



Solutions

4.1 Permutations

1. (a) 720
(b) 120
(c) 24
(d) 360
(e) 360
2. 24
(a) 12
(b) 12
(c) 12
3. (a) 5040
(b) 2160
(c) 2880
(d) 720
(e) 4320
4. 15
5. (a) 1404000
(b) 5400
6. (a) 5040
(b) 144
7. (a) 15120
(b) 12096
(c) 6048
(d) 2688

4.2 Combinations

1. i. 10
ii. 6
2. i. 210
ii. 28
iii. 84
iv. 21
3. 120
i. 10
ii. 31
iii. 37
iv. 83
v. 55
4. i. 715
ii. 180
5. i. 210
ii. 231
6. i. 3003
ii. 286
iii. 2002
iv. 2717
7. i. 75
ii. 95
iii. 145
iv. 70
8. 15750

4.3 The Addition Rule

1. i. $\frac{1}{2}$
ii. $\frac{1}{2}$
iii. $\frac{2}{3}$
2. i. $\frac{6}{25}$
ii. $\frac{1}{5}$
iii. $\frac{2}{5}$
3. i. $\frac{1}{2}$
ii. $\frac{1}{6}$
iii. $\frac{11}{18}$
4. i. $\frac{1}{4}$
ii. $\frac{1}{13}$
iii. $\frac{4}{13}$
5. i. $\frac{1}{2}$
ii. $\frac{7}{13}$
iii. $\frac{7}{26}$
iv. No
6. i. $\frac{7}{36}$
ii. $\frac{1}{2}$





4.4 Tree Diagrams

1. $\frac{9}{50}$
2. 0.21
3. 0.55
4. i. 0.118125
ii. 0.268125
iii. 0.72125

5. 0.71

6. i. 0.03
ii. 0.01

7. i. 0.14
ii. 0.18
iii. $\frac{2}{3}$

8. i. $\frac{6}{25}$
ii. $\frac{3}{4}$
iii. $\frac{11}{152}$

4.5 Combinatorics

1. $\frac{4}{7}$
2. i. $\frac{7}{24}$
ii. $\frac{7}{40}$
3. i. $\frac{9}{182}$
ii. $\frac{5}{182}$
iii. $\frac{31}{364}$
iv. $\frac{45}{182}$
v. $\frac{137}{182}$
4. i. 324632
ii. 150
iii. 4350
5. i. $\frac{1}{14}$
ii. $\frac{1}{210}$
iii. $\frac{19}{42}$

6. i. $\frac{2}{15}$
ii. $\frac{4}{9}$
iii. $\frac{8}{15}$

7. i. $\frac{3}{10}$
ii. $\frac{1}{12}$

4.6 Independent Events

1. i. $\frac{1}{8}$
ii. $\frac{7}{12}$
2. i. $x = 0.42$
ii. $y = 0.28$
3. i. $x = 6$ and $y = 3$
ii. No.
iii. $P(C|D) = \frac{3}{10}$ and $P(D|C) = \frac{1}{3}$
4. $x = 0.3$, $y = 0.2$ or $x = 0.2$, $y = 0.3$

5. (a) $x = 0.45$ and $y = 0.1$
(b) Yes.

4.7 Conditional Probability

1. $\frac{2}{3}$
2. $P(A|B) = \frac{2}{5}$ and $P(B|A) = \frac{1}{3}$
3. $\frac{3}{8}$
4. i. 7
ii. $\frac{9}{10}$
iii. $\frac{7}{19}$

5. i. $\frac{5}{24}$
ii. $\frac{5}{12}$
iii. $\frac{7}{12}$
iv. $\frac{3}{4}$
v. $\frac{7}{10}$

6. i. 0.18
ii. 0.66
iii. $\frac{3}{8}$

$$7. P(A \cap B) = \frac{1}{6} \text{ and } P(B) = \frac{5}{12}$$

$$8. \frac{77}{185}$$

$$9. \frac{17}{31}$$

$$10. \frac{3}{4}$$





4.8 Expected Values

1. 114.17
2. Yes
3. i. Three point shots have a higher expected result.
ii. 82 points.

4. i.

Sum	2	3	4	5	6	7	8	9	10	11	12
Probability	$\frac{1}{36}$	$\frac{2}{36}$	$\frac{3}{36}$	$\frac{4}{36}$	$\frac{5}{36}$	$\frac{6}{36}$	$\frac{5}{36}$	$\frac{4}{36}$	$\frac{3}{36}$	$\frac{2}{36}$	$\frac{1}{36}$

ii. $E(X) = 7$

5. $x = 0.05$ and $y = 0.5$

6. i.

Number of family members	2	3	4	5	6	7
Probability	$\frac{1}{35}$	$\frac{4}{35}$	$\frac{9}{35}$	$\frac{12}{35}$	$\frac{6}{35}$	$\frac{3}{35}$

ii. $E(X) = 4.77$

7. i. 0.101
ii. 0.086
iii. €616.91
iv. €520.39
v. €1071.09, €503.61
vi. €448

4.9 Binomial Distribution

1. i. $\frac{25}{72}$
ii. $\frac{5}{72}$
iii. $\frac{1}{216}$
iv. $\frac{91}{216}$
2. i. 0.16807
ii. 0.36015
iii. 0.47178
3. i. 0.01177
ii. 0.23549
4. i. $\frac{1}{1024}$
ii. $\frac{53}{512}$
5. i. $\frac{256}{3125}$
ii. $\frac{1024}{3125}$
iii. $\frac{256}{625}$
iv. $\frac{821}{3125}$
6. i. 0.05631
ii. 0.146
iii. 0.756
7. i. 0.3771
ii. 0.6229
8. Proof
9. 0.0299
10. $\frac{3}{16}$
11. 0.02
12. 0.0335
13. 0.08
14. $n = 3$
15. $n = 26$





4.10 Applications of Normal Distribution

1. i. 0.1587
ii. 0.648
iii. 0.1933
2. 0.73%
3. i. 0.3413
ii. 0.2486
iii. 0.5899
4. i. 0.9932
ii. 0.1587
5. i. 0.2743
ii. 0.1587
iii. 0.567
6. i. 0.8907
ii. 0.7967
iii. 0.6874
7. i. 0.5832
ii. 0.989
iii. 0.2817
8. 490 points.
9. 57.6
10. 67.62kg to 86.38kg
11. €134752 to €155786
12. 60.55 minutes

		Die 2					
		1	2	3	4	5	6
Die 1	1	L	L	L	L	L	L
	2	L	L	L	L	L	L
	3	L	L	L	L	L	W
	4	L	L	L	L	W	W
	5	L	L	L	W	W	W
	6	L	L	W	W	W	W

5 Exam Questions

1. (a) figure above
(b) i. $\frac{5}{18}$
ii. 0.3767
(c) 0.0791
2. (a) 0.448
(b) 0.072
(c) 0.748
(d) i. .
ii. .
(e) i. .
ii. $n = 7$
(f) i. 0.75
ii. Events are not independent





3. (a) 12
(b) Game B would be more successful at earning money for this charity as its expected earnings are higher per spin.
(c) 0.2
4. (a) i. .
ii. 0.04285
iii. 0.0693
iv. No. Only 6.93% of the people who test positive have the disease. Over 93% of those who test positive do not have the disease. This would cause unnecessary alarm in the community.
- (b) i. Drug A is not more successful than the generic drug, $P \leq 0.51$. Conclusion: We can say with 95% confidence that the percentage success of drug A is between 54.73% and 66.67%. Therefore, the null hypothesis is rejected. The company's claims are true.
ii. 277
5. (a) i. A sample space is the set of all possible outcomes of an experiment.
ii. If two events have no outcomes in common they are said to be mutually exclusive.
iii. Two events A and B are independent if knowing that one occurs does not change the probability that the other occurs.
- (b) i. .
ii. $P(E|F) = P(E) = \frac{2}{3}$
 $P(F|E) = P(E) = \frac{1}{5}$. E and F are independent events.
6. (a) i. 0.9452
ii. 0.8904
7. (a) i. The population is divided into different subgroups which have common characteristics. Random samples are drawn from each sub group according to their proportion of the population.
ii. Long Haul economy class passengers. Long haul business class passengers. Long haul executive class passengers. Short haul passengers.
- (b) i. 0.231, 0.238
ii. No, these are not independent events- a delayed flight would influence your satisfaction with the overall service.
- (c) i. .
ii. Most people would keep their baggage under 20 kg so they would not have to pay extra charges.
- (d) i. .
ii. The distribution is skewed right. This is because the median is less than the mean.





- (e) i. Null hypothesis: satisfaction level is unchanged, $P = 0.7$, reject the null hypothesis.
ii. No, margin of error = 0.02236
- (f) i. .
ii. $r = 0.8822$
iii. Strong positive correlation: Older people spend more.
8. (a) Trials are independent of each other. Probability of success is the same each time.
- (b) i. 0.311
ii. 0.092
9. (a) 124
(b) 94.35%
10. (a) i. They've gone up a lot - they were mostly between 1% and 5% in 2009, and mostly between 5% and 15% in 2011
ii. They've gone up a lot too - they were mostly between 2.3% and 4.1% in 2009, and mostly between 4% and 6% in 2011
iii. There seems to be a stronger relationship in 2011 than in 2009.
- (b) You would need to know how many mortgage holders are represented by each point on the relevant diagram.
- (c) It is a question of whether higher arrears rates cause interest rates to go up, or whether higher interest rates cause arrears rates to go up (assuming there is a casual relationship at all)
- (d) i. 0.31
ii. 0.08
iii. 0.48
- (e) Null hypothesis: $P(\text{property is in negative equity}) = 0.31$ Reject the null hypothesis and accept the alternative hypothesis that the proportion of properties in negative equity has changed
11. (a) Table: 0.038, $E(X) = 13.663$
(b) $E(X)$ means the mean age on 1 Jan 2010 of second level students in Ireland.
(c) 0.248

