Complex Numbers Solutions

1 Basic Arithmetic

1. i. $9 + 6i$
   ii. $5 + 4i$
   iii. $25 + 17i$
   iv. $8 + 7i$
2. i. $5 + 2i$
   ii. $1 - 6i$
   iii. $9 + 10i$
   iv. $-4 - 24i$
3. i. $0 + 7i$
   ii. $-4 + i$
   iii. $7 + 10i$
   iv. $3 + i$

2 Modulus and Argand Diagram

1. i. 10
   ii. 13
   iii. 25
   iv. 17
   v. 41
   vi. 5
2. i. 61
   ii. 5
   iii. 37
   iv. 85
   v. 65
   vi. 10

3. ± 5
4. ± 6
5. $\frac{7}{2} + \frac{1}{2}i$
6. $2\sqrt{5}, 4\sqrt{5}, 6\sqrt{5}$; yes.
7. i. $s = \pm 6$
   ii. $t = \pm 4\sqrt{21}$

3 Multiplication

1. i. $-35 + 21i$
   ii. $1 + 3i$
   iii. $14 + 2i$
   iv. $3 + 15i$
   v. $14 - 7i$
   vi. $-8 - 2i$
2. i. $41 + 11i$
   ii. $-18 + 13i$
   iii. $-15 + 16i$
   iv. 13
   v. 25
   vi. $10 + 4i$
3. i. $-12 + 6i$
   ii. 2
   iii. $30 - 5i$
   iv. 8
   v. $9 + 17i$
   vi. $14+21i$
4. i. $-1 + 3i$
   ii. no solution given
   iii. $90°$ anti-clockwise rotation about the origin.
5. i. $-2 - 3i$
   ii. No solution given.
   iii. $90°$ anti-clockwise rotation about the origin.
6. i. $4 + 3i$

ii. No solution given

iii. 90° clockwise rotation about the origin.

7. i. 126-32i

ii. 130

iii. No solution Given

4 Conjugate

1. i. $7 - 5i$

ii. $2 - i$

iii. 14

iv. 4

2. i. $5 - 2i$

ii. $3 + 4i$

iii. $8 + 2i$

iv. $8 - 2i$

v. $8+2i$

3. i. $-1 - 2i$

ii. $2 - 3i$

iii. $1 - 5i$

iv. $1 + 5i$

v. $1 - 5i$

5 Division

1. i. $3 + \frac{3}{2}i$

ii. $3 - 4i$

iii. $6 + 3i$

iv. $2 - i$

v. $\frac{5}{7} + \frac{12}{7}i$

2. i. $3 - i$

ii. $3 - 2i$

iii. $1 + 3i$

iv. $1 - 2i$

v. $2 + i$

6 Quadratic Equations with complex roots

1. $-2 \pm 3i$
2. i. $-3 - 4i$
   ii. $-2 + 4i$
3. No Solution Given
4. $4 \pm i$
5. $k = -6; 3 + 2i$
6. i. $p = 7$
   ii. $0 - 7i$
7. $2 + i, 0 + i$
8. $1 + 2i; 0 + 2i$

7 Polar Form

1. i. $\sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$
   ii. $2(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$
   iii. $6(\cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4})$
   iv. $\sqrt{6}(\cos(-\frac{3\pi}{4}) + i\sin(-\frac{3\pi}{4}))$
   v. $4(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$
   vi. $5(\cos\pi + i\sin\pi)$
   vii. $3(\cos(-\frac{\pi}{2}) + i\sin(-\frac{\pi}{2}))$
   viii. $1(\cos(-\frac{\pi}{3}) + i\sin(-\frac{\pi}{3}))$
2. i. $4(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$
   ii. $1(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$
3. i. $2(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$
   ii. $2\sqrt{3}(\cos(-\frac{5\pi}{6}) + i\sin(-\frac{5\pi}{6}))$
   iii. $\sqrt{2}(\cos(-\frac{3\pi}{4}) + i\sin(-\frac{3\pi}{4}))$
4. $\sqrt{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$
   $\sqrt{2}(\cos(-\frac{\pi}{4}) + i\sin(-\frac{\pi}{4}))$

8 de Moivre's Theorem

1. i. $-4 + 0i$
   ii. $-8 + 0i$
   iii. $-64 + 0i$
2. i. $1728 + 0i$
   ii. 4096
3. $\cos(-\frac{\pi}{6}) + i\sin(-\frac{\pi}{6}); -1$
4. i. $4096 + 0i$
   ii. $256 + 0i$
   iii. $\frac{1}{2} + \frac{\sqrt{3}}{2}i$
   iv. $-\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{2}}i$
   v. $1 + 0i$
   vi. $-1 + 0i$
   vii. $-8192 + 8192i$
   viii. $64 - 64i$

9 Finding the n^{th} root

1. $2, -1 + \sqrt{3}i, -1 - \sqrt{3}i$
2. $2, 1 + \sqrt{3}i, 1 - \sqrt{3}i$
3. $4(\cos\frac{\pi}{3} + isin\frac{\pi}{3}); \sqrt{3} + i, -\sqrt{3} - i$
4. i. $(\cos 2n\pi + i\sin 2n\pi), 1, -\frac{1}{2} + \frac{\sqrt{3}}{2}i, -\frac{1}{2} - \frac{\sqrt{3}}{2}i$
5. $-3i, \frac{3\sqrt{3}}{2} + \frac{3}{2}i, -\frac{3\sqrt{3}}{2} + \frac{3}{2}i$
6. i. $\frac{\sqrt{3}}{\sqrt{2}} + \frac{1}{2}i, -\frac{\sqrt{3}}{\sqrt{2}} - \frac{1}{2}i$
   ii. $\sqrt{3} - i, -\sqrt{3} + i$
   iii. $\sqrt{2} + \sqrt{2}i, -\sqrt{2} - \sqrt{2}i$
7. $\cos(\frac{2n\pi}{5}) + isin(\frac{2n\pi}{5}), n \in 0, 1, 2, 3, 4$. Proof.