



4 Probability

4.1 Permutations

1. How many ways are there of arranging the letters of the word LINEAR:
 - (a) If there are no restrictions
 - (b) If they must begin with N
 - (c) If they must begin with R and end with L
 - (d) If they must begin with a vowel
 - (e) If they must not begin with a vowel
2. How many four digit numbers can be made from the digits 2,3,5 and 8 (without repetition)?
 - (a) How many of these are even?
 - (b) How many are odd?
 - (c) How many are over 5,000?
3. How many ways are there of arranging the letters of the word IRELAND:
 - (a) If there are no restrictions
 - (b) If they must begin with a vowel
 - (c) If they must begin with a consonant
 - (d) If the vowels must be together
 - (e) if the vowels must be apart
4. How many natural numbers can be made using some or all of the digits 1, 2 and 3, with no repetitions?
5. A password for a safe consists of three different letters followed by two different digits.
 - (a) How many different passwords are possible?
 - (b) A person cannot remember his password. He remembers that the first letter is S and the second digit is 9. How many different passwords are possible now, given the above information.
6. Three men and four women line up for a photograph. How many different arrangements are possible:
 - (a) If there are no restrictions
 - (b) If no two women can stand beside each other.





7. A five digit number is to be formed from the digits 0, 1, 2, 3, ..., 9, where no digit is repeated in the number.

How many of these numbers

- (a) are even?
- (b) are greater than 60,000?
- (c) are divisible by 5?
- (d) are greater than 60,000 and divisible by 5?

4.2 Combinations

1. An exam consists of 5 questions.
 - i. In how many ways can 3 questions be selected?
 - ii. If question 1 is compulsory, in how many ways can 3 questions then be selected?
2. A bag contains 10 marbles, each of a different colour. Blue, green, red, black, white are some of the colours in the bag. In how many ways can 4 marbles be pick if:
 - i. if there are no restrictions?
 - ii. if black and white are always included?
 - iii. if green is always included
 - iv. if black and white are always included but red is excluded?
3. Two marbles are withdrawn from a bag of sixteen marbles. How many possible combinations are there?

The bag contains five red, seven green and four white marbles. Two marbles are withdrawn, without replacement.

How many possible combinations will have:

 - i. Two red marbles?
 - ii. Two red or two green marbles?
 - iii. Two marble of the same colour.
 - iv. Two marbles of different colour.
 - v. One red marble and one marble that is not red.
4. John has to choose four subjects to study for his Leaving Cert, from a list of Business, Accounting, Construction Studies, Engineering, French, Spanish, German, Art, Geography, History, Physics, Biology or Chemistry.
 - i. How many combinations are possible?
 - ii. John has decided his four choices will include one Science subject and one Language. How many possible combinations fit this criteria?
5. A management team of 8 persons is to be chosen from 6 men and 7 women. In how many ways can this be done;
 - i. when there are 5 men on each management team?





- ii. when there is a majority of men on each management team?
6. There are 15 students, including Jack and Jane, in a class. How many class committees of 5 students can be selected;
- i. if all students are eligible for the committee?
 - ii. if Jack and Jane are to be included on the committee?
 - iii. if Jack will be excluded from the committee?
 - iv. if Jack and Jane will not work together on the same committee?
7. In how many ways can a management team of 8 be selected from 5 men and 6 women if each management team consists of;
- i. An equal number of men and women?
 - ii. At least 4 men?
 - iii. At least 4 women?
 - iv. if there is more women than men on the committee?
8. A football squad has 3 goalkeepers, 7 defenders, 6 midfielders and 5 strikers. How many matchday selections are possible, if each team will be in a 4-4-2 format? (4-4-2 format means: 1 goalkeeper, 4 defenders, 4 midfielders and 2 strikers on each team.)

4.3 The Addition Rule

1. An unbiased six-sided die is thrown. Find the probability that the number obtained is:
- i. Even?
 - ii. Greater than 3?
 - iii. Even or greater than 3?
2. A number is selected at random from the integers 1 to 50 inclusive. Find the probability that the number is:
- i. a multiple of 4?
 - ii. a multiple of 5?
 - iii. a multiple of 4 or a multiple of 5?
3. Two unbiased dice are rolled. What is the probability of getting:
- i. a total which is odd?
 - ii. a total greater than 9?
 - iii. a total which is odd or greater than 9?
4. A card is selected at random from a pack of 52. Find the probability that the card drawn is:
- i. a Spade?
 - ii. an Ace?





- iii. a Spade or an Ace?
5. A card is selected at random from a pack of 52. Find the probability that the card is:
- i. a Heart or a Club?
 - ii. a King or a red card?
 - iii. a Spade or a black King?
 - iv. Are the events of getting a King and getting a red card mutually exclusive?
6. Two unbiased dice are rolled. What is the probability of getting:
- i. a total of three or a total of eight?
 - ii. a total less than six or a total which is prime?

4.4 Tree Diagrams

1. When a basketball player takes a free throw, the probability that he will score is $\frac{9}{10}$ each time. In a match, he takes two free throws. What is the probability that he will score exactly once?
2. 60% of smartphones in a store are Android phones. 40% are i-phones. 25% of the Android phones are waterproof, and 15% of the i-phones are waterproof. If you buy a phone in the store at random, what is the probability that it is waterproof?
3. A UFC fighter fights at two weights, lightweight and welterweight. $\frac{3}{4}$ of his fights are at lightweight, the rest at welterweight. In a lightweight fight, he knocks out his opponent 60% of the time, at welterweight he knocks out his opponent 40% of the time. If he takes on a fight at random (either lightweight or welterweight opponent), what is the probability he will knock out his opponent?
4. Three contestants, Grace, Ciara and Andrew, take on the Ninja Warrior obstacle course. The probability that each will complete the course are 0.75, 0.65 and 0.55 respectively. Find the probability that :
 - i. Grace completes the course, but Andrew and Ciara don't.
 - ii. They all complete the obstacle course.
 - iii. At least two complete the obstacle course.
5. Adam is an out-half on a rugby team. If a day is calm, he will score his first penalty 80% of the time. If the day is windy, his success rate on his first penalty drops to 50%. In Ireland, 30% of all days are windy.
In a rugby match on a random day in Ireland, what is the probability that Adam scores his first penalty?
6. When serving a point in tennis, a player can either serve an Ace (where the opponent fails to return the shot), serve a shot that was returned, or serve a fault (where the serve goes 'out'). If a player serves a fault on the first serve, they get a 2nd serve. If they fault again on the 2nd serve, they lose the point with a double fault.





Ciara plays tennis, and she gets an Ace on 40% of her first serves and 15% of her second serves, against a particular opponent. She gets a fault on 20% of her first serves and on 5% of her second serves.

- i. What is the probability that Ciara wins the point by an Ace on the 2nd serve?
 - ii. What is the probability of Ciara getting a double fault?
7. Niamh gets the bus to school everyday from Monday to Friday. The probability that the bus is late on Monday is 0.3 and the probability that the bus is late on any day from Tuesday to Friday is 0.15.
 - i. A day is chosen at random. Find the probability that
 - ii. It is Monday and the bus is not late.
 - iii. The bus is late.
 - iv. Given that the bus is late, what is the probability that it is not a Monday?
8. Maedhbh walks, cycles or drives to school with probabilities 0.6, 0.3 and 0.1, respectively. If she walks, she has a probability of 0.3 of being late. If she cycles, the probability of being late is 0.05. If she drives, the probability of being late is 0.45. Find the probability that:
 - i. She will be late on a particular day.
 - ii. She walked, given that she was late.
 - iii. She drove, given that she was not late.

4.5 Combinatorics

1. A box contains 4 red discs and three blue discs. Two discs are taken out, without replacement. What is the probability that one is red and the other is blue?
2. A bag contains seven red marbles and three blue marbles. Sam takes three marbles out.
What is the probability that:
 - i. All three are red.
 - ii. One is red and two are blue.
3. A bag contains six red sweets, three green sweets and five blue sweets. Aaron takes out three sweets.
Find the probability of getting:
 - i. Two green sweets and one red sweet.
 - ii. Three blue sweets.
 - iii. All three sweets being the same colour.
 - iv. All three sweets being different colours.
 - v. At least two sweets are the same colour.
4. For a lottery, 35 cards numbered 1 to 35 are placed in a drum. Five cards will be chosen at random from the drum as the winning combination.



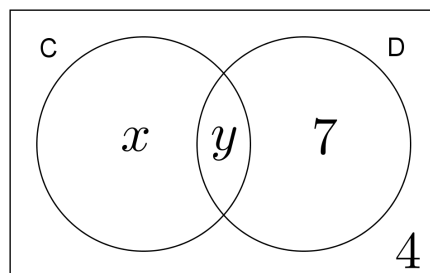
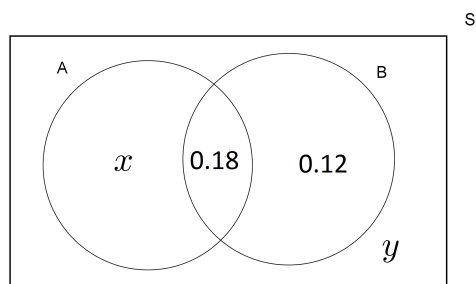


- i. How many different combinations are possible?
 - ii. How many of all the possible combinations will match exactly four numbers with the winning combination?
 - iii. How many of all the possible combinations will match exactly three numbers with the winning combination?
 - iv. Show that the probability of matching at least three numbers with the winning combination is approximately 0.014.
5. A team of four students is to be chosen at random to represent their school in a table quiz. The team is to be chosen from the ten students with top exam results, which include 6 girls and 4 boys. Find the probability that the team consists of:
 - i. Only girls.
 - ii. Only boys
 - iii. More girls than boys.
6. Ten balls, numbered 1 to 10 are placed in a drum. Two balls are taken out. Find the probability that :
 - i. Both are prime numbers (1 is not a prime number.)
 - ii. Both are even or both are odd.
 - iii. One is prime but the other is not.
7. A deck contains ten cards, each marked with a different whole number from 11 to 20. Three of the cards are drawn at random from the deck. Find the probability that:
 - i. A card with the number 15 is drawn?
 - ii. The three cards are odd?
 - iii. The product of the numbers on the three cards is odd?

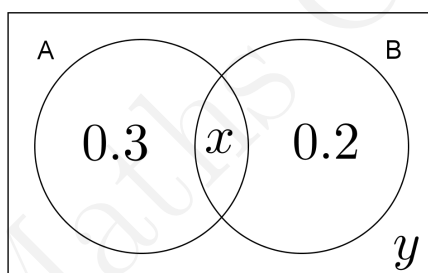
4.6 Independent Events

1. E and F are independent events, such that $P(E) = \frac{1}{3}$ and $P(F) = \frac{3}{8}$. Find:
 - i. $P(E \cap F)$
 - ii. $P(E \cup F)$
2. A and B are independent events. Find the value of :
 - i. x
 - ii. y
3. The Venn diagram shows the number of elements in each subset. An element is picked at random from S. If $P(C) = \frac{9}{20}$ and $P(D) = \frac{1}{2}$:
 - i. Find the values of x and y
 - ii. Investigate if C and D are independent.
 - iii. Find $P(C|D)$ and $P(D|C)$



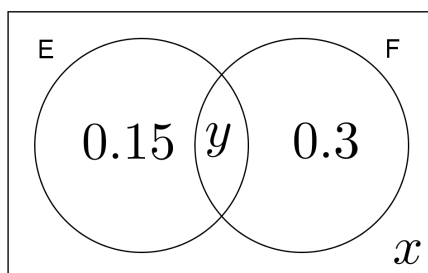


4. A and B are independent events. Using the information in the venn diagram, find the two sets of values for x and y .





5. (a) If $P(E|F) = \frac{1}{4}$, find the values of x and y .
 (b) Investigate if E and F are independent.



4.7 Conditional Probability

- Two events, A and B , are such that $P(A \cap B) = 0.5$ and $P(B) = 0.75$. Find $P(A|B)$.
- If $P(A) = \frac{1}{2}$ and $P(B) = \frac{5}{12}$ and $P(A \cup B) = \frac{3}{4}$.
Find $P(A|B)$ and $P(B|A)$
- In an all boys school, 40% of the students play rugby. 15% of the students play both rugby and hurling. If a student from the school is selected at random, what is the probability that he plays hurling, given that he plays rugby.
- In a class of 30 students, 19 of the students study French. 15 of the students study Business and 3 of the students study neither French nor Business.
 - Using a Venn Diagram, find how many of the students study both French and Business.
 - What is the probability that a student studies either French or Business?
 - What is the probability that a student studies Business, given that they study French?
- $P(E) = \frac{7}{12}$ $P(F) = \frac{1}{2}$ $P(E \cup F) = \frac{7}{8}$
Find:
 - $P(E \cap F)$
 - $P(E|F)$
 - $P(E'|F)$
 - $P(E|F')$
 - $P(F|E')$
- A and B are two events such that $P(A) = 0.48$, $P(B) = 0.36$, and $P(A|B) = 0.5$
Find:
 - $P(A \cap B)$
 - $P(A \cup B)$
 - $P(B|A)$





7. If $P(A) = \frac{1}{2}$, $P(A|B) = \frac{2}{5}$ and $P(A \cup B) = \frac{3}{4}$, find $P(A \cap B)$ and $P(B)$.
8. In a school, 55% of the students are girls and 45% are boys. 35% of the girls play sports and 60% of the boys play sports. A student is picked random, what is the probability that this student is a girl, given that they play sports?
9. In a certain county in Ireland, 60% of people are aged 35 years or more, and 40% of people are under 35 years old. In the General election, 34% of the 35 and over group voted for Fine Gael, 30% of the 35 and over group voted for Fianna Fail, and the rest of the 35 and over group voted for another party. Regarding the under 35 group, 42% of them voted for Fine Gael, 26% for Fianna Fail and the rest voted for another party. If a person is selected at random, what is the probability that person is over 35, given that they voted for Fine Gael.
10. You meet a man who tells you he has 3 children. He also tells you that two of his children are boys. What is the probability that the other child is a girl?

4.8 Expected Values

1. A €5, €10, €20, €50, €100 and €500 note are placed into a hat. You get to pick one note out at random as a prize. What is the expected value of your prize?
2. You play a game which involves rolling two dice and adding the scores. If the scores total to 4 or lower, or, if the scores total to 10 or higher, you will win €15. If the scores total to any other value you win €7.50. Is €10 a fair price to play the game?
3. In basketball, scoring a basket in play is worth either two points or three points, depending on how far from the basket the shot was taken. A certain basketball team shoots two point shots with 47% accuracy and they shoot three point shots with 36% accuracy.
 - i. Is the team better off shooting more two point or three point shots?
 - ii. A free throw is worth one point. The team shoots free throws with 80% accuracy. In a certain game, the team shoots 44 two point shots, 23 three point shots, and 20 free throws. What is their expected score in the game?
4. A pair of fair six-sided dice is rolled and the total on the two dice is noted.

- i. Copy and complete the following probability distribution table:

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability				$\frac{4}{36}$					$\frac{2}{36}$		

- ii. Calculate the expected value.
5. The members of a local minor football team were asked their ages. The probability distribution of their ages is outlined in the table below:

Age	15	16	17	18
$P(X=\text{Age})$	x	0.15	0.3	y

Given that the expected age of a randomly selected member of the team is 17.25, find the value of x and y .





6. Every student in a class of 35 were asked how many people were in their family (including parents). The results are displayed in the table below.

Number of family members	2	3	4	5	6	7
Frequency	1	4	9	12	6	3

- Create a probability distribution table for the variable X , where X is the number of people in the family of a randomly selected student.
 - If a student is selected at random, what is the expected number of people in his/her family?
7. The table below gives motor insurance information for fully licensed, 17-20 year old drivers in Ireland in 2007. All drivers who had their own insurance policy are included.

	Number of drivers	Number of claims	Average cost per claim
Male	9634	977	€6108
Female	6743	581	€6051

Questions (i) to (v) refer to drivers in the table above only.

- What is the probability that a randomly selected male driver made a claim during the year? Give your answer correct to three decimal places
- What is the probability that a randomly selected female driver made a claim during the year? Give your answer correct to three decimal places.
- What is the expected value of the cost of claims on a male driver's policy?
- What is the expected value of the cost of claims on a female driver's policy?
- The male drivers were paying an average of €1688 for insurance in 2007, and the female drivers were paying an average of €1024. Calculate the average surplus for each group, and comment your answer. (Note: the surplus is the amount paid for the policy minus the expected cost of claims)
- A 40-year-old female driver with a full license has a probability of 0.07 of making a claim during the year. The average cost of such claims is €3900. How much should a company charge such drivers for insurance in order to show a surplus of €175 per policy?

(Leaving Cert. Project Maths Paper 2, 2010)

4.9 Binomial Distribution

- A dice is rolled three times. What is the probability of obtaining the following?
 - Exactly one 6.
 - Exactly two 6's.
 - Exactly three 6's.
 - At least one 6.
- Darragh is late for school 30% of days. In one week of five consecutive schooldays, find the probability that Darragh is late for school:
 - No times.
 - Only one time.





- iii. More than one time.
3. Jake plays FIFA on the X-box. In an online game of FIFA against an opponent, the probability that Jake wins is 65%. He plays six games against this opponent, where no draws are allowed. What is the probability that Jake will win:
- The first, second and fifth game?
 - Exactly three games?
4. 25% of all Leaving Cert students sit higher level maths. If you select a group of 5 Leaving Cert students at random, what is the probability that:
- All five study higher level maths?
 - The majority of the group study higher level maths?
5. Jane is a busy woman, who usually can only answer her phone 20% of the time. One afternoon, her phone is called five times. What is the probability that:
- She answers the first call but misses all of the rest?
 - She misses all the calls?
 - She answers exactly one of the calls?
 - She answers at least two of the calls?
6. A multiple choice test has 10 questions. Each question has four choices. Rebecca didn't study for the test so she guesses every answer. What is the probability she gets:
- All questions wrong?
 - Exactly four questions correct?
 - At least two questions correct?
7. According to an extensive survey, it was revealed that 15% of teenagers smoke. If you take a sample of six teenagers, what is the probability that:
- None of them smoke?
 - At least one of them smokes?
8. A fair coin is flipped n times. Show that the probability of getting exactly two heads is $\frac{n^2 - n}{2^{n+1}}$.
9. Michael is a very good hurler. Over the course of the season it was found that his scoring accuracy from the 65 yard line is 85%. Michael is practicing his 65 yard shots at training. He must score 10 times before he can finish. Find the probability that this takes him 15 shots.
10. A fair coin is flipped repeatedly until exactly three heads appear. Find the probability that this will take exactly five flips.
11. Aebfhinn is practicing her hockey penalties, which she usually scores with 90% accuracy. At the end of training she decides she will take penalty flicks until she scores 5 penalties. Find the probability that she scores her fifth penalty;





- i. On her fifth shot?
 - ii. On her 8th shot?
12. An unbiased dice is rolled repeatedly until 6 appears for the third time. Find the probability that this takes 7 rolls of the dice.
13. 30% of students in sixth year in your school have a part time job. During lunch hour you ask random people in sixth year in your school if they have a part time job. This process is repeated until three people with part time jobs are found. Find the probability that this will take exactly 10 people.
14. Matthew, a basketball player, shoots free throws with 70% accuracy. How many free throws must he shoot to ensure that there is a greater than 95% chance that he scored at least once?
15. What is the minimum number of times an unbiased dice must be rolled in order to ensure that the probability of getting at least one 6 is greater than 99%?

4.10 Applications of Normal Distribution

1. The average height of a man in Ireland is 177 cm, with a standard deviation of 8 cm. Assume that heights are normally distributed. An Irish man is selected at random. What is the probability that his height is:
 - i. Over 185 cm?
 - ii. Less than 180 cm?
 - iii. Between 180 and 185 cm?
2. Leaving Certificate points are normally distributed with a mean 375 and standard deviation 90. If 595 points are required to be eligible to study medicine in college, what percentage of Leaving Cert students will be eligible for medicine, based on their points?
3. The average life of a timing belt in the engine of a certain car is 80,000 miles, with a standard deviation of 12,000 miles. If a timing belt is selected and tested, find the probability that it will have the following lifetime. Assume the variable is normally distributed.
 - i. Between 68,000 and 80,000 miles.
 - ii. Between 80,000 and 88,000 miles.
 - iii. Between 68,000 and 88,000 miles.
4. The journey time for a bus trip from Limerick to Dublin is normally distributed with an average of 185 minutes, and a standard deviation of 10 minutes. If one of these trips is selected at random, find the probability that the bus journey will take:
 - i. At least 170 minutes.
 - ii. At least 3 hours and 15 minutes.





5. The average annual rainfall in Ireland is normally distributed with a mean of 1,150 mm and a standard deviation of 250 mm. What is the probability that next year there will be:
 - i. Less than 1,000 mm of rainfall in Ireland?
 - ii. More than 1,400 mm of rainfall in Ireland?
 - iii. Between 1,000 and 1,400 mm of rainfall in Ireland?
6. A round of golf burns an average of 1,500 calories. If the standard deviation of the distribution is 120 calories, find the probability that a person who plays one round of golf will burn the following calories. Assume the variable is normally distributed.
 - i. More than 1,350 calories?
 - ii. Less than 1,600 calories?
 - iii. Between 1,400 and 1,650 calories?
7. In July, the average temperature of Lough Derg is 15.3° and the standard deviation is 1.4° . Assume the variable is normally distributed. For a randomly selected day in July, find the probability that the temperature will be as follows:
 - i. Above 15°
 - ii. Below 18.5°
 - iii. Between 16° and 18°
8. If Leaving Cert points are normally distributed with a mean of 375 points and a standard deviation of 90 points, how many points does a student need to be in the top 10%?
9. Any student who scores in the bottom 15% of a particular reading test is eligible for assistance in their final exams. If scores on the reading test are normally distributed with an average of 68, with a standard deviation of 10, below what score will a student be eligible for assistance?
10. The average weight of an Irish man is 77 kg, with a standard deviation of 14 kg. For a health study, a volunteer must have a weight that is in the middle 50%. If the variable is normally distributed, between what weights should a man be in order to enable him to volunteer to participate in the study?
11. The average price house in Co. Limerick is €145,269, with a standard deviation of €2520. Find the maximum and minimum prices of the houses a developer will build to include the middle 60% of the market. Assume that the variable is normally distributed.
12. A 10 km road race has an average finishing time of 65.25 minutes with a standard deviation of 5.6 minutes. Any athlete who finishes in the top 20% of times will win a free pair of trainers from the race sponsor. Assuming the times are normally distributed, what finishing time is required for an athlete to win a free pair of trainers?





4.11 Exam Questions

1. 2015 Paper 2

An experiment consists of throwing two fair, standard, six-sided dice and noting the sum of the two numbers thrown. If the sum is 9 or greater it is recorded as a win (W). If the sum is 8 or less it is recorded as a loss (L).

- (a) Complete the table below to show all possible outcomes of the experiment.

		Die 2					
		1	2	3	4	5	6
Die 1	1		L				
	2						
	3						
	4						
	5						W
	6						

- (b) i. Find the probability of a win on one throw of the two dice.
 ii. Find the probability that each of 3 successive throws of the dice results in a loss. Give your answer correct to four decimal places.
- (c) The experiment is repeated until a total of 3 wins occur. Find the probability that the third win occurs on the tenth throw of the two dice. Give your answer correct to four decimal places.

2. 2015 Paper 2

In basketball, players often have to take free throws. When Michael takes his first free throw in any game, the probability that he is successful is 0.7.

For all subsequent free throws in the game, the probability that he is successful is:

- 0.8 if he has been successful on the previous throw.
- 0.6 if he has been unsuccessful on the previous throw.

- (a) Find the probability that Michael is successful (S) with all three of his first three free throws in a game.

$$P(S, S, S) =$$

- (b) Find the probability that Michael is unsuccessful (U) with his first two free throws and successful with the third.

$$P(U, U, S) =$$

- (c) List all the ways that Michael could be successful with his third free throw in a game and hence find the probability that Michael is successful with his third free throw.

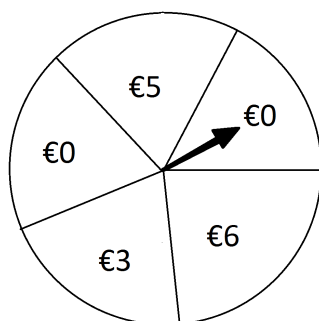
- (d) i. Let p_n be the probability that Michael is successful with his n^{th} free throw in the game (and hence $(1 - p_n)$ is the probability that Michael is unsuccessful with his n^{th} free throw).

$$\text{Show that } p_{n+1} = 0.6 + 0.2p_n.$$

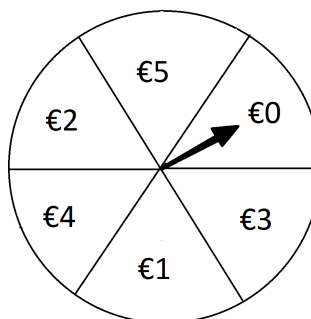




- ii. Assume that p is Michael's success rate in the long run; that is, for large values of n , we have $p_{n+1} \approx p_n \approx p$.
Using the result from part (d)(i) above, or otherwise, show that $p = 0.75$.
- (e) For all positive integers n , let $a_n = p - p_n$, where $p = 0.75$ as above.
- Use the ratio $\frac{a_{n+1}}{a_n}$ to show that a_n is a geometric sequence with common ratio $\frac{1}{5}$.
 - Find the smallest value of n for which $p - p_n < 0.00001$.
- (f) You arrive at a game in which Michael is playing. You know that he has already taken many free throws, but you do not know what pattern of success he has had.
- Based on this knowledge, what is your estimate of the probability that Michael will be successful with his next free throw in the game?
 - Why would it **not** be appropriate to consider Michael's subsequent free throws in the game as a sequence of Bernoulli trials?
3. **2014 Paper 2** Two different games of chance, shown below, can be played at a charity fund-raiser. In each game, the player spins an arrow on a wheel and wins the amount shown on the sector that the arrow stops in. Each game is fair in that the arrow is just as likely to stop in one sector as in any other sector on that wheel.



Game A



Game B

- John played Game A four times and tells us that he has won a total of €8. In how many different ways could he have done this?
- To spin either arrow once, the player pays €3. Which game of chance would you expect to be more successful in raising funds for charity? Give reason for your answer.
- Mary plays Game B six times. Find the probability that the arrow stops in the €4 sector exactly twice.

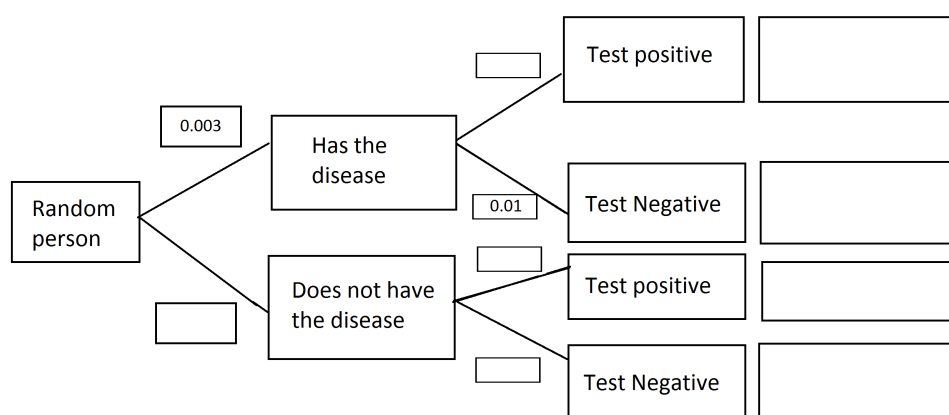




4. **2014 Paper 2** Blood tests are sometimes used to indicate if a person has a particular disease. Sometimes such tests give an incorrect result, either indicating the person has the disease when they do not (called a false positive) or indicating that they do not have the disease when they do (called a false negative).

It is estimated that 0.3% of a large population have a particular disease. A test developed to detect the disease gives a false positive in 4% of tests and a false negative in 1% of tests. A person picked at random is tested for the disease.

- (a) i. Write the probability associated with each branch of the tree diagram in the blank boxes provided.



- ii. Hence, or otherwise, calculate the probability that a person selected at random from the population tests positive for the disease.
- iii. A person tests positive for the disease. What is the probability that the person actually has the disease? Give your answer correct to three significant figures.
- iv. The health authority is considering using a test on the general population with a view to treatment of the disease. Based on your results, do you think that the above test would be an effective way to do this? Give a reason for your answer.
- (b) A generic drug used to treat a particular condition has a success rate of 51%. A company is developing two drugs, A and B, to treat the condition. They carried out clinical trials on two groups of 500 patients suffering from the condition. The results showed that Drug A was successful in the case of 296 patients. The company claims that Drug A is more successful in treating the condition than the generic drug.
- i. Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to justify the company's claim. State the null hypothesis and state your conclusion clearly.

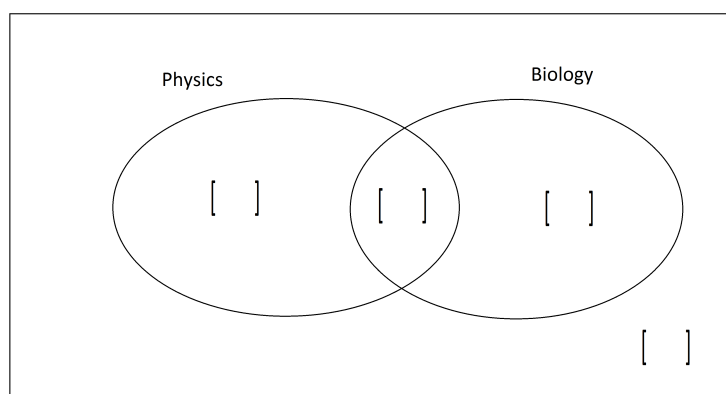




- ii. The null hypothesis was accepted for Drug B. estimate the greatest number of patients in the trial who could have been successfully treated with Drug B.

5. 2013 Paper 2

- (a) Explain each of the following terms:
- Sample space.
 - Mutually exclusive events.
 - Independent events.
- (b) In a class of 30 students, 20 study physics, 6 study biology and 4 study both Physics and Biology.
- represent the information on the Venn Diagram.



A student is selected at random from this class. The events E and F are:

E: The student studies Physics

F: The student studies Biology.

- ii. By calculating probabilities, investigate if the events E and F are independent.

6. 2013 Paper 2

- (a) A random variable X follows a normal distribution with mean 60 and standard deviation 5.
- Find $P(X \leq 68)$
 - Find $P(52 \leq X \leq 68)$.
- (b) The heights of a certain type of plant, when ready to harvest, are known to be normally distributed, with a mean of μ . A company tests the effects of three different growth hormones on this type of plant. The three hormones were used on a different large sample of the crop. After applying each hormone,

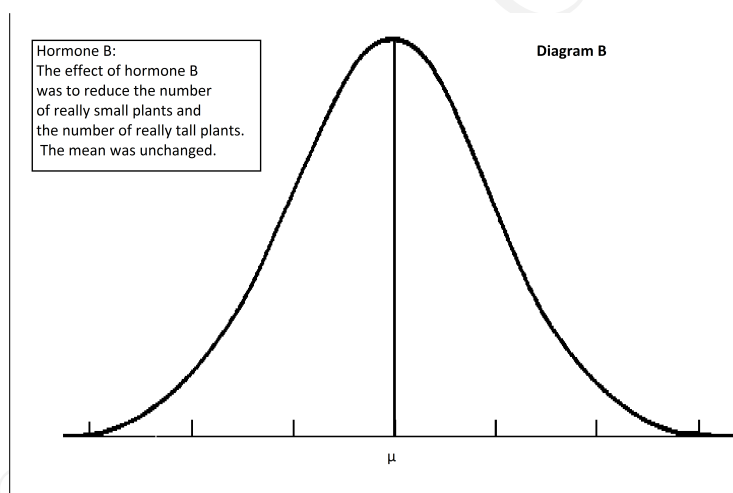
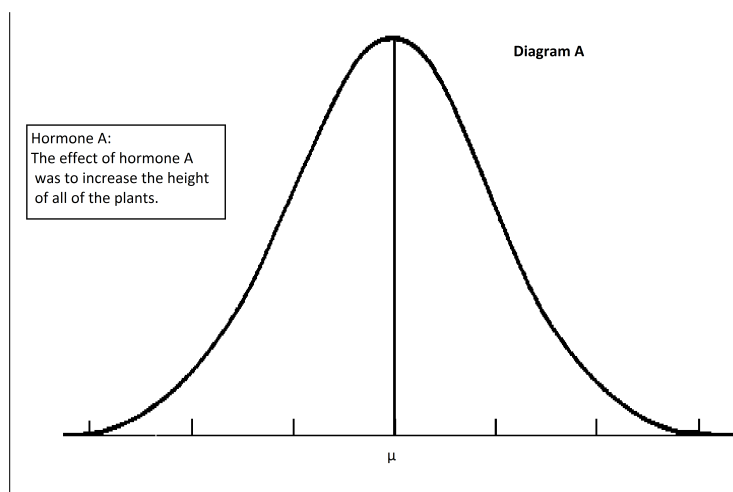


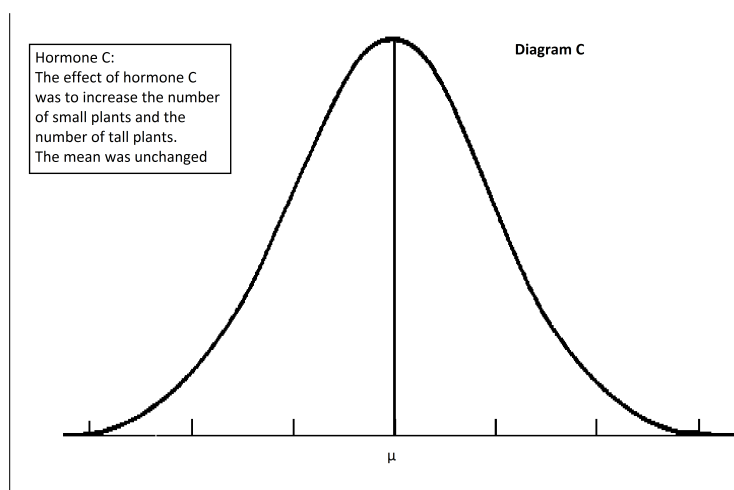


it was found that the heights of the plants in the samples were still normally distributed at harvest time.

The diagrams A,B and C show the expected distribution of the heights of the plants, at harvest time, without the use of the hormones.

The effect, on plant growth, of each of the hormones is described beside each diagram. Sketch, on each diagram, a new distribution to show the effect of the hormones.





7. 2013 Paper 2

Go Fast Airlines provides internal flights in Ireland, short haul flights to Europe and long haul flights to America and Asia. On long haul flights the company sells economy class, business class and executive class tickets. All passengers have a baggage allowance of 20 kg and must pay a cost per kg for any weight over the 20 kg allowance. Each month the company carries out a survey among 1000 passengers. Some of the results of the survey for May are shown below.

Gender	Male: 479	Female: 521
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Previously flown with <i>Go Fast Airlines</i>	Yes: 682	No: 318
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Would fly again with <i>Go Fast Airlines</i>	Yes: 913	No: 87
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Passenger Age	Mean age: 42	Median age: 31
Spend on in-flight facilities	Mean spend: €18.65	Median spend: €32.18

Was flight delayed?	Yes	No	Don't know
	231	748	21

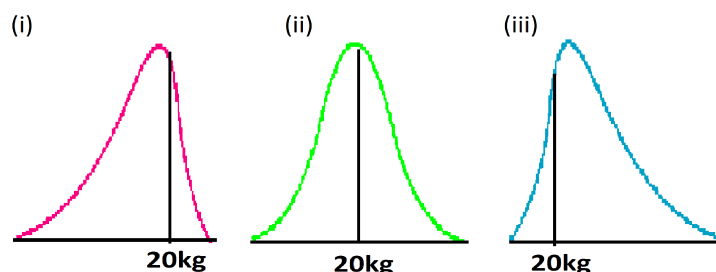
Passenger satisfaction with overall service	Satisfied	Not satisfied	Don't know
	664	238	98

- (a) *Go Fast Airlines* used a **stratified random sample** to conduct the survey.
- Explain what is meant by a **stratified random sample**.
 - Write down 4 different passenger groups that the company might have included in their sample.
- (b)
- What is the probability that a passenger selected at random from this sample
 - had his/her flight delayed Answer:
 - was not satisfied with the overall service Answer:
 - An employee suggest that the probability of selecting a passenger whose flight was delayed and who was not satisfied with the overall service should be equal to the product of the two probabilities in (i) above. Do you agree with the employee?





- (c) Which of the graphs below do you think is most likely to represent the distribution of the weight of passengers baggage?



Give a reason for your answer.

- (d) i. Draw a sketch of the possible distribution of the ages of the passengers based on the data in the survey.
ii. Explain your answer
- (e) i. The company repeatedly asserts that 70% of their customers are satisfied with their overall service. Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to conclude that their claim is valid in May. Write the null hypothesis and state your conclusion clearly.
ii. A manager of the airline says: "If we survey 2000 passengers from June on, we will halve the margin of error in our surveys." Is the manager correct? Explain your answer.
- (f) The response of ten individual passengers to the question on age and in-flight spend are given below.

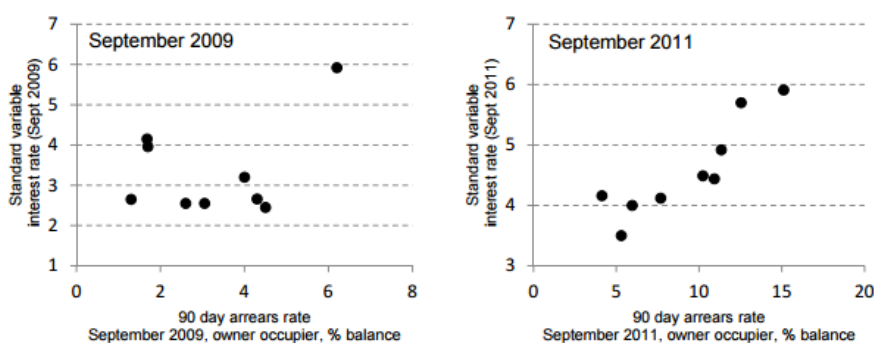
Age(years)	46	29	37	18	25	75	52	35	40	31
In-flight spend (euro)	30	15	20	0	10	45	25	20	20	30

- i. Draw a scatter plot of the data.
ii. Calculate the correlation coefficient between passengers age and in-flight spend.
iii. What can you conclude from the completed scatter plot and the correlation coefficient?
iv. Sketch the line of best fit in the completed scatter plot above.
8. **2012 Paper 2** A certain basketball player scores 60% of the free-throw shots she attempts. During a particular game, she gets six free throws.
- (a) What assumption(s) must be made in order to regard this as a sequence of Bernoulli trials?
- (b) Based on such assumption(s), find, correct to three decimal places, the probability that:





- i. she scores on exactly four of the six shots.
 - ii. she scores for the second time on the fifth shot.
9. **2012 Paper 2** A company produces calculator batteries. The diameter of the batteries is supposed to be 20 mm. The tolerance is 0.25 mm. Any batteries outside this tolerance are rejected. You may assume that this is the only reason for rejecting the batteries.
 - (a) The company has a machine that produces batteries with diameters that are normally distributed with mean 20 mm and standard deviation 0.1 mm. Out of every 10,000 batteries produced by this machine, how many, on average, are rejected?
 - (b) A setting on the machine slips, so that the mean diameter of the batteries increases to 20.05 mm, while the standard deviation remains unchanged. Find the percentage increase in the rejection rate for batteries from this machine.
10. **2012 Paper 2** To buy a home, people usually take out loans called *mortgages*. If one of the repayments is not made in time, the mortgage is said to be *in arrears*. One way of considering how much difficulty the borrowers in a country are having with their mortgages is to look at the percentage of all mortgages that are in arrears for 90 days or more. For the rest of this question, the term *in arrears* means in arrears for 90 days or more.
 The two charts below are from a report about mortgages in Ireland. The charts are intended to illustrate the connection, if any, between the percentages of mortgages that are in arrears and the interest rates being charged for mortgages. Each dot on the chart represents a group of people paying a particular interest rate to a particular lender. The arrears rate is the percentage in arrears.



(Source: Goggin et al. *Variable Mortgage Rate Pricing in Ireland*, Central Bank of Ireland, 2012)

- (a) Paying close attention to the scales on the charts, what can you say about the change from September 2009 to September 2011 with regard to:
 - i. the arrears rates?
 - ii. the rates of interest being paid?
 - iii. the relationship between the arrears rate and the interest rate?
 - (b) What additional information would you need before you could estimate the median interest rate being paid by mortgage holders in September 2011?





- (c) Regarding the relationship between the arrears rate and the interest rate for September 2011, the authors of the report state: "The direction of causality ... is important" and they go on to discuss this.
Explain what is meant by the "direction of causality" in this context.

- (d) A property is said to be in "negative equity" if the person owes more mortgage than the property is worth. A report about mortgaged properties in Ireland in December 2010 has the following information:

- Of the 475136 properties examined, 145414 of them were in negative equity.
- Of the ones in negative equity, 11644 were in arrears.
- There were 317355 properties that were neither in arrears or negative equity.

- i. What is the probability that a property selected at random (from all those examined) will be in negative equity? Give your answer correct to two decimal places.
- ii. What is the probability that a property selected at random from all those in negative equity will also be in arrears? Give your answer correct to two decimal places.
- iii. Find the probability that a property selected at random from all those in arrears will also be in negative equity. Give your answer correct to two decimal places.

- (e) The study in part (d) was so large that it can be assumed to represent the population. Suppose that, in early 2012, researchers want to know whether the proportion of properties in negative equity has changed. They analyse 2000 randomly selected properties with mortgages. They discover that 522 of them are in negative equity. Use a hypothesis test at the 5% level of significance to decide whether there is sufficient evidence to conclude that the situation has changed since December 2010.

Be sure to state the null hypothesis clearly, and to state the conclusion clearly.

11. **2014 Sample Paper 2** The random variable X has a discrete distribution. The probability that it takes a value other than 13, 14, 15 or 16 is negligible.

- (a) Complete the probability distribution table below and hence calculate $E(X)$, the expected value of X .

x	13	14	15	16
$P(X = x)$	0.383	0.575		0.004

- (b) If X is the age, in complete years, on 1 January 2013 of a student selected at random from among all second-year students in Irish schools, explain what $E(X)$ represents.
- (c) If ten students are selected at random from this population, find the probability that exactly six of them were 14 years old on 1 January 2013. Give your answers correct to three significant figures.

