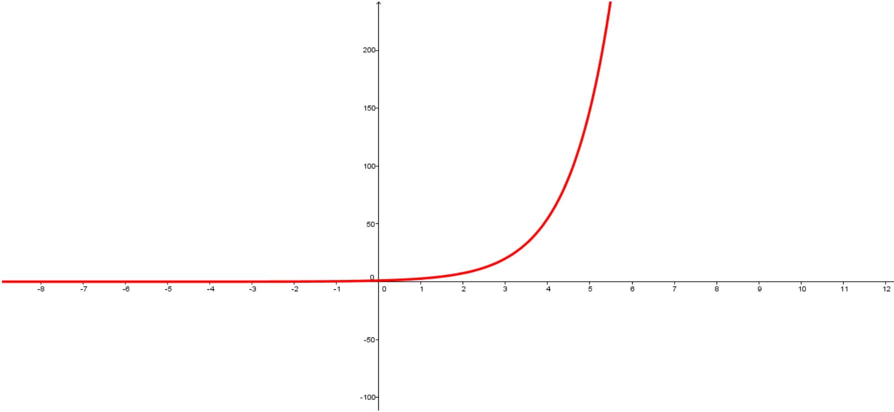
Quartic



Trigonometric



exponential



Properties of Functions

For a relation to be a function, there must be only one output that corresponds to a given input.
Which of the following relations can be classified as functions?

1. $f(x) = 3x + 1$
2. $f : x \to \frac{1}{x+1}$
3. $2y = x^2 + 1$
4. $y^2 = x + 1$
5. $f(x) = 8$

1. $f(x) = 3x + 1$ is a function
2. $f : x \to \frac{1}{x+1}$ is a function
3. $2y = x^2 + 1$ is a function
4. $y^2 = x + 1$ is NOT a function
5. $f(x) = 8$ is a function

Graphing Functions

For each value of x in the domain of the function, the function gives us a value for y. These are the coordinates of a point (x, y) in the plane.
Plotting all these points gives us the graph of the function.(i.e. joining the dots)

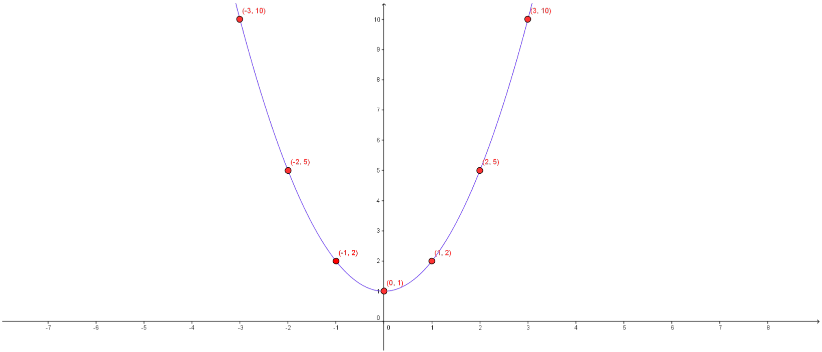
Example: Plot the function $f(x) = x^2 + 1$ in the domain $-3 \leq x \leq 3$

$f(x) = x^2 + 1$

Input(x)	$x^2 + 1$	Output(y)	(x, y)
-3	$(-3)^2 + 1$	10	$(-3, 10)$
-2	$(-2)^2 + 1$	5	$(-2, 5)$
-1	$(-1)^2 + 1$	2	$(-1, 2)$
0	$(0)^2 + 1$	1	$(0, 1)$
1	$(1)^2 + 1$	2	$(1, 2)$
2	$(2)^2 + 1$	5	$(2, 5)$
3	$(3)^2 + 1$	10	$(3, 10)$

Graphing Functions

Plotting these points and joining the dots results in the following graph.



Injective Functions

An injective function is a function where every element of the co-domain is mapped onto by **at most** one element of the domain; ie. every output has at most one input.

$$f : \mathbb{R} \mapsto \mathbb{R}, f(x) = x^3 \text{ is injective}$$

$$f : \mathbb{R} \mapsto \mathbb{R}, f(x) = x^2 \text{ is not injective}$$

$$f : \mathbb{R}^+ \mapsto \mathbb{R}^+, f(x) = x^2 \text{ is injective}$$

Surjective Functions

A surjective function is a function where every element of the co-domain is mapped onto by **at least** one element of the domain; ie. every output has at least one input.

$f:\mathbb{R}\mapsto\mathbb{R}, f(x)=x^3$ is surjective

$f:\mathbb{R}\mapsto\mathbb{R}, f(x)=x^2$ is not surjective

Bijective Functions

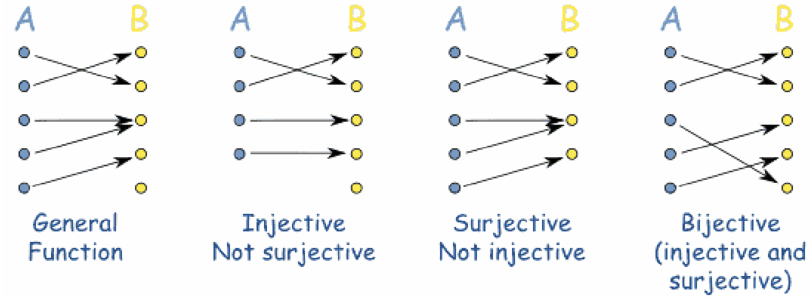
A bijective function is a function which is both injective and surjective. Every element on the co-domain is mapped onto by **exactly** one element of the domain. Bijective functions are often called **one to one** functions.

$$f : \mathbb{R} \mapsto \mathbb{R}, f(x) = x^3 \text{ is bijective}$$

$$f : \mathbb{R} \mapsto \mathbb{R}, f(x) = x^2 \text{ is not bijective}$$

$$f : \mathbb{R} \mapsto \mathbb{R}, f(x) = x + 2 \text{ is bijective}$$

All linear functions are bijective


A
B
General
Function
A
B
Injective
Not surjective
A
B
Surjective
Not injective
A
B
Bijective
(injective and
surjective)

Composite Functions

Composition of functions is simply when the output of one function is used as the input of another.

$$f(x) = 3x + 2 \quad \text{and} \quad g(x) = x^2 - 1$$

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$$\begin{aligned} g(3) &= (3)^2 - 1 = 8 \\ f \circ g(3) = f(8) &= 3(8) + 2 \\ f \circ g(3) &= 26 \end{aligned}$$

$f(x) = 3x + 2 \quad g(x) = x^2 - 1$

Find $f \circ g(x)$

$$f \circ g(x) = 3(x^2 - 1) + 2$$

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Find $f \circ g(x)$

$$
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f \circ g(x) =& 3(x^2 - 1) + 2 \\
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f \circ g(x) =& 3x^2 - 1
\end{aligned}
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Find $g \circ f(x)$

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$$g \circ f(x) = 9x^2 + 12x + 4 - 1$$

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$$g \circ f(x) = 9x^2 + 12x + 4 - 1$$
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Notice that $f \circ g(x) \neq g \circ f(x)$

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$$f(x) = 2^x$$
$$\text{Input: } x = 3$$
$$f(3) = 2^3 = 8$$

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$$f(x) = 2^x$$
Input: $x = 3$
$$f(3) = 2^3 = 8$$

$$f^{-1}(x) = \log_2(x)$$
Input: $x = 8$
$$f^{-1}(8) = \log_2 8 = 3$$

Calculating the Inverse

To calculate the inverse of a function, let $f(x) = y$ and solve the resulting equation for x. The expression on the other side of the equals to sign will represent the inverse function.

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$$\begin{aligned} y &= 3x + 4 \\ y - 4 &= 3x \end{aligned}$$

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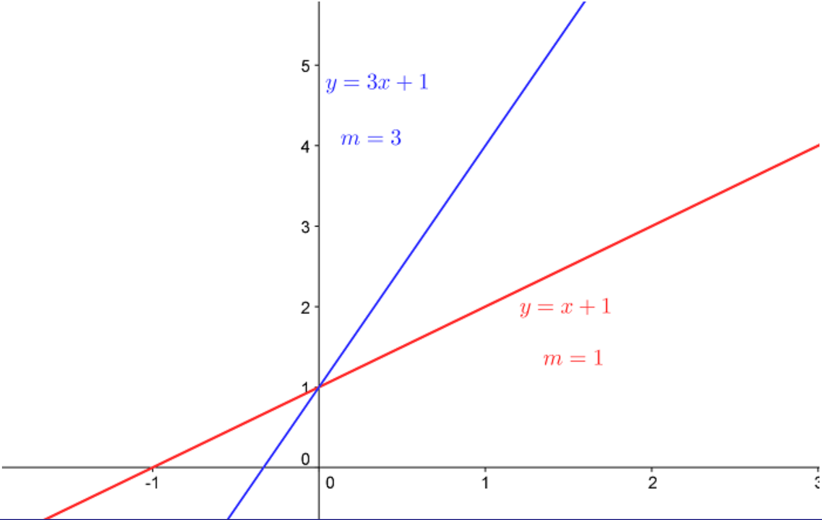
$$\begin{aligned} y &= 3x + 4 \\ y - 4 &= 3x \\ \frac{y-4}{3} &= x \end{aligned}$$

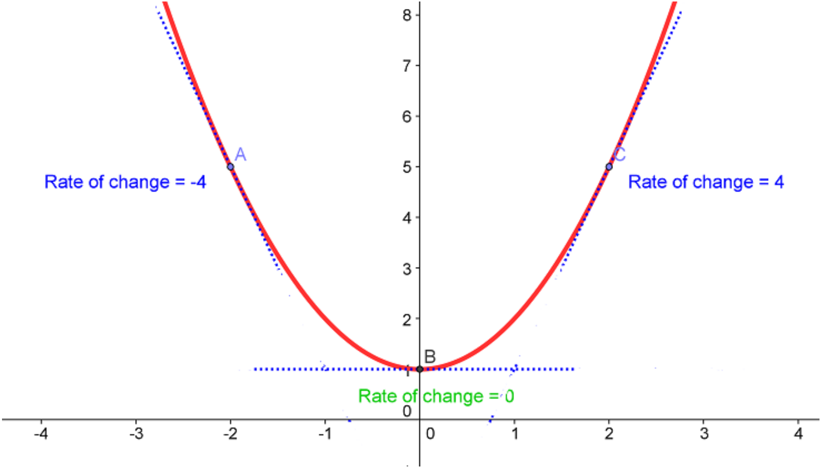
The left hand side contains a representation of the inverse. We will express the invverse function in terms of x.

$$f^{-1}(x) = \frac{x-4}{3}$$

Rate of Change - Linear Functions



Rate of Change - Quadratic Functions



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- The derivative is the slope of the tangent line to the graph.
- The derivative can change in value from point to point.

Slopes of Tangents

Find the equation of the tangent to the function $f(x) = x^2 - 2x - 8$ at the point where $x = 2$.

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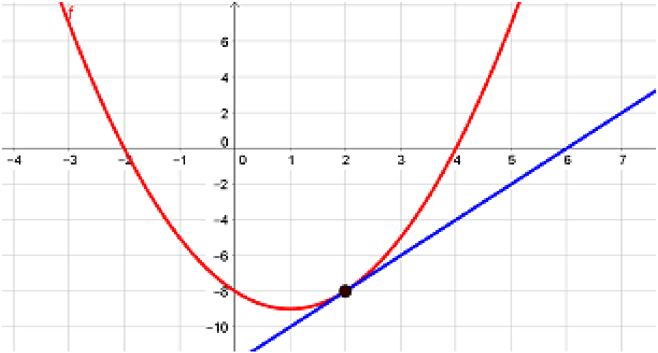

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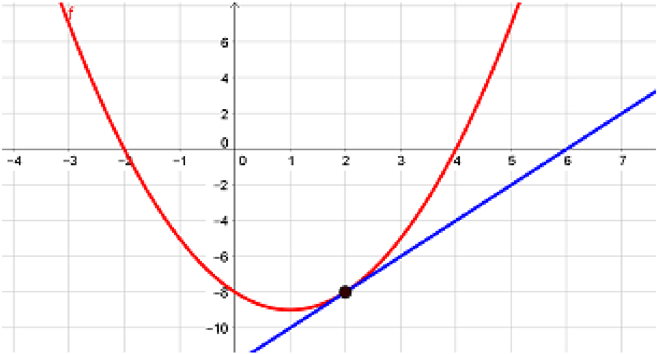

Tangent: $y - y_1 = m(x - x_1)$

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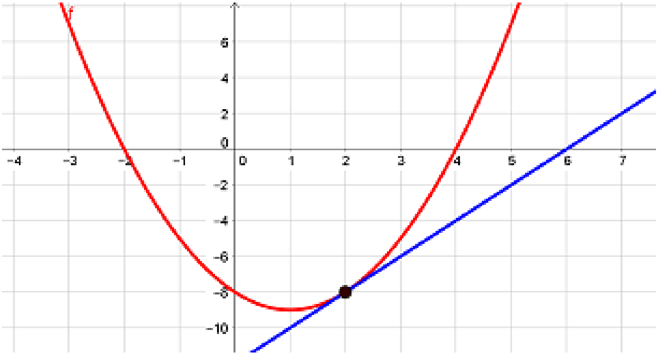

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$$f(2) = (2)^2 - 2(2) - 8$$

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Tangent: $y - y_1 = m(x - x_1)$

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$$f(2) = (2)^2 - 2(2) - 8$$
$$f(2) = -8$$

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Slope: Find $f'(x)$

$$f'(x) = 2x - 2 \text{ @ } x = 2$$

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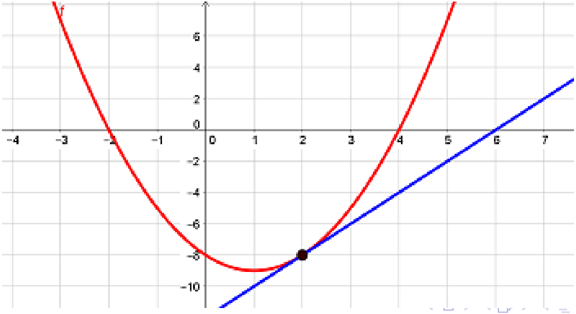
$$f'(x) = 2x - 2 \text{ @ } x = 2$$
$$m = f'(2) = 2(2) - 2$$
$$m = 2$$

Point $(2, -8)$ and Slope $= 2$

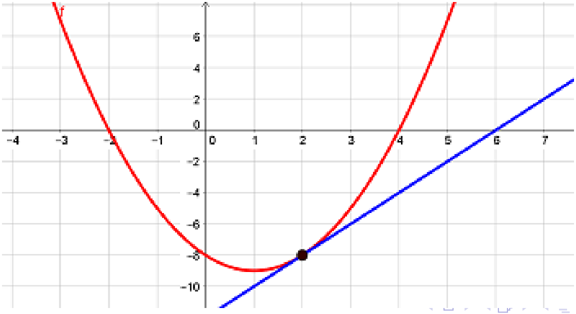
$$y - y_1 = m(x - x_1)$$



Point $(2, -8)$ and Slope $= 2$

$$y - y_1 = m(x - x_1)$$

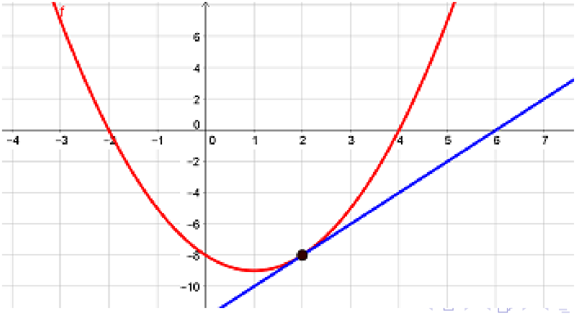
$$y - (-8) = 2(x - 2)$$



Point $(2, -8)$ and Slope $= 2$

$$y - y_1 = m(x - x_1)$$

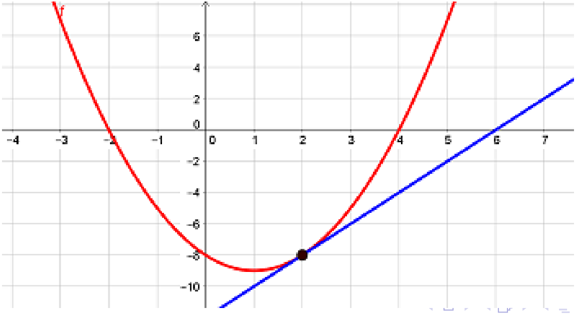
$$\begin{aligned} y - (-8) &= 2(x - 2) \\ y + 8 &= 2x - 4 \end{aligned}$$



Point $(2, -8)$ and Slope $= 2$

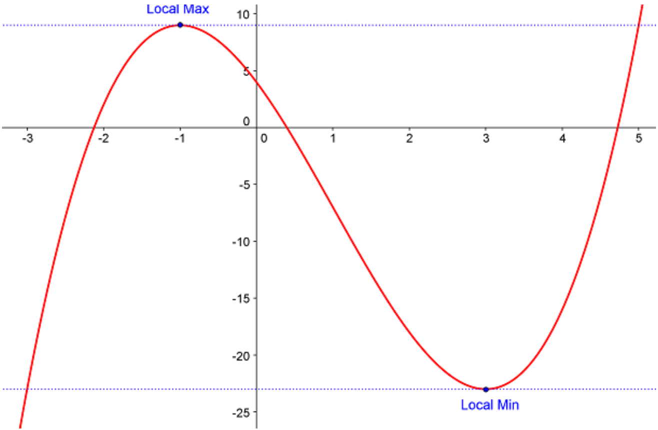
$$y - y_1 = m(x - x_1)$$

$$\begin{aligned} y - (-8) &= 2(x - 2) \\ y + 8 &= 2x - 4 \\ 2x - y - 12 &= 0 \end{aligned}$$



Introduction

The rate of change is zero at a turning point.



Calculating Turning Points

- Differentiate the function (Calculate $\frac{dy}{dx}$).

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- Differentiate the function (Calculate $\frac{dy}{dx}$).
- Let $\frac{dy}{dx} = 0$ and solve for x.
- Once we have found our turning point, we classify it by filling it into $\frac{d^2y}{dx^2}$

Local Maximum

- $\frac{dy}{dx} = 0$
- $\frac{d^2y}{dx^2} < 0$

Local Minimum

- $\frac{dy}{dx} = 0$
- $\frac{d^2y}{dx^2} > 0$

Example

Find the Local Maximum and Local Minimum of the function $y = x^3 - 3x^2 - 9x + 4$

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$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

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Find the Local Maximum and Local Minimum of the function
$y = x^3 - 3x^2 - 9x + 4$

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- Let $\frac{dy}{dx} = 0$ and solve for x.
$$3x^2 - 6x - 9 = 0$$

Example

Find the Local Maximum and Local Minimum of the function $y = x^3 - 3x^2 - 9x + 4$

- Differentiate the function (Calculate $\frac{dy}{dx}$).

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

- Let $\frac{dy}{dx} = 0$ and solve for x.

$$3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

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- Let $\frac{dy}{dx} = 0$ and solve for x.

$$3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

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- Differentiate the function (Calculate $\frac{dy}{dx}$).

$$\frac{dy}{dx} = 3x^2 - 6x - 9$$

- Let $\frac{dy}{dx} = 0$ and solve for x.

$$3x^2 - 6x - 9 = 0$$

$$x^2 - 2x - 3 = 0$$

$$(x - 3)(x + 1) = 0$$

$$x = 3 \quad x = -1$$

To find the corresponding y-values we fill our x-values into the **original** equation.

Example Continued

$$y = x^3 - 3x^2 - 9x + 4$$

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Turning point:$(3, -23)$

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Turning point:$(3, -23)$

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$$y = 9$$

Example Continued

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Turning point:$(3, -23)$

$x = -1$

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Turning Point: $(-1, 9)$

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We have found the two turning points, $(-1, 9)$ and $(3, -23)$. Which one is the max and which one is the min?

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For $x = 3$, $\frac{d^2y}{dx^2} = 6(3) - 6 = 12$
$12 > 0$, so the point is a local minimum.

Max/Min

So, $(-1, 9)$ is the local max and $(3, -23)$ is the local min.