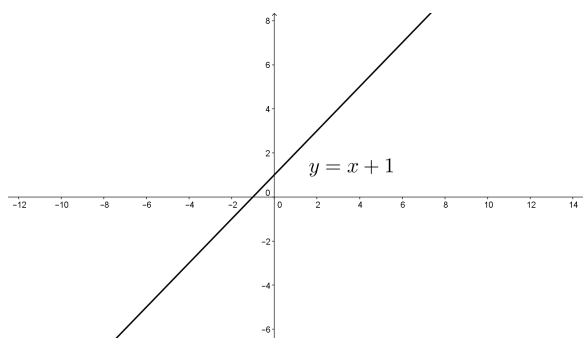




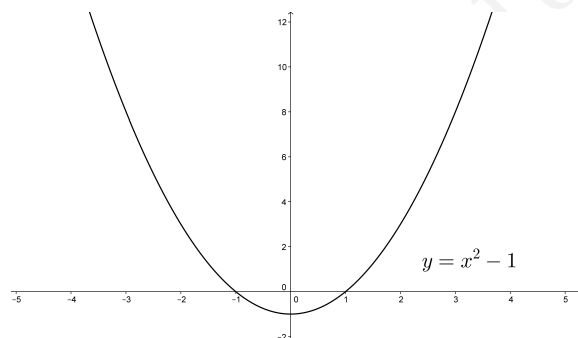
3 Functions

3.1 Introduction

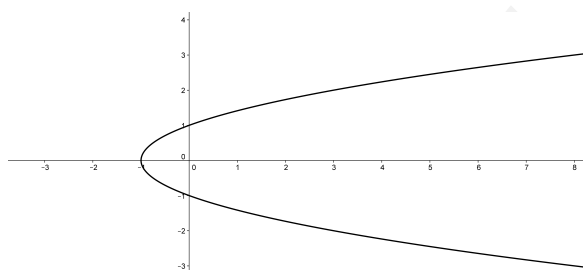
1. Which of the following relations are functions?



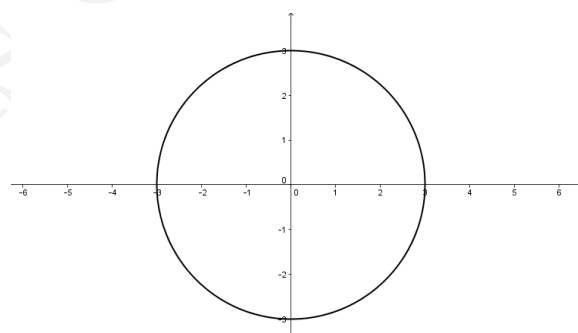
(a)



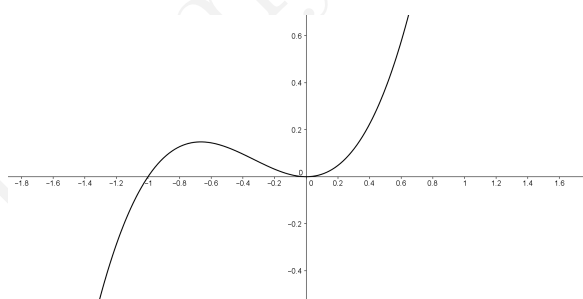
(b)



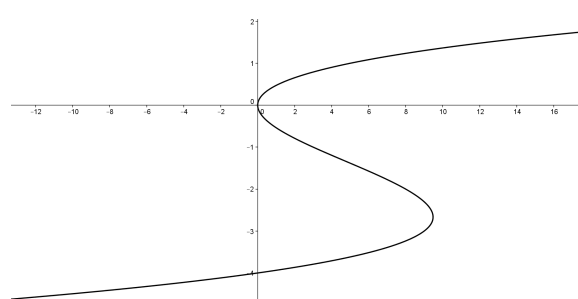
(c)



(d)



(e)

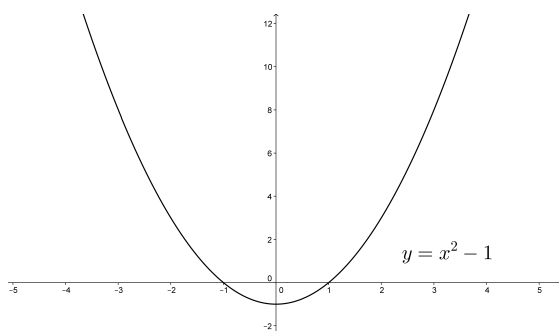


(f)

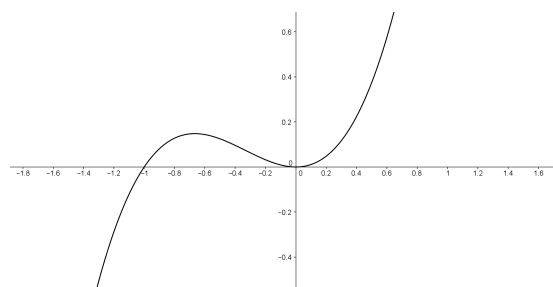




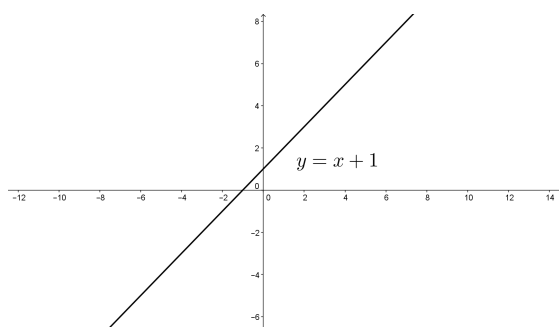
2. State whether the following functions injective, surjective, bijective or neither.



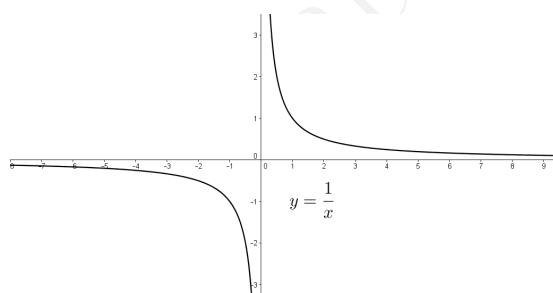
(a)



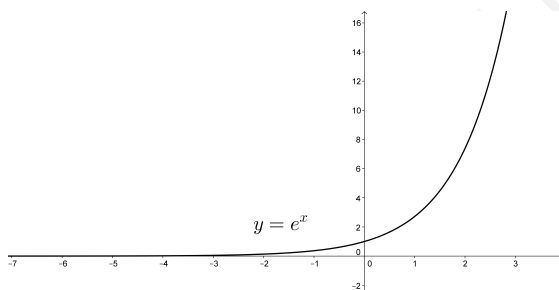
(b)



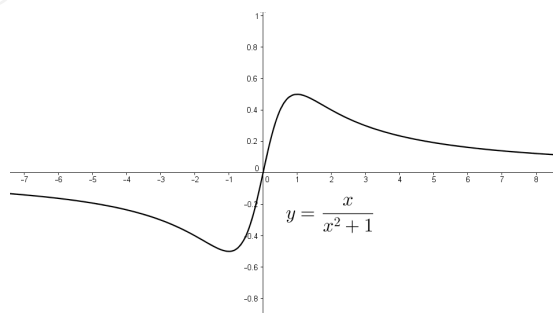
(c)



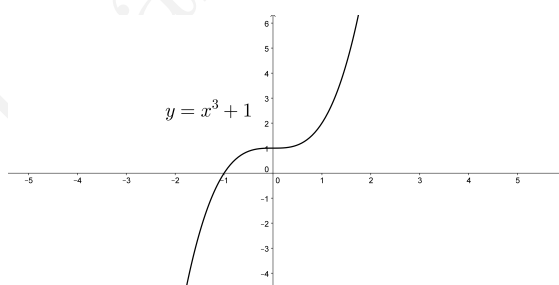
(d)



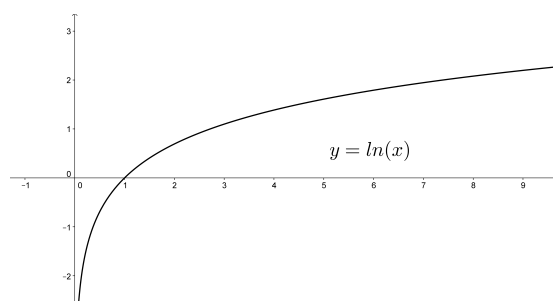
(e)



(f)



(g)



(h)





3.2 Composition of Functions

1. $f : x \rightarrow 2x - 1$ and $g : x \rightarrow 3 - 5x$ are two functions.

Find

- i. $f(4)$
- ii. $g \circ f(4)$
- iii. $f \circ g(-1)$
- iv. $g \circ f(x)$.

For what value of x is $f \circ g(x) = 25$?

2. If $f(x) = x^2 - 1$ and $g(x) = 3x + 2$, find the value of each of the following:

- i. $f(2)$
- ii. $g \circ f(2)$
- iii. $g(4)$
- iv. $f \circ g(4)$
- v. $f^2(3)$
- vi. $g^2(3)$
- vii. $g \circ f(-5)$
- viii. $f \circ g(\frac{2}{3})$

3. A stone is dropped into a pond creating a circular ripple. The radius of this ripple increases at the rate of 1.5 m/s.

- (a) Find an expression for r (radius) in terms of t (time).
- (b) A is the area of the circular ripple. Find $A \circ r(t)$. What does this function represent?
- (c) After how many seconds will the area be $225\pi \text{ m}^2$?

4. A spherical balloon is being inflated. Its radius is increasing at a rate of 3 cm/s.

- (a) Find an expression for r in terms of t .
- (b) Let $V(t)$ describe the volume of the balloon at any time. Find the expression for $V(t)$.
- (c) After how many seconds will the volume be $972\pi \text{ cm}^3$?

5. If $f(x) = 4x - 3$, find

- i. $f^2(x)$
- ii. $f^3(x)$
- iii. $f^4(x)$.

Hence, find an expression for $f^n(x)$ in terms of n .





3.3 Inverse Functions

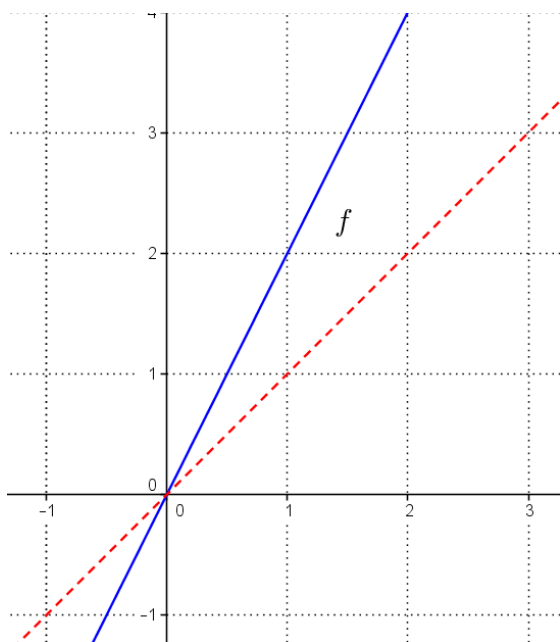
1. Find the inverse of each of the following functions.
 - i. $f(x) = 2x$
 - ii. $g(x) = x + 4$
 - iii. $h(x) = 2x - 1$
 - iv. $k(x) = 3x + 5$
 - v. $f(x) = 2 - 3x$
 - vi. $g(x) = \frac{4}{x}, x \neq 0$
 - vii. $h(x) = \frac{1}{x-2}, x \neq 2$
 - viii. $k(x) = \frac{x-5}{x}, x \neq 0$
 - ix. $g(x) = \frac{2x}{3x-2}, x \neq \frac{2}{3}$
 - x. $f(x) = 1 + \frac{1}{x}, x \neq 0$
2. Find the inverse function of each of the following by completing the square:
 - i. $f(x) = x^2 + 6x - 10, x \geq -3$
 - ii. $f(x) = x^2 - 4x - 5, x \geq 2$
 - iii. $f(x) = x^2 - 10x + 13, x \geq 5$
 - iv. $f(x) = x^2 + 8x + 8, x \geq -4$
3. Given $f(x) = \frac{4-x}{3}, -1 \leq x \leq 4$.
Sketch the graph of the given function and on the same set of axes, sketch the graph of the inverse function.
State the domain and range of the inverse function.
4. Let $f: A \rightarrow R, f(x) = \sqrt{2-x}$.
If A is the set of all real values of x for which $f(x)$ is defined, find A .



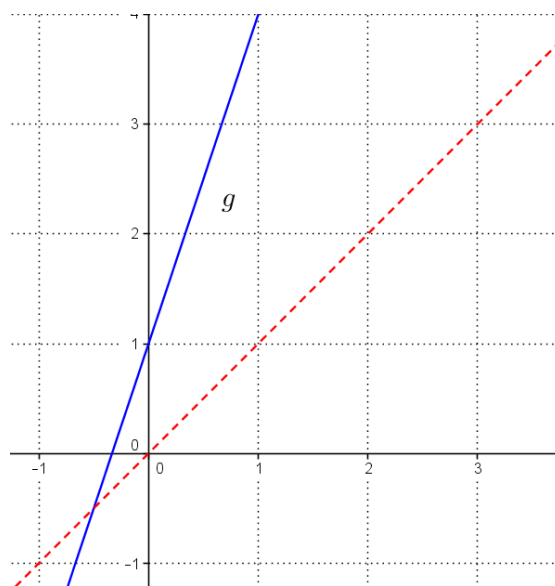


3.4 Graphing Inverse Functions

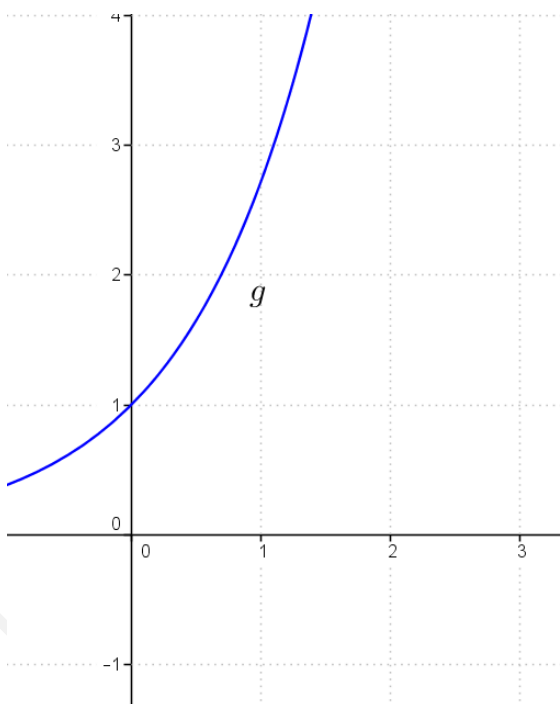
1. Graph the inverse of the following functions.



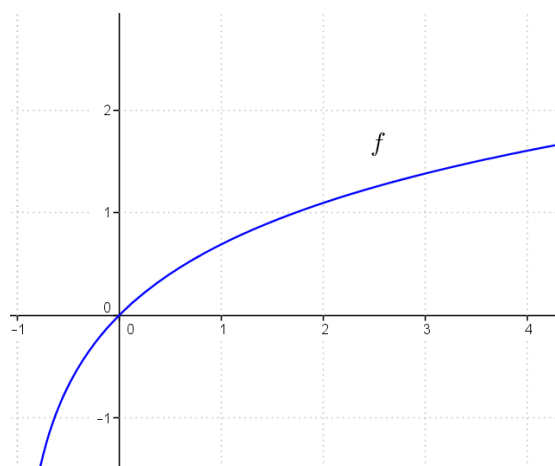
(a)



(b)



(g)



(h)





3.5 Turning Points

1. The graph of $y = a(x - p)^2 + q$ has a turning point (p, q) . By **completing the square**, find the turning point of each of the following quadratic functions.

(a) $f(x) = x^2 + 6x + 10$

(b) $g(x) = x^2 + 8x + 4$

(c) $h(x) = 5 + 8x - x^2$

(d) $f(x) = x^2 + 11x + 10$

(e) $g(x) = 2x^2 + 4x + 12$

(f) $h(x) = 1 + 6x - 3x^2$

3.6 Asymptotes And Curve Sketching

1. Evaluate each of these limits:

(a) $\lim_{n \rightarrow \infty} \frac{n^2+5}{7n^2+2n}$

(b) $\lim_{n \rightarrow \infty} \frac{3n^2+2n}{2n^2-5n-7}$

(c) $\lim_{n \rightarrow \infty} \frac{n^2+2}{3n^2-1}$

(d) $\lim_{n \rightarrow \infty} \frac{3n-4}{4n+3}$

(e) $\lim_{n \rightarrow \infty} \frac{5n}{n^3-2n}$

(f) $\lim_{n \rightarrow \infty} \frac{n}{n^2-3n+2}$

2. Find the horizontal and vertical asymptotes of each of the following curves:

(a) $y = \frac{3}{x-5}$

(b) $y = \frac{3x+2}{2x}$

(c) $y = \frac{2x+1}{x-3}$

(d) $y = \frac{5x}{3x-2}$

3. Given the function $f(x) = \frac{x}{x-4}$. Find:

- (a) the point at which the curve crosses the x-axis
(b) the equations of the horizontal and vertical asymptotes.

Hence draw a rough sketch of the function.

4. Find the equations of the horizontal and vertical asymptotes of the function

$$f(x) = \frac{5+x}{x-3}$$

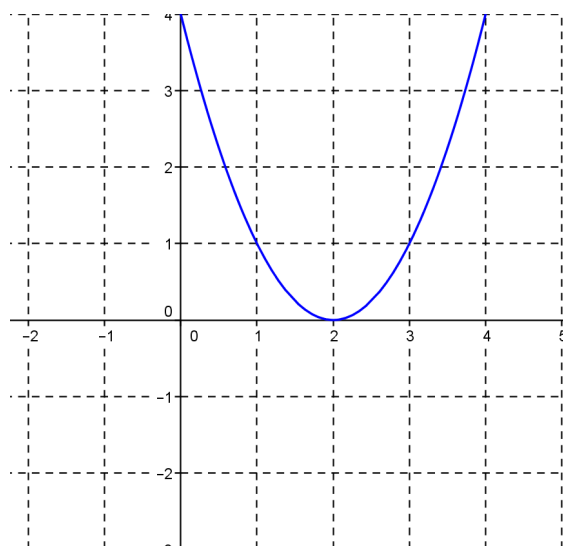
Find where the curve cuts the x-axis and y-axis and hence draw a rough sketch of the curve.



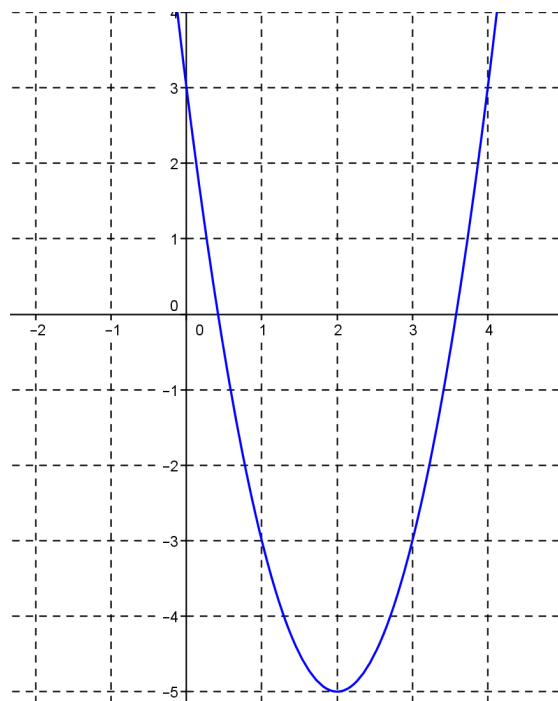
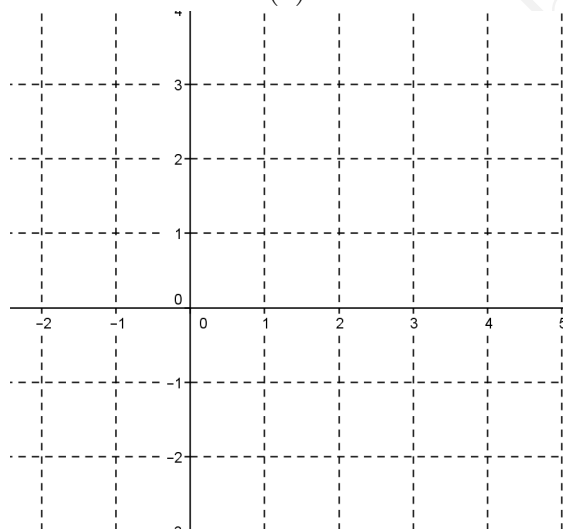


3.7 Graphing $\frac{dy}{dx}$

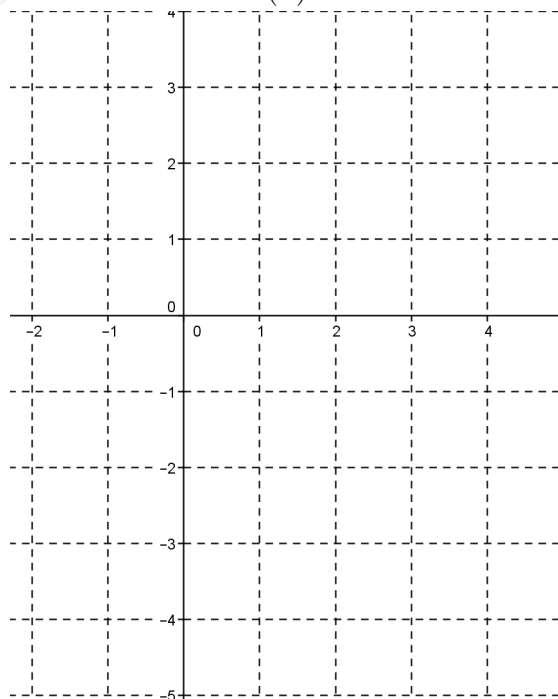
1. Plot the derivative of each of the following functions, on the graph below each function.

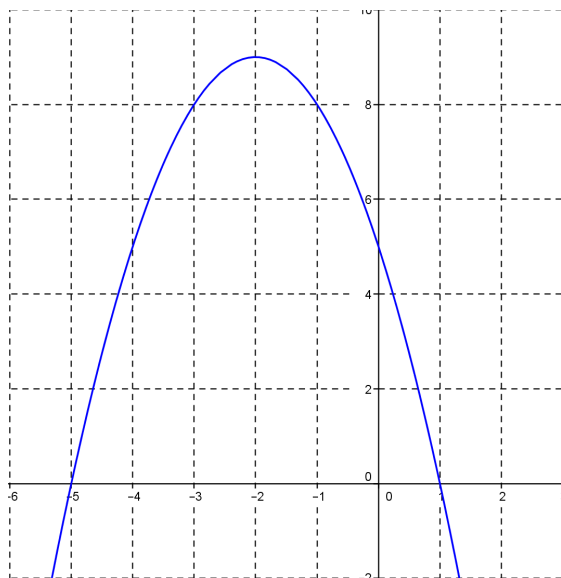


(a)

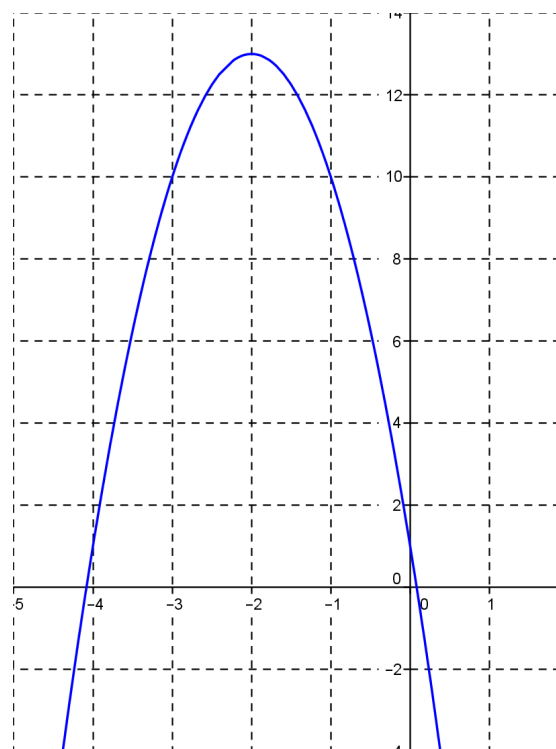


(b)

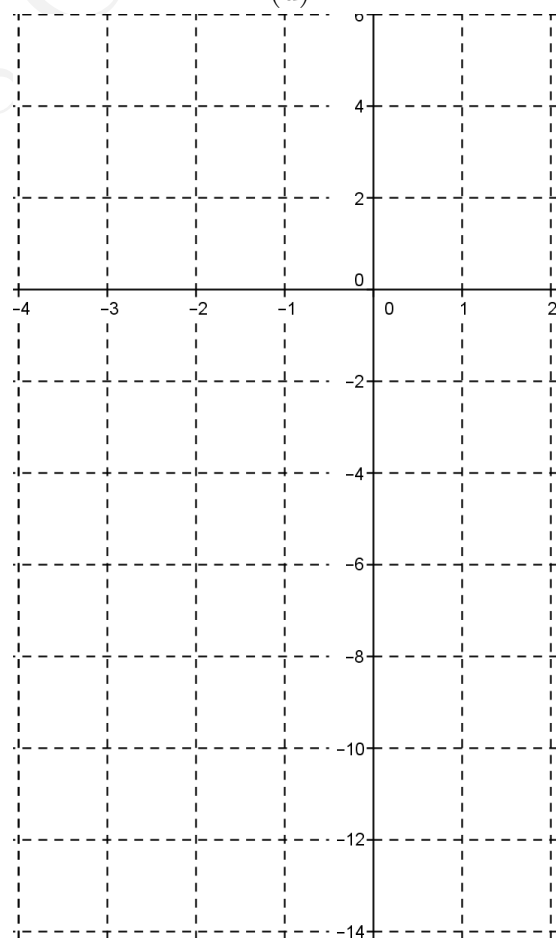


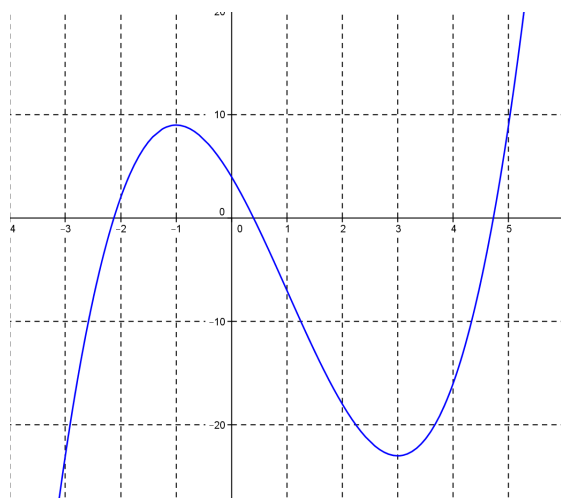


(c)

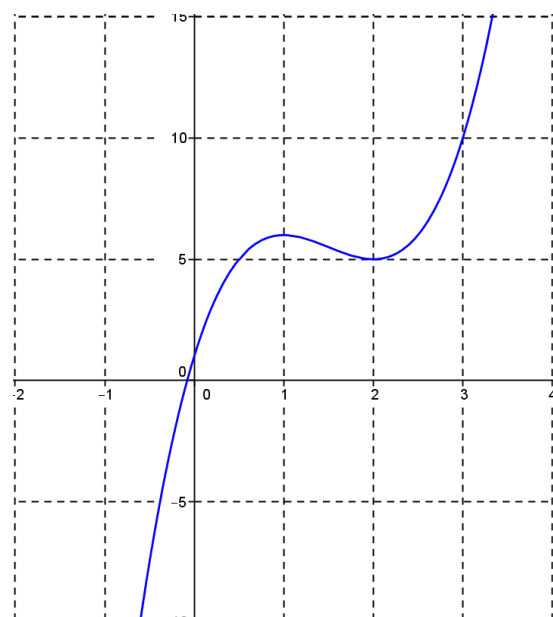
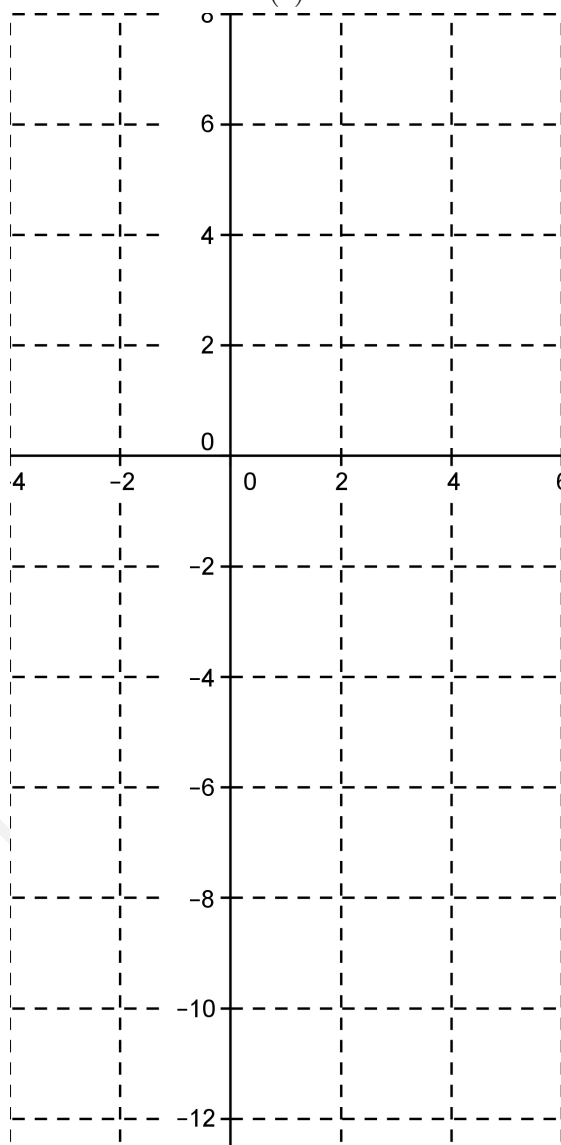


(d)



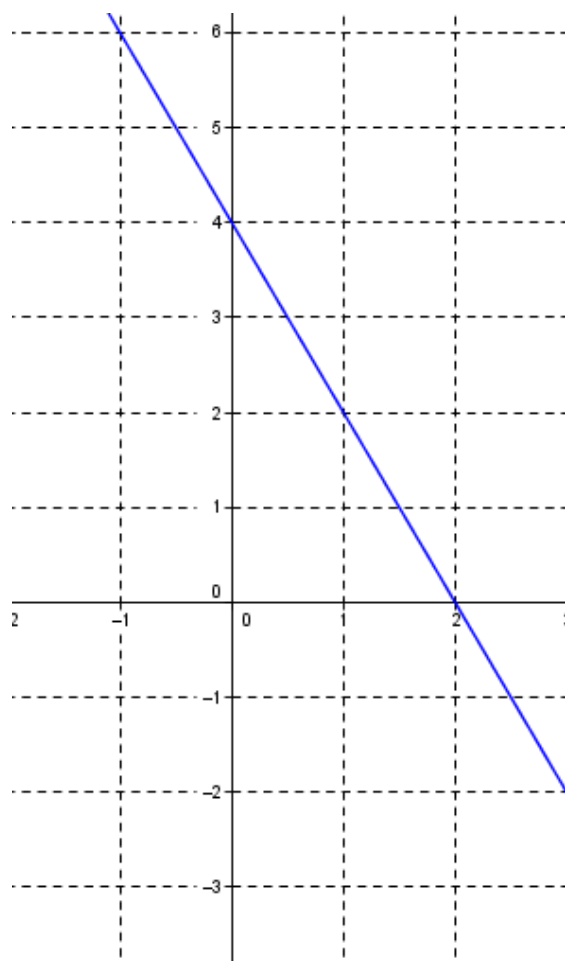


(e)

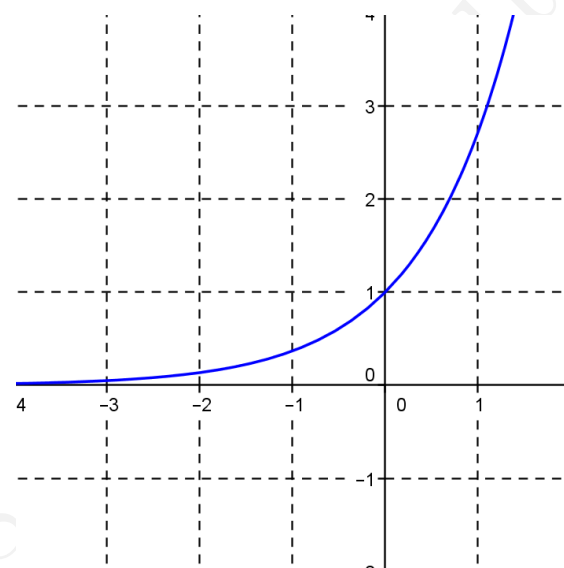


(f)

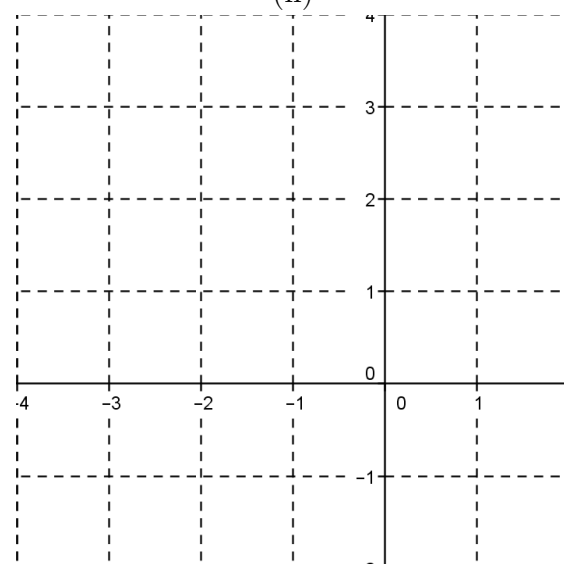
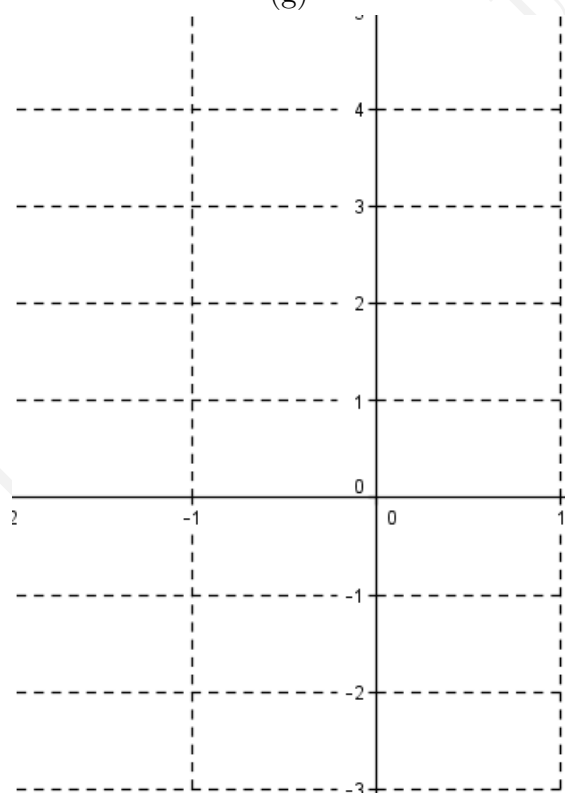


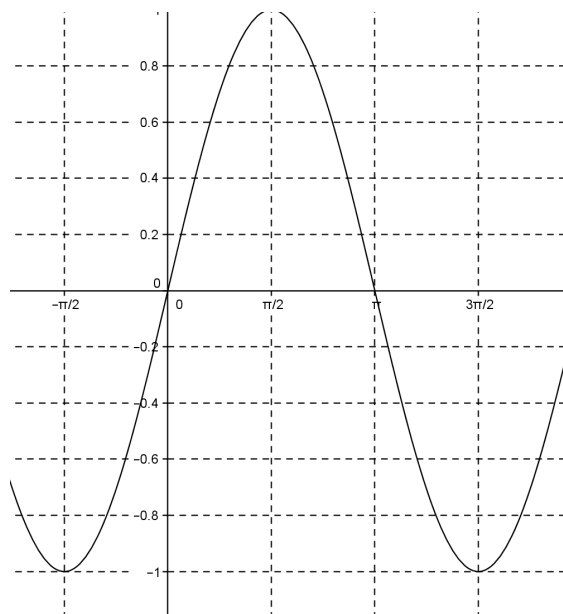


(g)

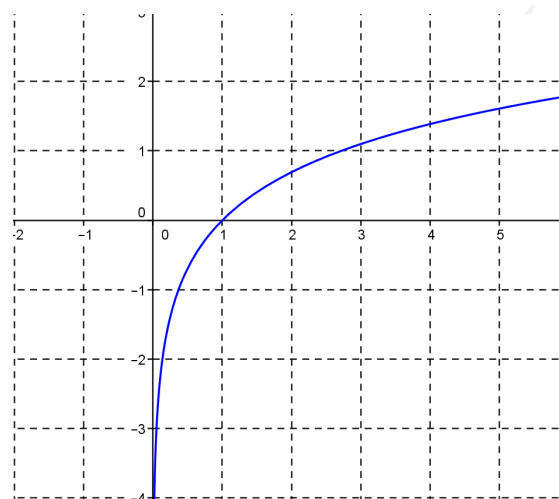
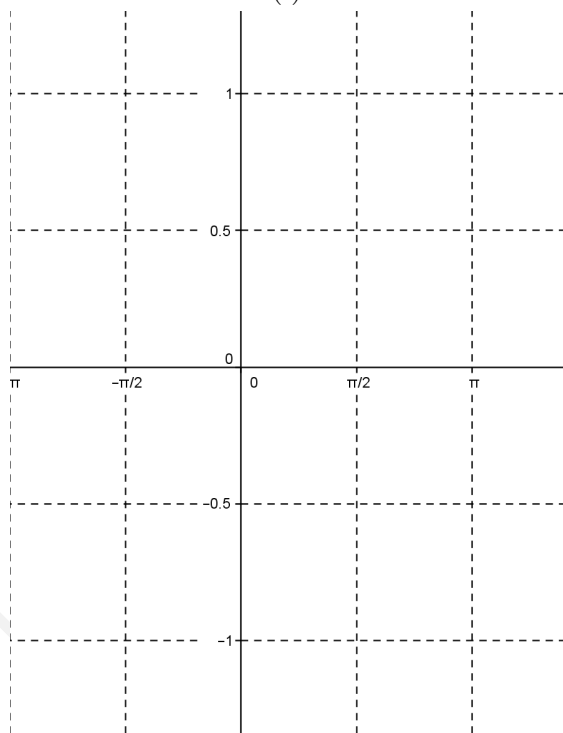


(h)





(i)



(j)

