Solutions

8.1 Arithmetic Series

1. i. $T_n = 3n - 1$
   ii. $T_n = 4n - 12$
   iii. $T_n = 13 - 4n$
2. $a = 1, d = 3$
3. $T_{n+1} - T_n \neq$ a constant
4. $T_n = 2n + 1$
5. $T_n = 3n - 2$
6. $S_n = 2n^2 + n$
   $n = 10$
7. $a = 2, d = 5$
8. (a) 6, 14, 24
   (b) $a = 6$, $d = 2$
   (c) $T_n = 2n + 4$
9. $x = \frac{7}{2}$
10. $T_n = 3n - 1$
11. $x = 2$, $y = 4$
12. i. 38
    ii. 11
    iii. 55
13. i. €10.5m
    ii. €973.75m

8.2 Geometric Series

1. i. $T_n = 3(2)^{n-1}$
   ii. $72(\frac{1}{2})^{n-1}$
   iii. $18(-\frac{1}{3})^{n-1}$
2. 11 terms
3. i. $T_n = 3(4)^{n-1}$
   ii. $T_n = \frac{27}{2}(\frac{1}{3})^{n-1}$
4. $x = 9$, $x = -1$
5. i. $3^n - 1$
   ii. $32(1 - \frac{1}{4}^n)$
   iii. $\frac{1}{3}[1 - (-2)^n]$
6. First term =18; Common ratio = $\frac{1}{3}$ OR First term = 2; Common ratio =3
7. $p = 24$, $q = 36$ OR $p = 8$, $q = 4$
8. $T_n = 4(5)^{n-1}$
9. $a = 3$, $r = 2$
10. i. $A = 50(0.8)^t$
    ii. 25.6 mg
    iii. $t = 6$ hours
    iv. 168 mg
    v. 7 hours (8th dose).
11. i. $P = 40(1.02)^t$
    ii. $ 72.45
    iii. $ 2173.5
    iv. $ 550.78

8.3 Infinite Series

1. i. 2
   ii. Doesn't exist
   iii. $\frac{27}{2}$
   iv. $\frac{16}{3}$
   v. Doesn't exist
2. $\frac{2}{3}$
3. $\frac{58}{99}$
4. $\frac{43}{9}$
5. i. $S_\infty = \dfrac{x}{1-\frac{1}{1-x}}$
   ii. $x = 4$
6. i. $S_\infty = \dfrac{x}{1-\frac{1}{x+2}}$
   ii. $x = 3$
7. i. $-4 > x > 2$
   ii. $-\frac{1}{3} > x > 1$
   iii. $-3 < x < 3$
8. i. $\theta = 15°, 45°, 75°$
   ii. $\theta = 0°, 60°$
9. $p = 3$

8.4 Quadratic Sequences and Series

1. i. $n^2 + 1$
   ii. $n^2 + 3n + 2$
   iii. $2n^2 - n + 3$
2. i. $n^2 - 2n + 18$
   ii. 98
3. i. No solution given
   ii. $T_n = 0.5n^2 + 0.5n$
   iii. 66
   iv. 20

8.5 Exam Questions

1. (a) i. 7 stages
   ii. 8 metres
   iii.

	1	2	3	4	5	6	7	8	9
Chg x	+4	0	-1	0	$\frac{1}{4}$	0	$-\frac{1}{16}$	0	$\frac{1}{64}$
Chg y	0	2	0	$-\frac{1}{2}$	0	$\frac{1}{8}$	0	$-\frac{1}{32}$	0

   (b) i. $G_5 =$ F, M, F, F, M.
   ii. $G_6 = 8$ $G_7 = 13$
   iii. $G_3 = 2$
2. (a)
   (b) $10\frac{13}{64}$ m

Bounce	0	1	2	3	4
Height (m)	$\frac{2}{1}$	$\frac{3}{2}$	$\frac{9}{8}$	$\frac{27}{32}$	$\frac{81}{128}$

(c) 14 m

3. (a) i. .
   ii. $\ln a$
   (b) $a = e^2$

4. (a) i. .
   ii. Number of small triangle: 1,4,9,16
   Number of matchsticks: 3,9,18,30
   (b) n^2
   (c) $3n$
   (d) $a = \frac{3}{2}$, $b = \frac{3}{2}$
   (e) 2704

5. (a) .
   (b) $\frac{172}{33}$

6. (a) i. Discrete means that only a countable number of independent variables are taken : $h = 0, 1, 2, 3, 4, 5$. Her model does not take account of $h = 2.5km$ because a geometric series can only take whole number values for $n = h \in \mathbb{N}$. Continuous means that the independent variable h can take on all possible $h \in \mathbb{R}^+$.
   ii. The continuous model can predict p for any h value, e.g. $h = 7.62km$. Discrete models cannot do this.
   (b) $33.7kPa$
   (c) $5.57km$
   (d) 27 floors.