Trigonometry
Revision Series 2017

LEAMY
MATHS COMMUNITY

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Right Angled Triangles

Pythagoras Theorem

$$a^2 + b^2 = h^2$$

Sin, Cosine and Tangent

$$\sin A = \frac{opp}{hyp}$$

$$\cos A = \frac{adj}{hyp}$$

$$\tan A = \frac{opp}{adj}$$

Example

(a) A tower that is part of a hotel has a square base of side 4 metres and a roof in the form of a pyramid. The owners plan to cover the roof with copper. To find the amount of copper needed, they need to know the total area of the roof.

A surveyor stands 10 metres from the tower, measured horizontally, and makes observations of angles of elevation from the point O as follows:

The angle of elevation of the top of the roof is 46°.
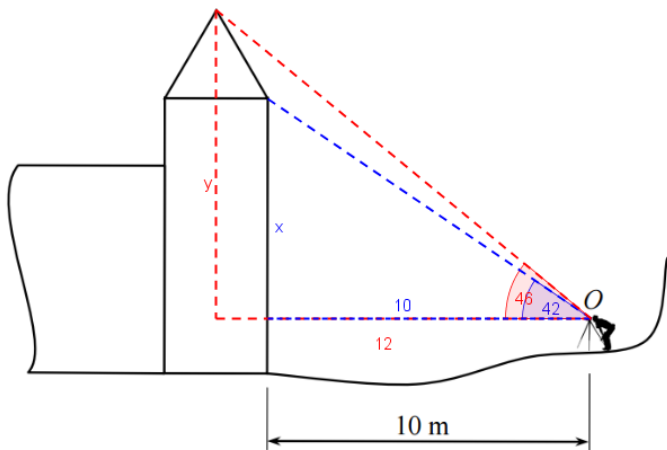The angle of elevation of the closest point at the bottom of the roof is 42°.
The angle of depression of the closest point at the bottom of the tower is 9°.

10 m
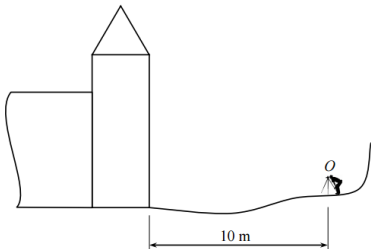
(i) Find the vertical height of the roof.

Example

$$\tan 42 = \frac{x}{10}$$
$$x = 10 \tan 42$$
$$x = 9.004$$

$$\tan 46 = \frac{y}{12}$$
$$y = 12 \tan 46 = 12.426$$

So the height $= 12.42 - 9 = 3.42$

Non-Right Angled Triangles

Sine Rule

$$\frac{A}{\sin A} = \frac{B}{\sin B}$$

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Example
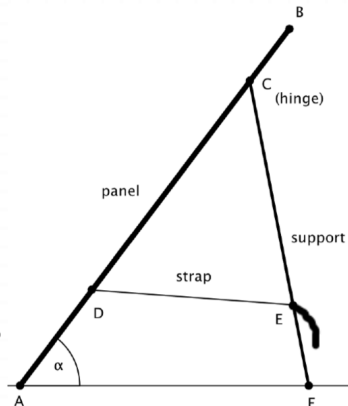

11 17

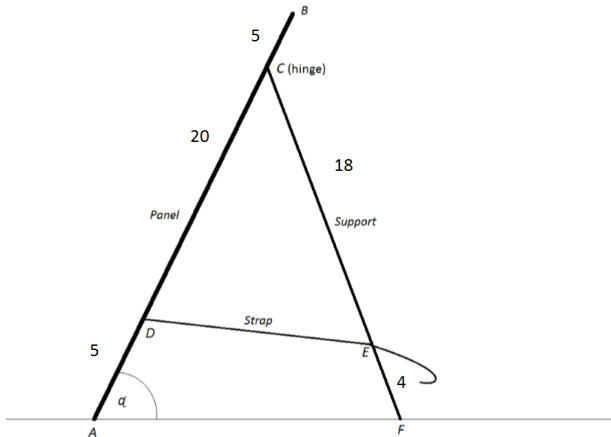The dimensions are as follows:

$|AB| = 30\ cm$

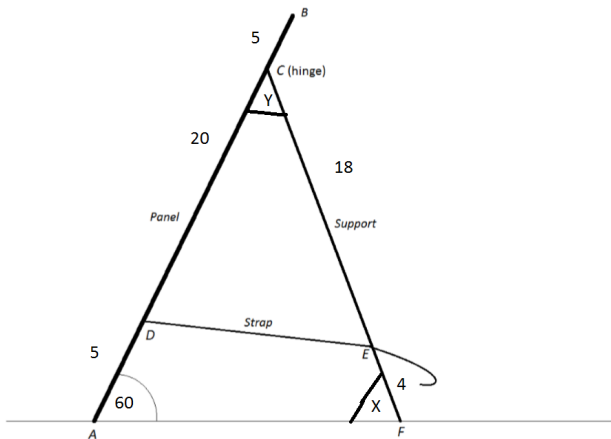$|AD| = |CB| = 5\ cm$

$|CF| = 22\ cm$

$|EF| = 4\ cm$

Find the length of the strap [DE]
such that the angle between
the panel and the ground (α) is 60°

Example

Example

Sine Rule

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$
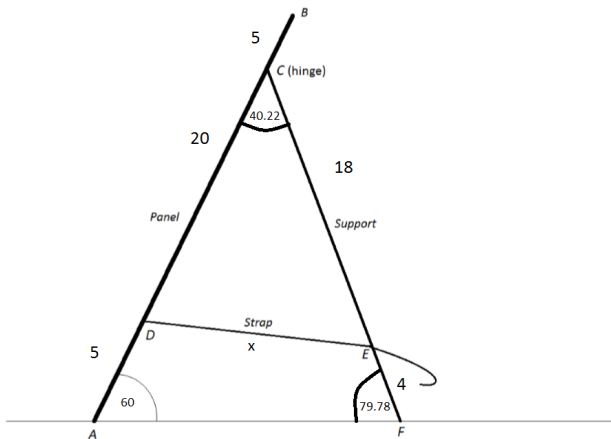
$$\frac{25}{\sin X} = \frac{22}{\sin 60}$$

$$\sin X = \frac{25 \sin 60}{22}$$

$$\sin X = .9841$$

$$X = 79.78^\circ$$

$$\text{so } Y = 40.22^\circ$$

Cosine Rule

Cosine Rule

$$a^2 = b^2 + c^2 - 2bc\cos A$$
$$x^2 = (20)^2 + (18)^2 - 2(20)(18)\cos(40.22)$$
$$x^2 = 174.23$$
$$x = 13.2$$

3-D Problems

- ▶ Where a horizontal line meets a vertical line, this creates a right angle. (Even if it doesn't look like a right angle in the diagram)
- ▶ You should break up the 3-D problem into a set of 2-D problems.

3-D Problems

|AE| is a vertical mast in a level field, supported by cables, |EB| and |ED|. These cables are anchored to the ground at the points B and D respectively. Using the measurements given in the diagram, calculate the total length of cable anchoring the mast, correct to 2 decimal places.
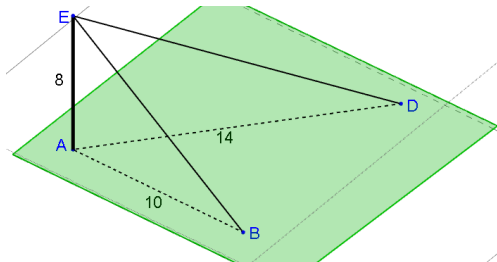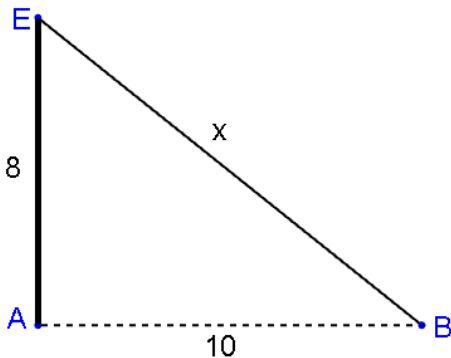

In the figure above, both $\triangle EAB$, $\triangle EAD$ are right angled.
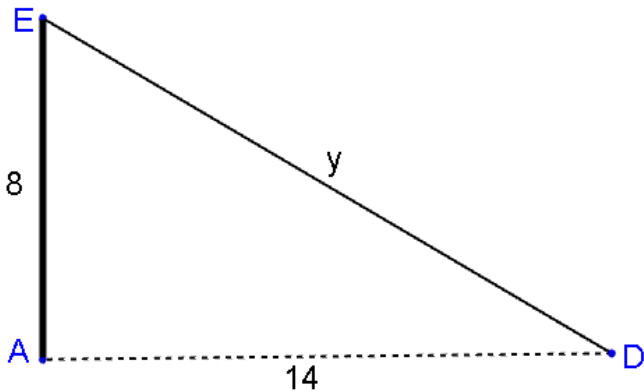
$\triangle EAB$

$$x^2 = 8^2 + 10^2$$
$$x = 12.806$$
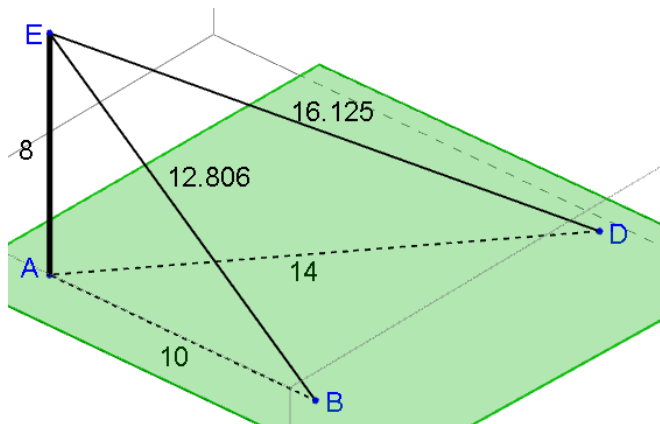
$\triangle EAD$

$$y^2 = 8^2 + 14^2$$
$$y = 16.125$$

3-D Problems

The total length of cable is:

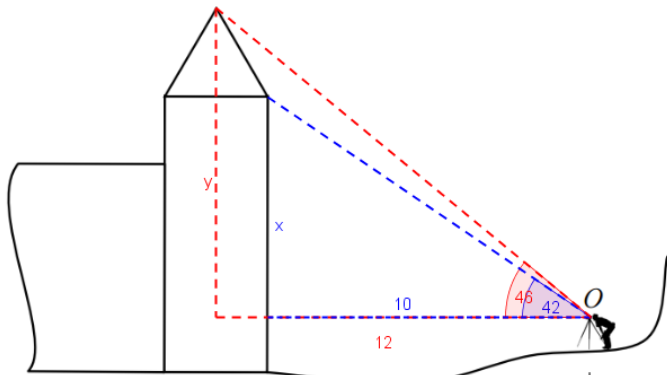$$12.806 + 16.125 = 28.93 \text{ m} \quad \text{(Correct to 2 d.p.)}$$
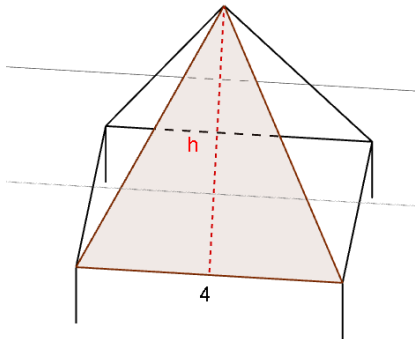
3-D Problems

3-D Problems

Question: Find the Area of the Roof. Give your answer correct to 1 decimal place.

We already know that the vertical height of the roof is 3.42 metres.

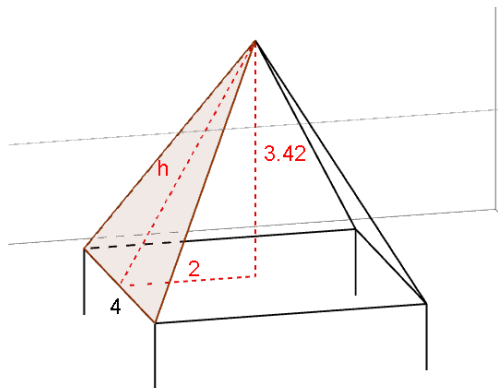# Area $= \frac{1}{2}$ base $\times$ height

The perpendicular height of one of the triangular faces **is not** the vertical height of the roof!

3-D Problems

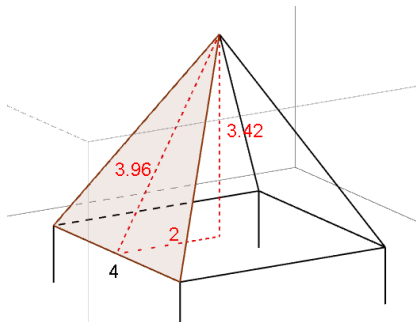
$$h^2 = 2^2 + 3.42^2$$
$$h = 3.967$$

3-D Problems

The area of one triangular face is:

$$\tfrac{1}{2} \times 4 \times 3.96 = 7.92$$

Total Area of roof:

$$4 \times 7.92 = 31.7 \text{ (Correct to 1 d.p.)}$$

Trigonometric Functions

Functions of the form:

$$f(x) = a\sin(bx) \text{ and } g(x) = a\cos(bx)$$

$$\text{Period: } = \frac{2\pi}{b}$$

$$\text{Range: } = [-a, a]$$

Trigonometric Equations

Solve the following equation for $0 \leq x \leq 360$:

$$\cos x = -\tfrac{1}{2}$$

Calculate Reference, identify quadrants, and calculate angles.

$$R = \cos^{-1}\left(\tfrac{1}{2}\right) = 60^\circ$$
$$x = 180 - 60 = 120^\circ$$
$$x = 180 + 60 = 240^\circ$$

Unit Circle