Junior Certificate Examination, 2015

Sample paper prepared by Leamy Maths Community

Mathematics

Project Maths - Phase 3

Paper 2

Solutions

Higher Level

Saturday 9 May

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Leamy Maths Community

300 marks

Question 1 **Suggested maximum time: 10 minutes**

In some games, you need an eight faced die. The faces are numbered 1, 2, 3, 4, 5, 6, 7 and 8.

(a) What is the probability of getting a 6?

$$p = \frac{1}{8} = 0.125 = 12.5\%$$

Marks: 0, 5

(b) The die is tossed 1000 times and the following results are found

Number on die	1	2	3	4
Frequency	100	95	110	93
Relative frequency	0.1	0.095	0.11	0.093
Number on die	5	6	7	8
Frequency	91	111	110	290
Relative frequency	0.091	0.111	0.11	0.29

(i) Calculate the missing frequency

$$x = 1000 - (100 + 95 + 110 + 93 + 91 + 110 + 290) = 111$$

Marks: 0, 5

(ii) Calculate the relative frequencies
Marks: 0, 2, 5

(iii) Do you think the die is fair? Justify your answer.
No, all relative frequencies are equal except for 8
Marks: 0, 2, 5

Question 2 **Suggested maximum time: 20 minutes**

In a fair, there is a new game: you can spin a three colour wheel twice. The three colours on the wheel are blue, red and yellow. If you get the same colour on both spins, you will be a winner.

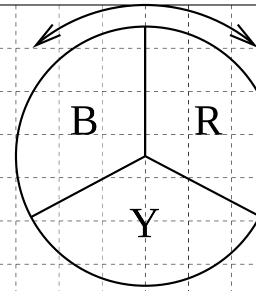
(a) List the all the possible outcomes using B for blue, R for red and Y for yellow Marks: 0, 2, 5

(b) Find the probability of winning

$$p = \frac{3}{9} = \frac{1}{3}$$

Marks: 0, 5

BB	BR	BY
RB	RR	RY
YB	YR	YY



(c) Jessica says that you are equally likely to get two Blues or two Reds. Is she correct? Justify your answer.
She is correct if the three colours are equally likely
Marks: 0, 2, 5

(d) James says that the probability of getting the same colour twice or at least one Blue in your combination is the same. He is correct? Justify your answer?
3 possibilities of getting the same colour (BB, RR, YY), 5 possibilities of getting at least one blue (BB, YB, BY, RB, BR). James is wrong
Marks: 0, 2, 5

(e) The owner considers an alternative rule. You will be a winner if your spin combination does not include any Red. What is the new probability of winning?

$$p = \frac{4}{9}$$

(BB, BY, YY, YB)
Marks: 0, 4, 7, 10

(f) Which rule is more favorable to the player? Give a reason for your answer.
The second rule as your probability o winning is higher
Marks: 0, 2, 5

(g) If you were spinning the wheel three times, what would be the probability of getting Yellow three times? Justify your answer.

$$p = \frac{1}{27}$$

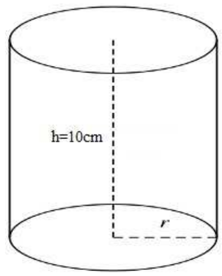
BBB, BBR, BBY, BRB, BRR, BRY, BYB, BYR, BYY, RBB, RBR, RBY, RRB, RRR, RRY, RYB, RYR, RYY, YBB, YBR, YBY, YRB, YRR, YRY, YYB, YYR, YYY
Marks: 0, 5, 10

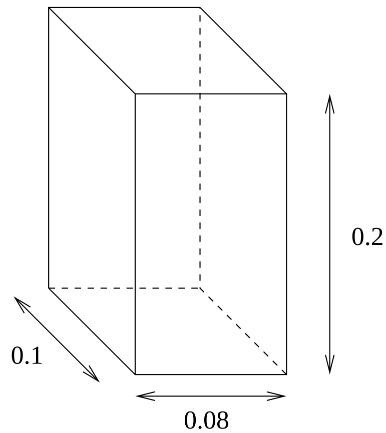
Question 3 **Suggested maximum time: 20 minutes**

(a) A cylindrical tin contains exactly a litre of soup. The tin is 10cm high. Find the diameter of the tin. Give your answer to the nearest millimetre.

$$V = \pi r^2 h \Longrightarrow r = \sqrt{\frac{V}{\pi h}} = \sqrt{\frac{1000}{10\pi}} = 56mm = 5.6cm$$

Marks: 0, 2, 5





(b) A rectangular container may be used instead. It length is 0.1m, it width is 0.08m and its height is 0.2m, see figure on the previous page. Find its capacity in litres.

$$V = 10 \times 8 \times 20 = 1600cm^3 = 1.6l$$

Marks: 0, 2, 5

(c) If you pour one litre of soup in the rectangular container, what will be the height of liquid?

$$1000 = 10 \times 8 \times h \Longrightarrow h = \frac{1000}{80} = 12.5cm$$

Marks: 0, 2, 5

(d) Calculate the surface of metal necessary to manufacture the cylindrical tin in cm^2. Give your answer correct to the nearest whole number.

$$S = 2\pi r^2 + 2\pi rh = 549cm^2$$

Marks: 0, 4, 7, 10

(e) If the height of the rectangular container is reduced to 12.5 cm, calculate the surface of metal necessary to manufacture the container in cm^2.

$$S = 2 \times (12.5 \times 8) + 2 \times (10 \times 12.5) + 2 \times (8 \times 10) = 610cm^2$$

Marks: 0, 4, 7, 10

(f) If the outside area of the cylinder was 610cm^2 and the circle at the bottom and top of the cylinder had the same area as the rectangle at the bottom of the rectangular container, what would be the height of the cylinder?

$$\pi r^2 = 80 \Longrightarrow r = \sqrt{\frac{80}{pi}} \approx 5cm$$

$$h = \frac{610 - 2 \times 80}{5\pi} \approx 28.6cm$$

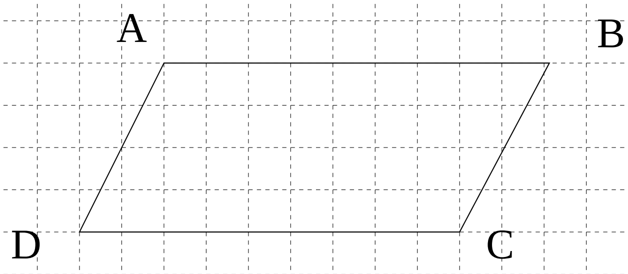
Marks: 0, 2, 5

(g) Why do you think most tins are cylindrical?
Comparing d and e, you need less metal in a cylindrical tin for an equivalent volume
Marks: 0, 5

Question 4 **Suggested maximum time: 10 minutes**

Prove that in a parallelogram, opposite sides are equals and opposite angles are equal.

Diagram

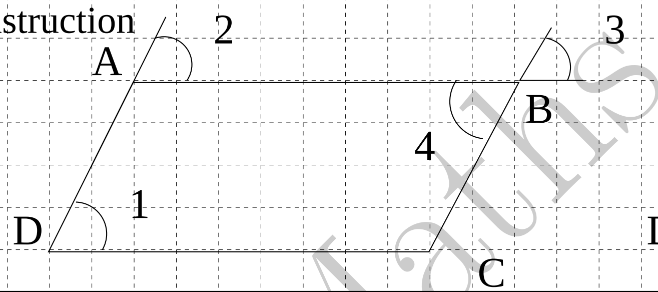


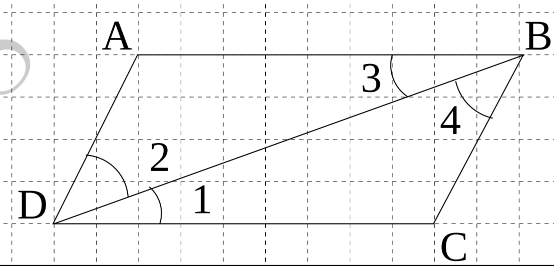
Given

(AB)//(CD) and (AD)//(BC)

To prove AD=BC AB=CD <DAB=<BCD <ABC=<CDA

Construction





Proof

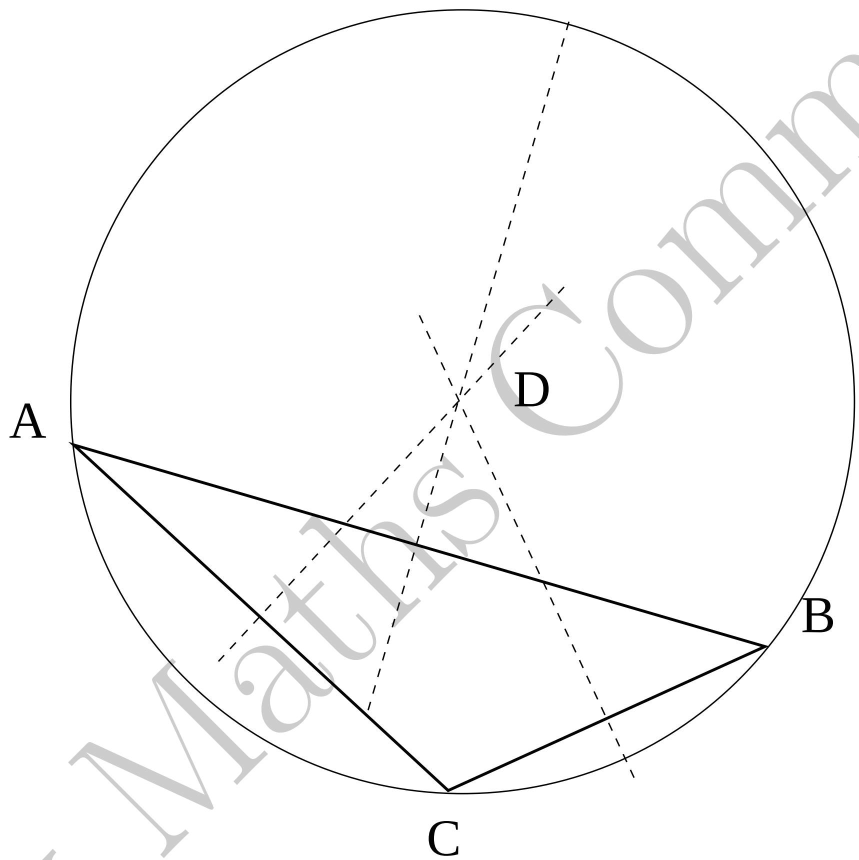
As (AB)//(CD) <1=<2, then as (AD)//(BC) <2=<3 and using internal external angles <3=<4 so <CDA=ABC. Using a similar technique, you can prove that <DAB=<BCD

As (AB)//(CD), <1=<3 and <2=<4
The two triangles ABD and BCD have two angles equal and a common side They are congruent and so AB=CD. You can prove AD=BC using the other diagonal

Marks: 0, 5, 10, 15, 20

Question 5 **Suggested maximum time: 15 minutes**

(a) Construct the perpendicular bisectors of segments [AB] and [BC] below using only a compass and a straight edge. Show all construction work.



Marks: 0, 4, 7, 10

(b) Prove that point D common to the two perpendicular bisectors also belongs to the perpendicular bisector of segment [AC].
A point M on the perpendicular bisector of [AB] verifies MA=MB. If the point is on the perpendicular bisector of [AC] as well, it verifies MA=MC so this implies MB=MC: the point is also on the perpendicular bisector of [BC]
Marks: 0, 4, 7, 10

(c) Draw the circle centred at point D with radius $R = |DA|$. Explain why points B and C also belong to the circle.
DA=DB=DC as D in on the three perpendicular bisectors.
Marks: 0, 4, 7, 10

Question 6 **Suggested maximum time: 15 minutes**

30 people are training for long distance running. At the beginning of the training, they make a test run to see for how long they can run. The results are given in minutes in the table below.

40	45	38	54	55	70	62	67	54	79
46	48	55	41	59	61	64	58	52	72
52	63	65	59	49	62	54	47	58	62

After two months of training, a new test run is organised with the following times (in minutes)

81	72	42	57	64	65	64	51	59	79
75	60	64	57	92	68	65	51	85	74
74	73	53	82	66	77	72	62	75	61

(a) Represent this data on a back-to-back stem leaf diagram.

First run		Second run
8	3	
9 8 7 6 5 1 0	4	2
9 9 8 8 5 5 4 4 4 2 2	5	1 1 3 7 7 9
7 5 4 3 2 2 2 1	6	0 1 2 4 4 4 5 5 6 8
9 2 0	7	2 2 3 4 4 5 5 7 9
	8	1 2 5
	9	2
8|3=38		Key: 4|7=47

Marks: 0, 2, 5

(b) Use the diagram to identify the median in each case.

$$M_1 = \frac{55+58}{2} = 56.5 \qquad M_2 = \frac{65+66}{2} = 65.5$$

Marks: 0, 2, 5

(c) Find the mean running time for each run.

$$\mu_1 = 56.37 \qquad \mu_2 = 67.3$$

Marks: 0, 4, 7, 10

(d) Compare running times for the two runs. Refer to **at least one** measure of central tendency in your answer. What other parameter should you study to make a more precise description of the data?
Second run is about 10 minutes longer if you compare both mean and median. Range is extended for the second run and the data is more skewed to the right. Data are more centred in the first run. You would need to include the standard deviation or IQ range to complete the study.
Marks: 0, 2, 5

(e) Deidre says that everyone has improved between the two runs. Is she correct? Justify your answer.
not necessarily true. The lowest time of the first run and the lowest time of the second run do not necessarily correspond to the same runner and this is valid at any rank. Some people might be slower run time c54 in the first run could become 42 in the second run
Marks: 0, 2, 5

Question 7 **Suggested maximum time: 10 minutes**

The students in a class are asked in which supermarket their parents do their shopping. The answers are listed below:

Tesco	Dunnes	Tesco	SuperValu	Lidl
SuperValu	Tesco	SuperValu	Tesco	Dunnes
Tesco	Dunnes	Lidl	Lidl	SuperValu
Tesco	Lidl	Tesco	Dunnes	Tesco
Lidl	SuperValu	Dunnes	Dunnes	Tesco

(a) What type of data is this: numerical continuous, numerical discrete, categorical nominal, categorical ordinal?
Categorical nominal

(b) Fill in the frequency table

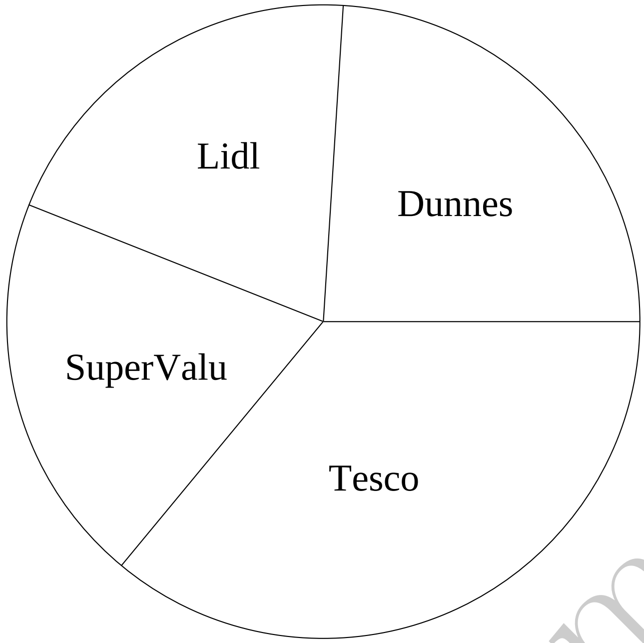
Supermarket	Dunnes	Lidl	SuperValu	Tesco
Frequency	6	5	5	9

Marks: 0, 2, 5

(c) Calculate the mode and the average of the data if they exist. If they do not exist, explain why.
Mode is Tesco, no mean as we are not dealing with numbers here
Marks: 0, 2, 5

(d) Display the data in a pie chart. Show all of your calculations clearly.
Dunnes: 86.4°, Lidl : 72°, SuperValu : 72°, Tesco: 129.6°
Marks: 0, 4, 7, 10

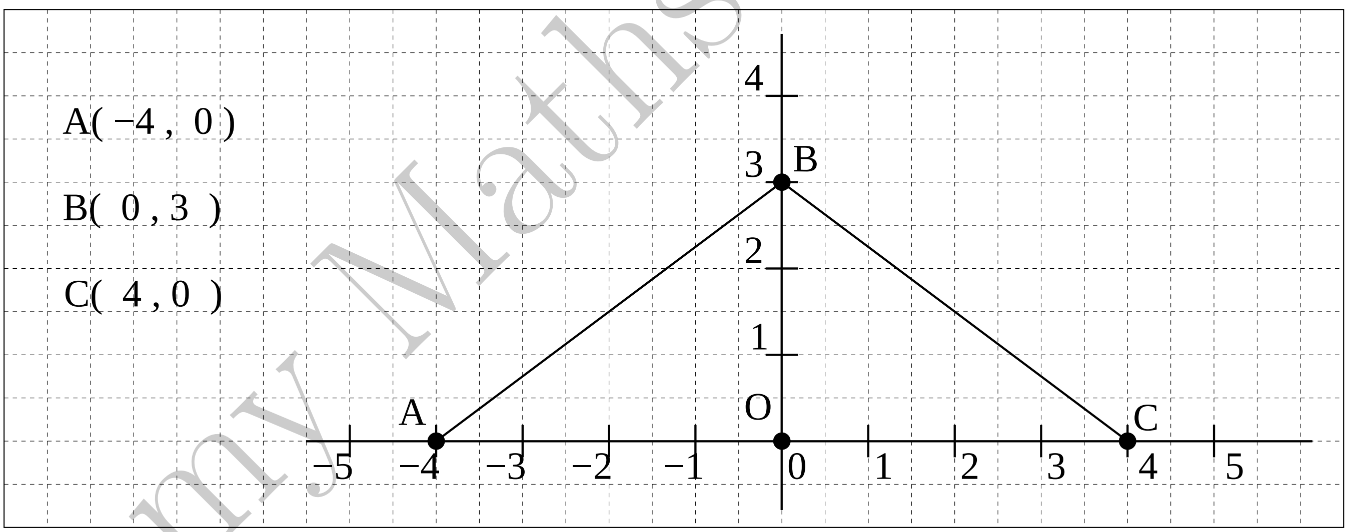
(e) Mark wants to get another set of data. He stands outside his local SuperValu store and ask people where they go shopping. Do you think his method is correct?
no the method is biased: people will most certainly answer SuperValu.
Marks: 0, 2, 5



Question 8 **Suggested maximum time: 10 minutes**

The roof of a house is described by the the following diagram

(a) What are the coordinates of points A, B and C?



(b) Calculate the slope of line (AB).

$$Slope = \frac{3-0}{0-(-4)} = \frac{3}{4} = 0.75$$

Marks: 0, 2, 5

(c) Calculate the distances AO, BO and AB. *There is room for working out on the next page.*

$$OA = 4 \qquad OB = 3 \qquad AB = 5$$

Marks: 0, 4, 7, 10

(d) Calculate the $\tan\theta$ where $\theta =< OAB$. What do you notice?

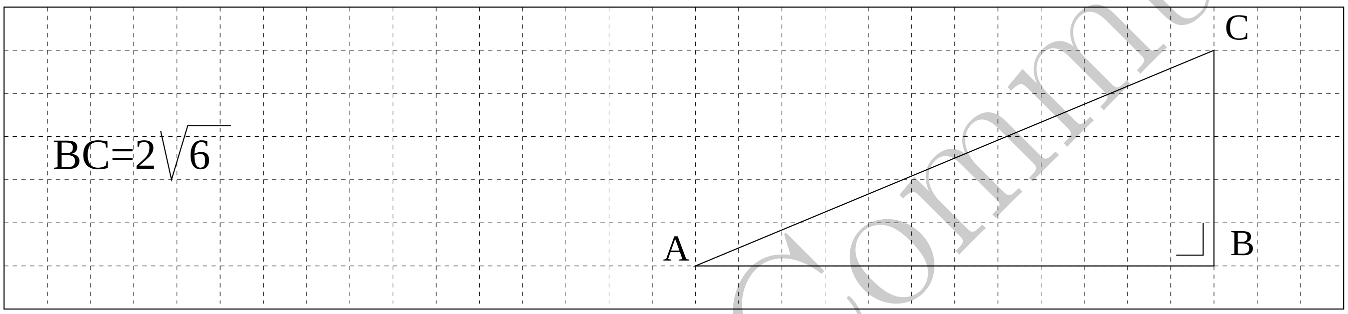
$$\tan\theta = \frac{3}{4} = slope$$

Marks: 0, 2, 5

Question 9 **Suggested maximum time: 5 minutes**

In the triangle ABC, $|AB| = 5$ and $|AC| = 7$.

(a) Calculate $|BC|$ leaving your answer in surd form



Marks: 0, 2, 5

(b) Calculate $\cos < BAC$ and $\sin < BAC$, leaving your answers in surd form.

$$\cos < BAC = \frac{AB}{AC} = \frac{5}{7} \qquad \sin < BAC = \frac{BC}{AC} = \frac{2\sqrt{6}}{7}$$

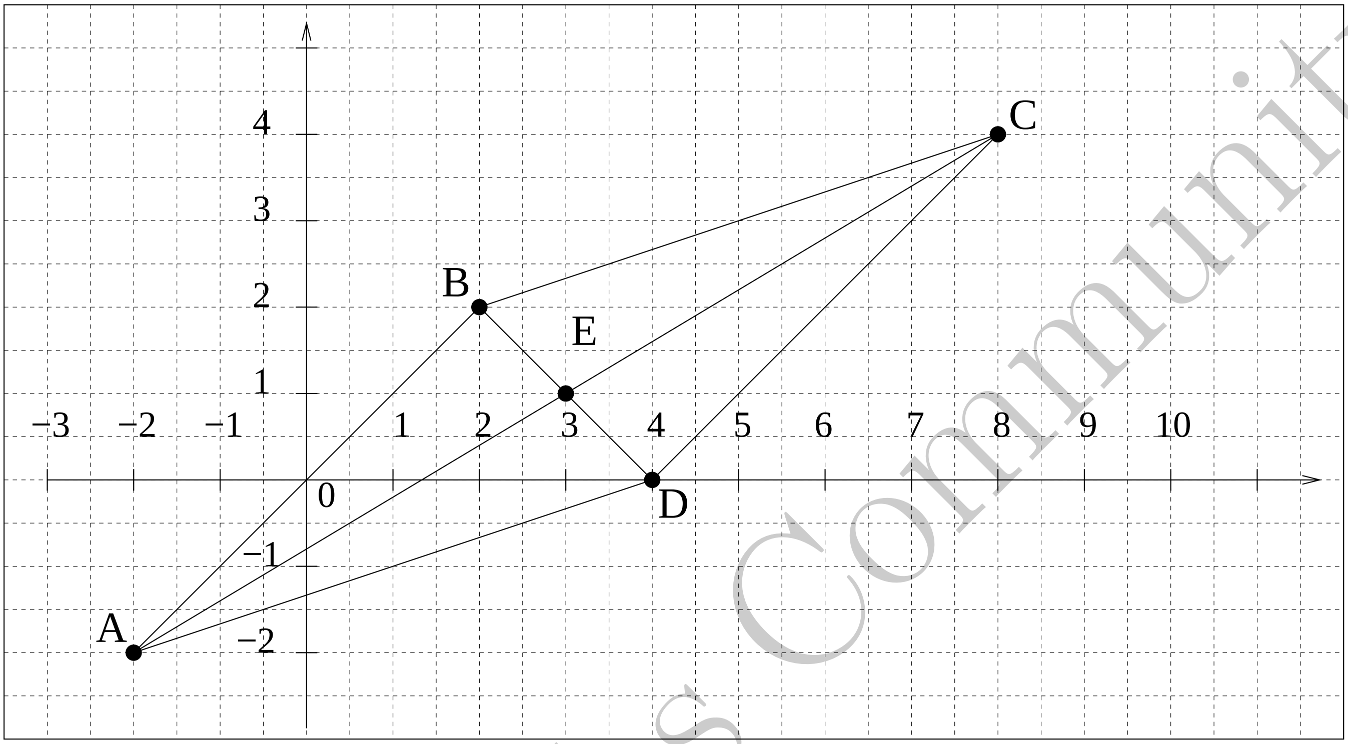
(c) Verify that $\cos^2\theta + \sin^2\theta = 1$ where $\theta =< BAC$.

$$\cos^2\theta + \sin^2\theta = \frac{25}{49} + \frac{24}{49} = \frac{49}{49} = 1$$

Marks: 0, 2, 5

Question 10 **Suggested maximum time: 5 minutes**

(a) Place the points A(-2, -2), B(2, 2), C(8, 4), D(4, 0) on the graph below.



(b) Read on the graph the coordinates of point E where the two lines (AC) and (BD) intersect.
E(3,1)
Marks: 0, 2, 5

(c) Show that the point E is the midpoint of [AC] and is also the midpoint of [BD].
E(3,1)
Marks: 0, 2, 5

(d) What can you say about the quadrilateral ABCD?
The two diagonal have the same moiddle point, this is a paralellogram
Marks: 0, 2, 5

Question 11 **Suggested maximum time: 20 minutes**

The equation of **line l** is x-2y=8.

(a) What are the slope and the y intercept of the line?
Slope=1/2 y intercept=-4
Marks: 0, 2, 5

(b) Is the point A(2,2) on **line l**? Justify your answer.

$$y = \frac{2}{2} - 4 = -3$$

The point is not on the line
Marks: 0, 2, 5

(c) **Line k** is parallel to **line l** and passes through point A. Calculate the equation of this line

$$y = \frac{x}{2} + 1$$

Marks: 0, 4, 7, 10

(d) **Line m** is perpendicular to **line l** and passes through point A. Calculate the equation of this line

$$y = -2x + 6$$

Marks: 0, 4, 7, 10

(e) Calculate the coordinates of the point common to **line l** and **line m**.
x=4, y=-2
Marks: 0, 4, 7, 10