Sample paper prepared by Leamy Maths Community

Mathematics

Paper 1

Higher Level

Sunday 23 April

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Leamy Maths Community

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8
Q9	Q10	Q11	Q12	Q13	Q14	Total	

300 marks

Sample Instructions

There are 14 questions on this examination paper. Answer all questions:

Questions do not necessarily carry equal marks. To help you manage your time during this imagination, a maximum time for each question is suggested. If you remain within these times, you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of Formulae and Tables. You must return it at the end of the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1 (Suggested maximum time 5 minutes)

(a) Paul has read that all numbers of the form $2^n - 1$ are prime numbers. Check if this is true for n=2, 3, 4, 5.

(b) Paul wonders if numbers of the form $3^n - 2$ are also prime numbers. Try the formula for n=2,3,4 and explain why Paul is right or wrong.

Question 2 (Suggested maximum time 5 minutes)

In a survey of 100 people, it was found that 75 own a car and 35 own a bicycle. 10 people owned **neither** a bicycle nor a car.

(i) Represent the above information on a Venn diagram.

(ii) How many people owned **both** a bicycle and a car?

(iii) How many people owned a bicycle, but not a car?

Question 3 (Suggested maximum time 10 minutes)

$U=$ the set of all natural numbers from 1 to 12.
$A =$ the set of all factors of 12. $B =$ the set all even numbers from 1 to 12.

(a) Represent this information on a Venn Diagram.

(b) List the elements of the following sets:

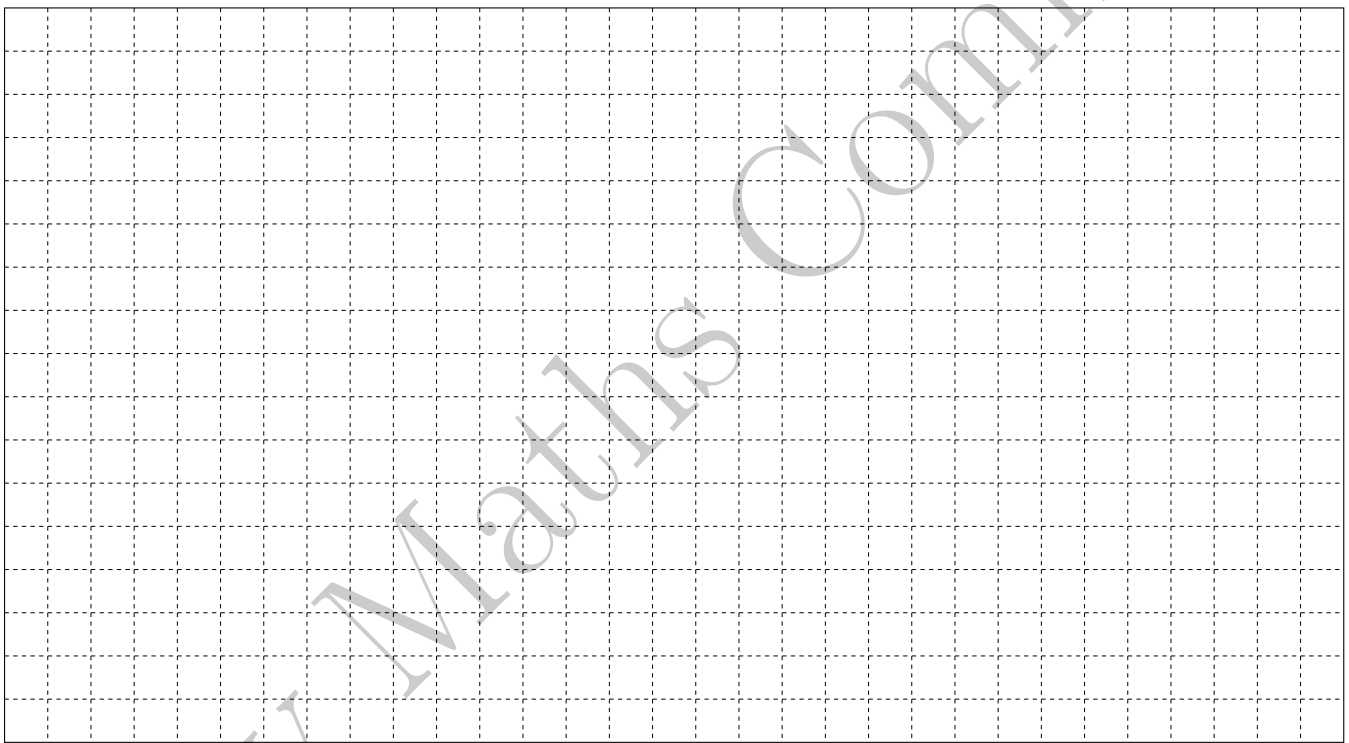   **(i)** $A \cup B$ _____

   (ii) $A \cap B$ _____

  (iii) A' _____

  (iv) B'_____

   (v) $(A \cup B)'$_____

  (vi) $(A \cap B)'$_____

(c) What is $\#B'$?_____

(d) What is $\#(A \cap B)'$?_____
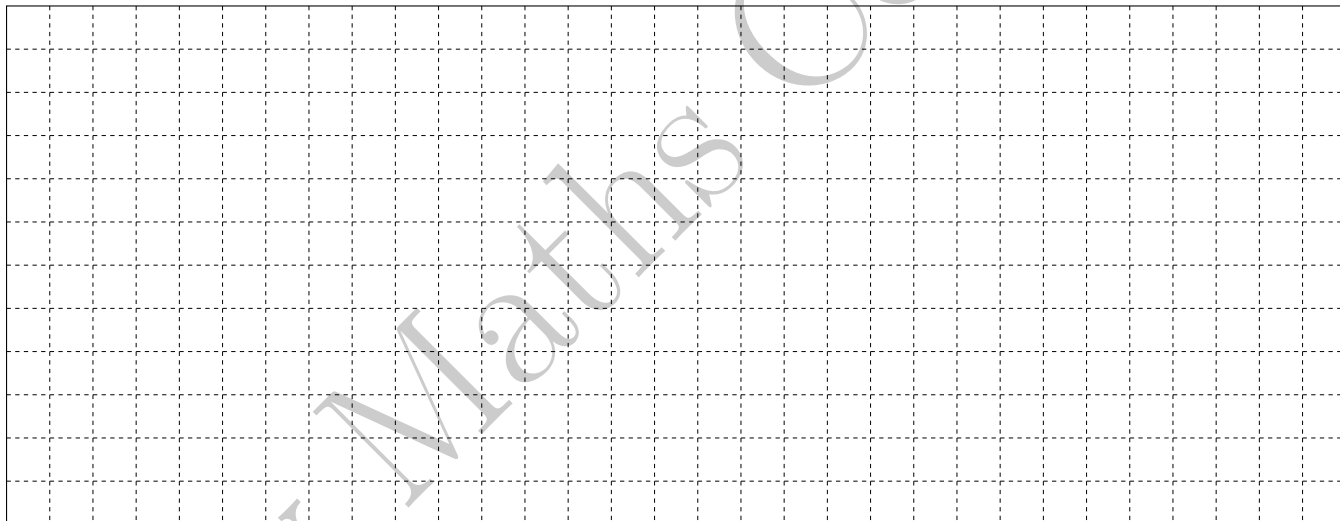
Question 4 (Suggested maximum time 10 minutes)

(a) Factorise $25a^2 - 16b^2$
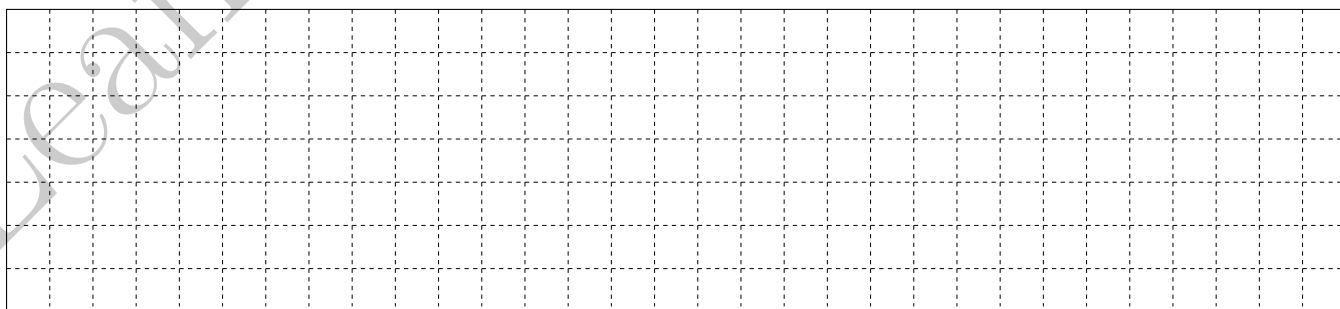
(b) Use factors to simplify

$$\frac{x^2 - 7x + 12}{x^2 - 16}$$

(c) Factorise $3a^2 + 2ab - 3ac - 2bc$

Question 5 (Suggested maximum time 10 minutes)

(a) **(i)** What is the **percentage** increase if 40 is increased to 50?

(ii) What is the **percentage** decrease if 50 is decreased to 40?

(b) Rodney owns a car dealership. He buys a car for €13,000. He wants to sell it for a profit. He advertises the car for sale for €16,000.

(i) If a customer buys the car form Rodney, for €16,000, what **percentage** profit will Rodney make on the deal? (Give your answer to the nearest whole number.)

(ii) During a "Summer Sale", Rodney 's dealership is selling all cars with a 10% discount on the selling price. What will the selling price of the car be during the sale?

(iii) If a customer buys the car during the "Summer Sale", then what **percentage** profit will Rodney make on the deal? (Give your answer correct to two decimal places.)

Question 6 (Suggested maximum time 5 minutes)

Aoife travels from Dublin to Limerick in her car at an average speed of 80 km/h while Mark also travels but at an average speed of 75 km/h. There are 200km between Dublin and Limerick. How far will Mark be from Limerick when Aoife arrives?

Question 7   (Suggested maximum time 5 minutes)

Solve the following inequality and plot the results on the number line.

$$-2 \leq 4 - 3n < 13 \qquad n \in \mathbb{N}$$
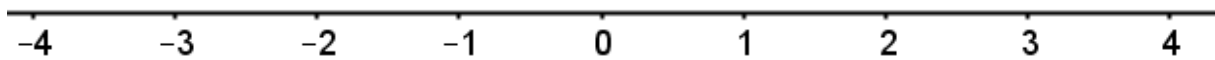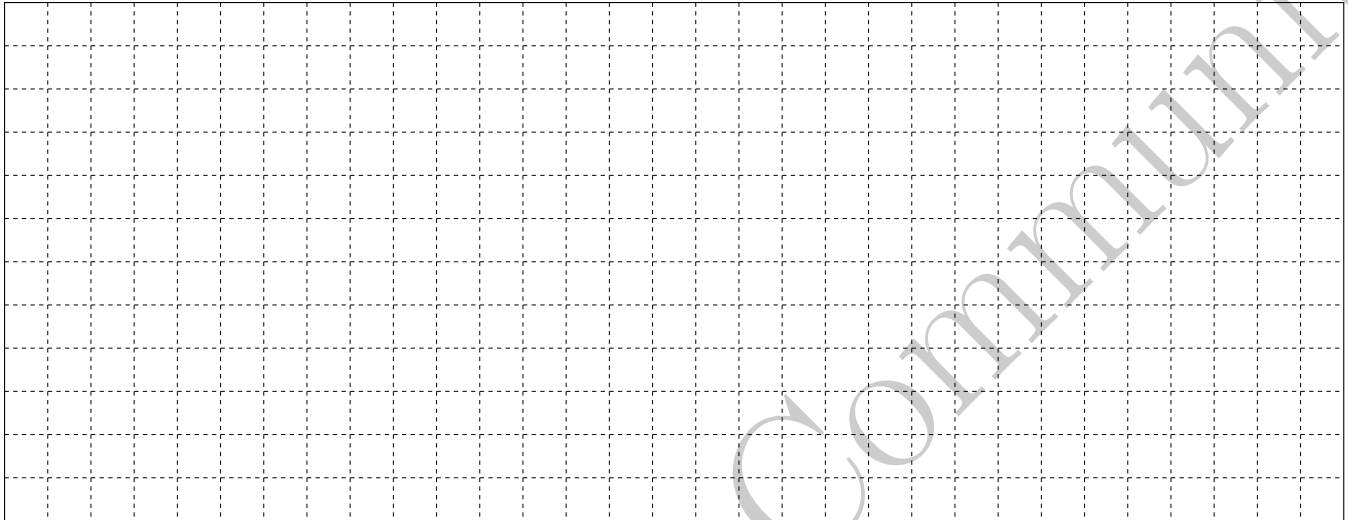
Question 8   (Suggested maximum time 15 minutes)

(a) Simplify (2x-3)(-5x+4)
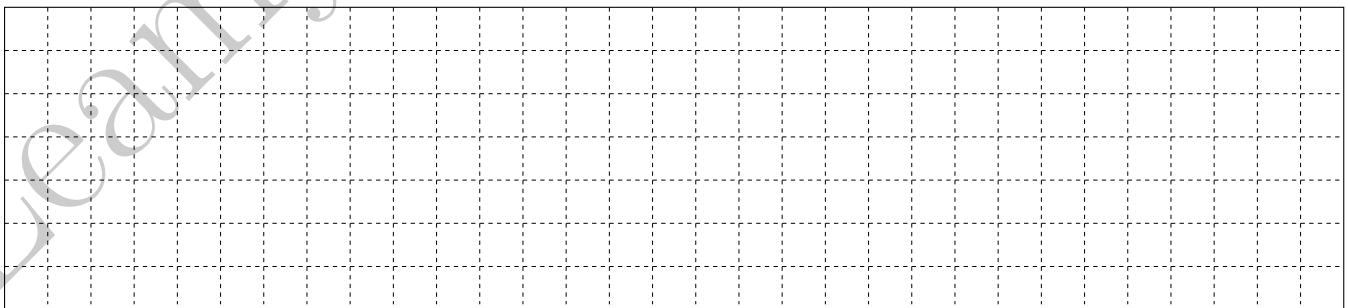
(b) Simplify $(3x^3 - 8x^2 - 41x + 30) \div (3x - 2)$
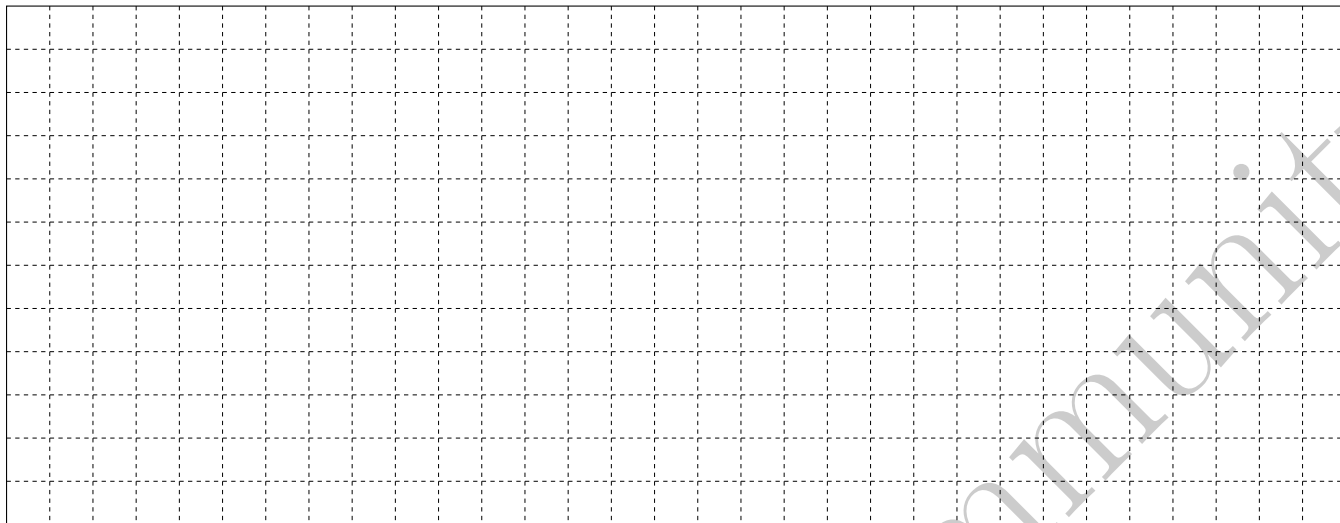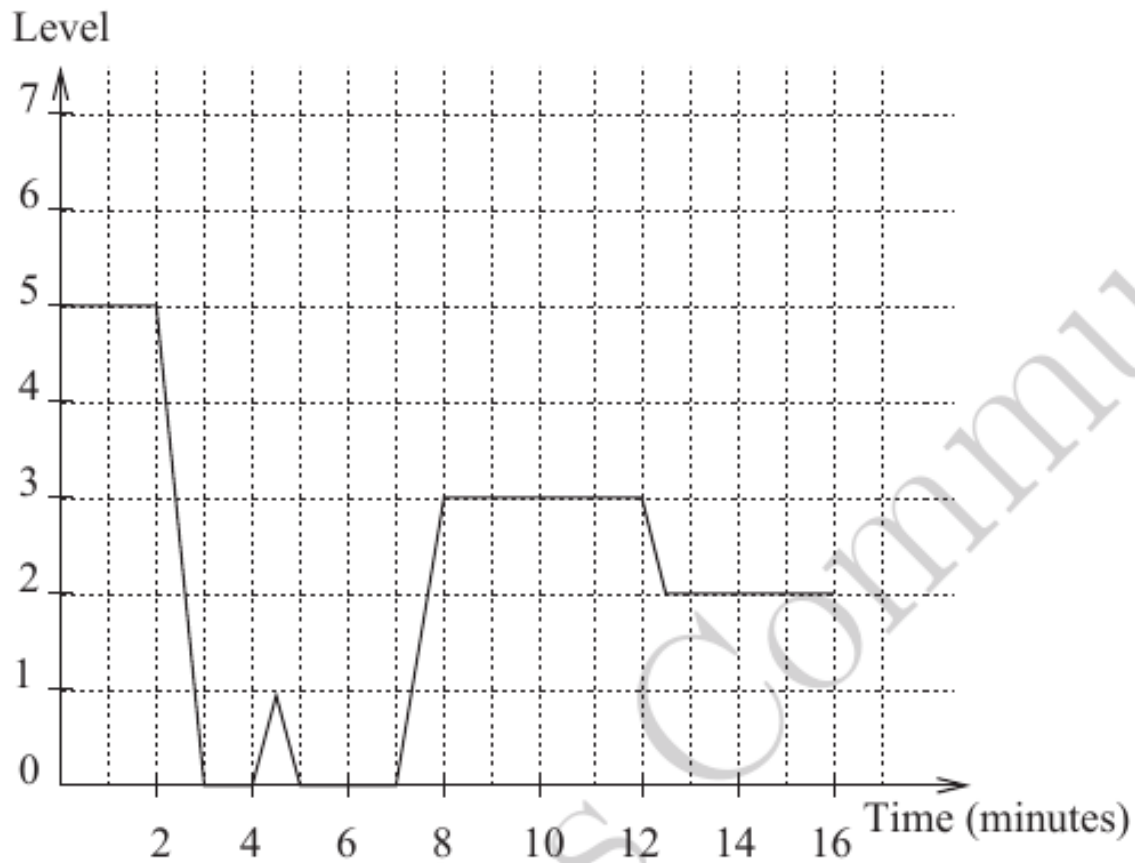
(c) Solve the simultaneous equations

$$2x + 3y = 13$$
$$-3x + 4y = 6$$
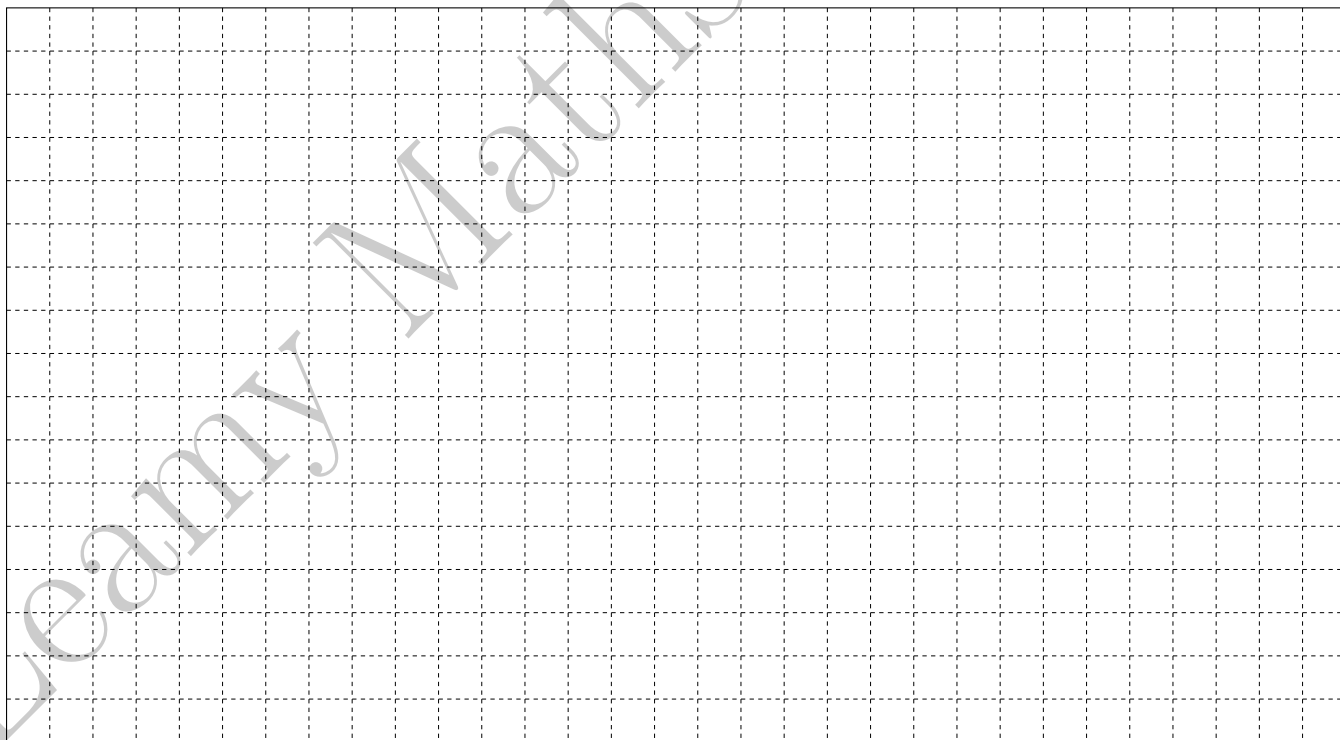
The curve below represents the position of a lift in a block of flats:

(a) Complet the following description:

- Between $t = 0$ and $t = 2$, the lift is at the 5th level.
- Between $t = 2$ and $t = 3$, the lift travles from level 5 to level 0.
- _____
- _____
- _____
- _____
- _____
- _____
- _____
- _____

(b) Floors are 3 metre high. How many metres has the lift travelled up and down beween $t = 0$ and $t = 16$
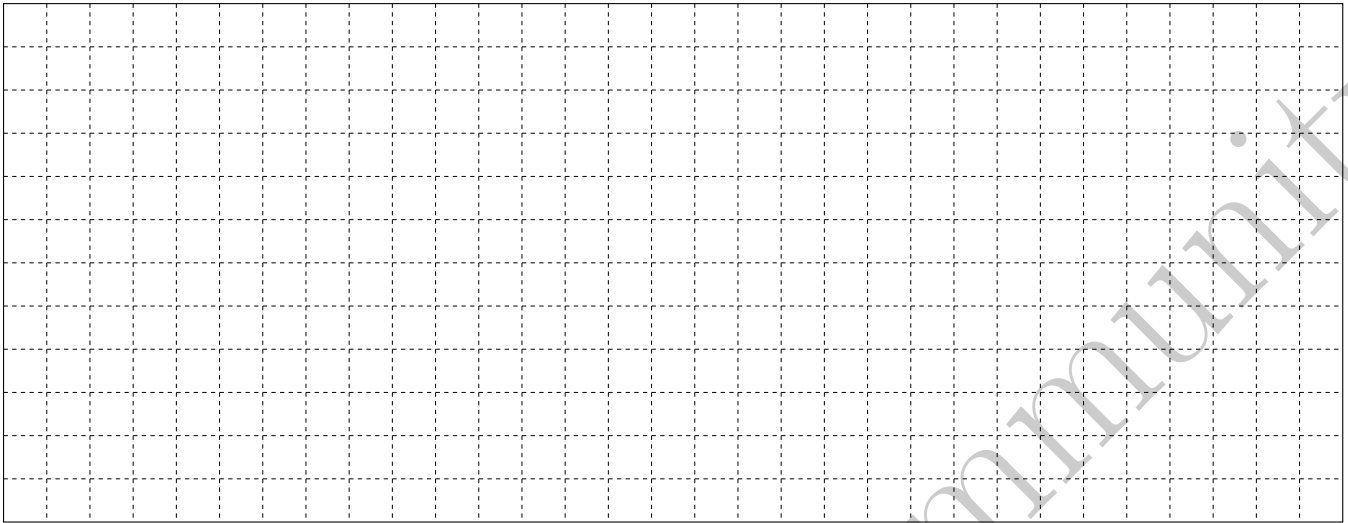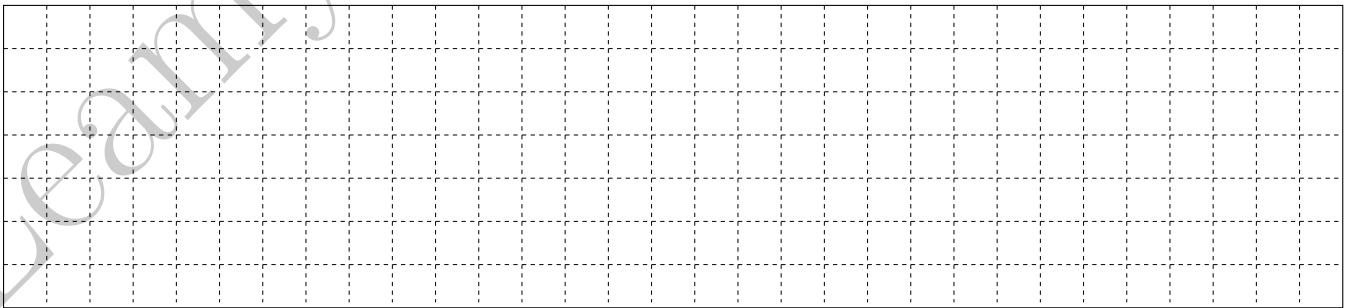
(c) Jack and Mary live in the block of flat. Jack lives on the fifth floor and Mary on the fourth floor. Jack takes the lift to go down while Mary uses the stairs: she needs 30 seconds to go down one floor.

    (i) Which section of the curve corresponds to Jack going down with the lift?
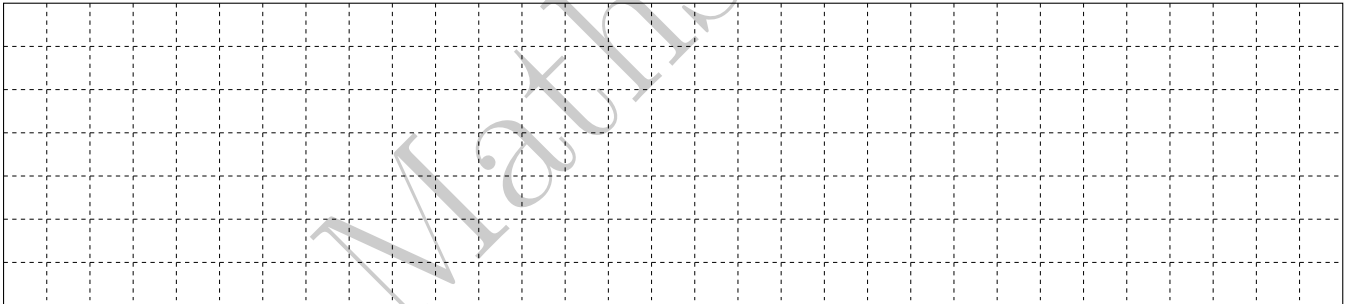
    (ii) How long will Mary need to reach the ground floor?

(iii) When should Mary start going down so she reaches the ground floor at the same time as Jack?

Question 10 (Suggested maximum time 5 minutes)

A pendulum is a weight suspended by a rod of length L which can oscillate. When moving, it takesthe pendulum T seconds to come back to its original position. The time T may be calculated as

$$T = 2\pi\sqrt{\frac{L}{g}}$$

where g=9.8 m/s^2 is the acceleration of gravity.

(a) Calculate the time T when L=0.6m. Give your answer correct to 3 decimal places.

(b) Express L in terms of g and T.

Question 11 (Suggested maximum time 15 minutes)

Eimear decides to change her gas provider. She goes for a plan with the following characteristics

Units used	Cost in Euro
500	58
1000	92
1500	126
2000	160
2500	194
3000	228
3500	262
4000	296

(a) Use the data above to investigate whether the pattern in the table above is linear, quadratic or exponential. Explain your conclusion

(b) The prices above do not include VAT. Eimear used 2000 units and her total bill was €181.6. Find the VAT rate on gas, correct to one decimal place

(c) The VAT rates will change in the near future. The new rates will work as follows:

- 15% on the first €125
- 20% on the balance

If Eimear uses 1000 units, what will be her total bill?

(d) If Eimear uses 2000 units, what will be her total bill with the new VAT rates?
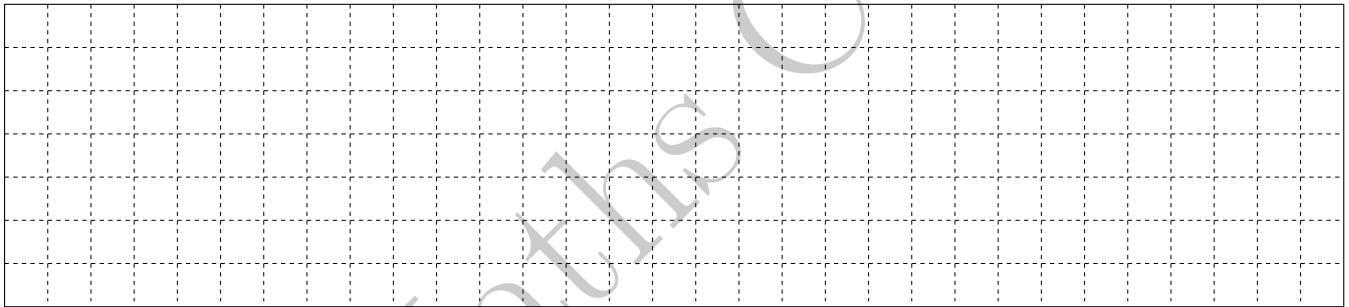
Question 12   (Suggested maximum time 10 minutes)

(a) Use your pythagoras theorem to find the value of the unknown side
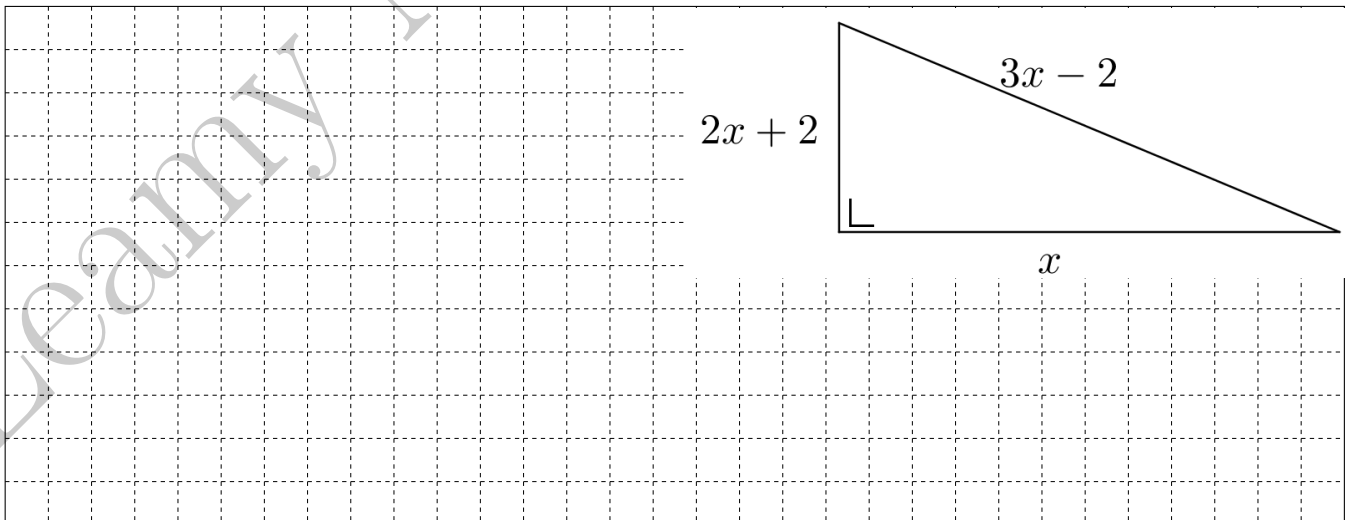in the triangle below.
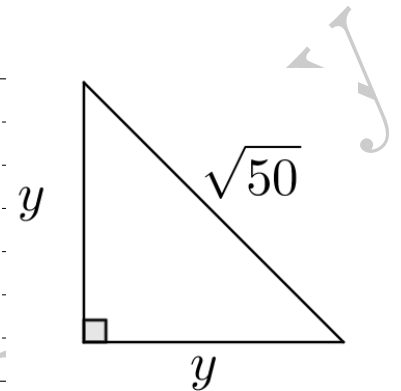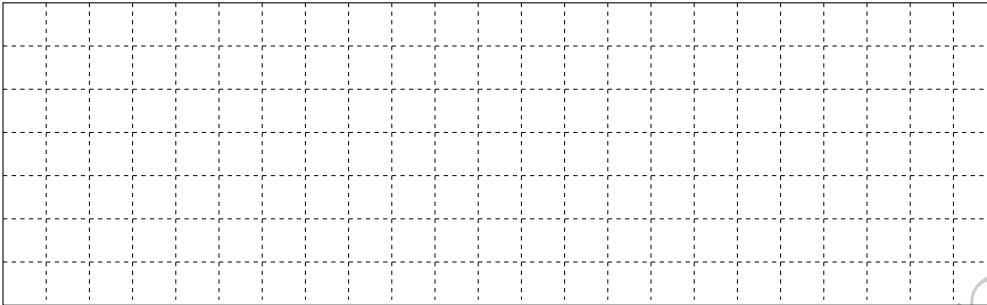
(b)   **(i)** Multiply out and simplify:

$$(3x - 2)^2$$

**(ii)** Hence, find the correct value for x in the diagram below.
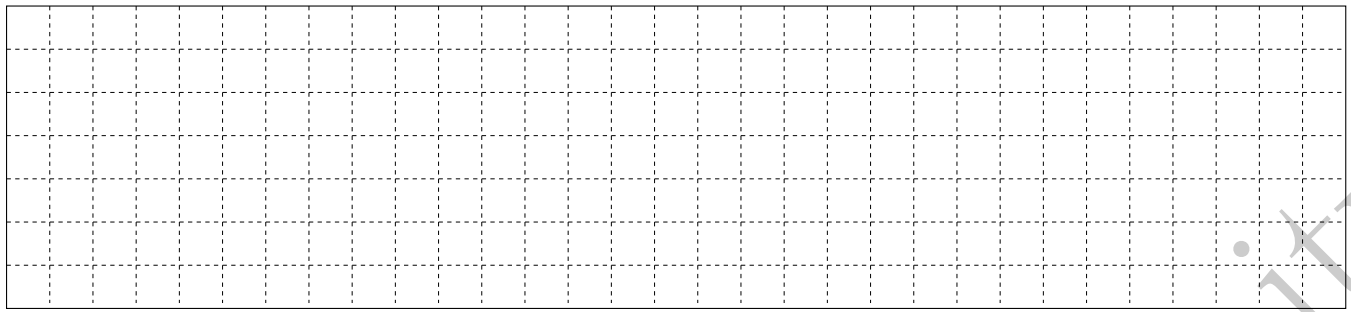
Question 13   (Suggested maximum time 15 minutes)

Frank and Greg are computer programmers. The have each created an app for mobile phone users. Frank's app has 1,000 users on 1st April 2017, whereas Greg's app has 5,000 users. They have also created functions which they use to describe the predicted growth in the number of users of their app:

$$\text{Frank's App:} \quad f(x) = 2^x$$
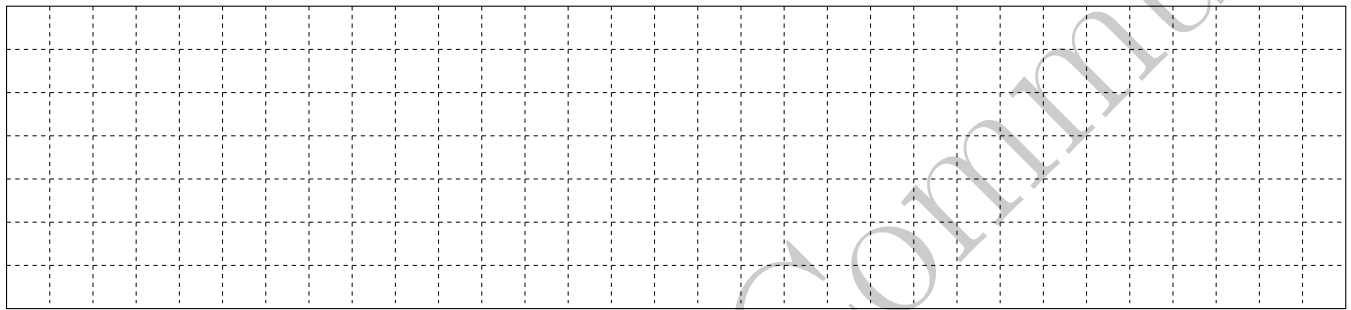$$\text{Greg's App:} \quad g(x) = 5 + 3x$$

where x is the number of months since April 1st 2017, and $f(x)$ and $g(x)$ describe the number of users each app has, in thousands. Label each graph clearly.

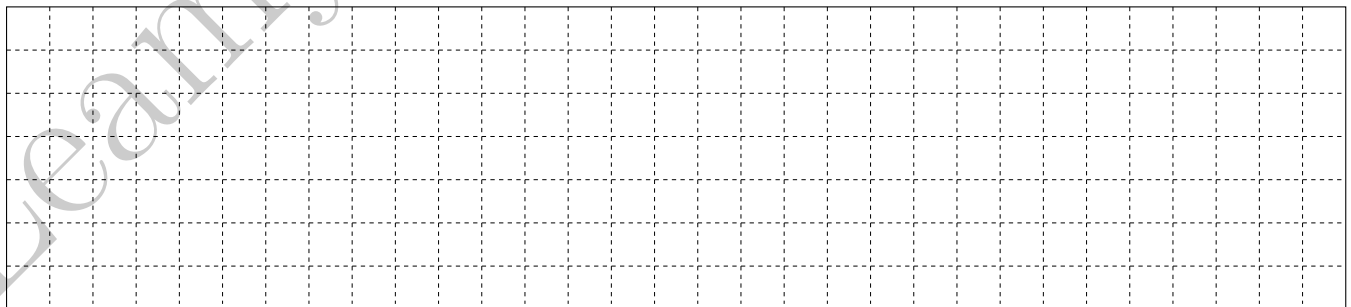**(a)** On the grid below, draw the graphs of $y = f(x)$ and $y = g(x)$ in the domain $0 \leq x \leq 5$
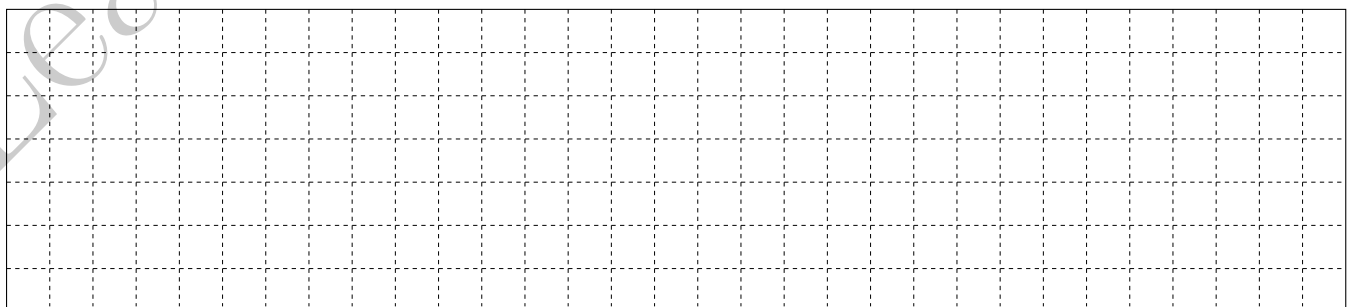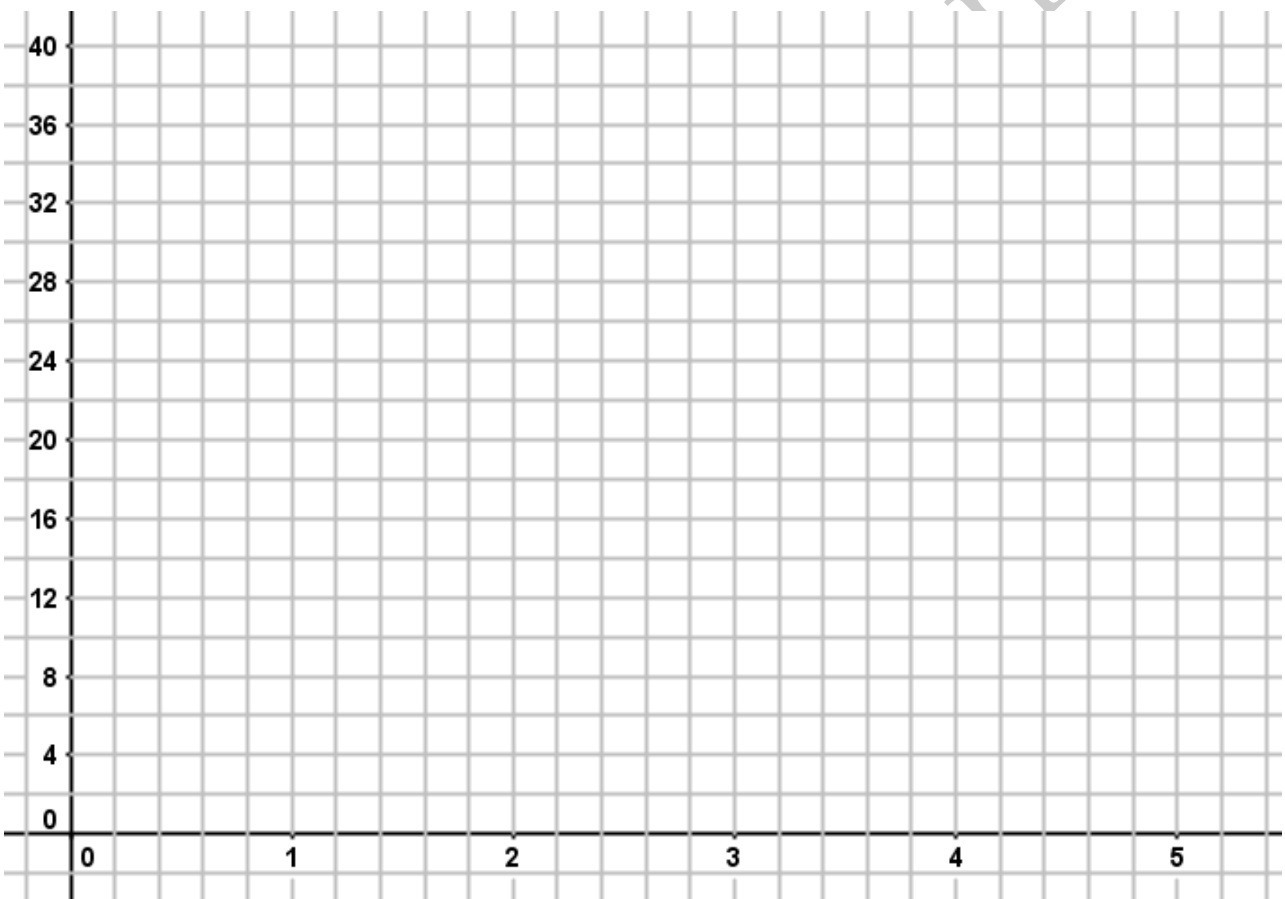
(b) How many users will each app have on the 1st June 2017.

(c) During what month Franks app reach 10,000 users?
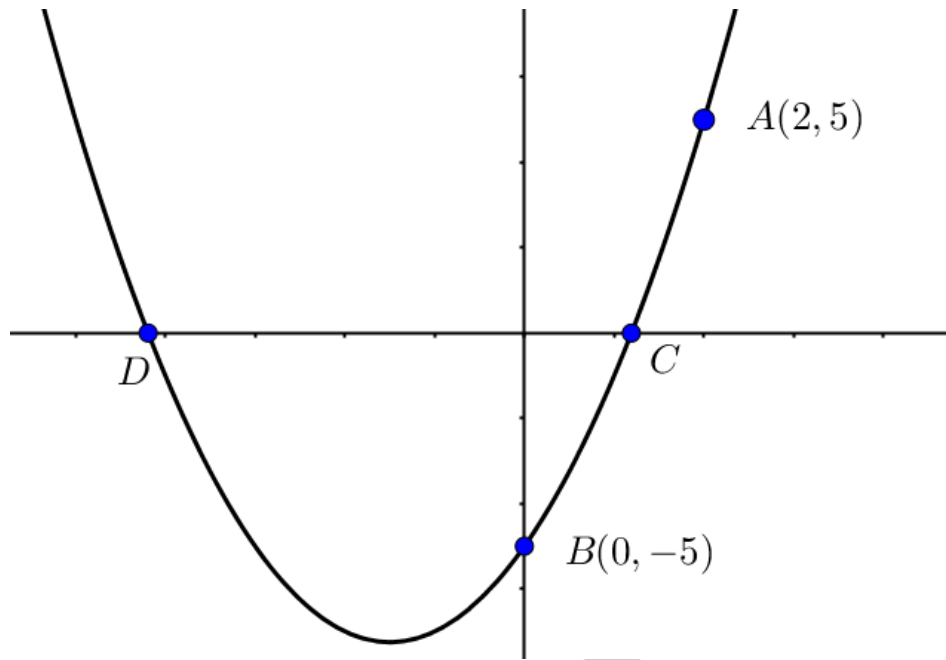
(d) During what month will the number of users for Frank's app overtake the number of users for Greg's app.
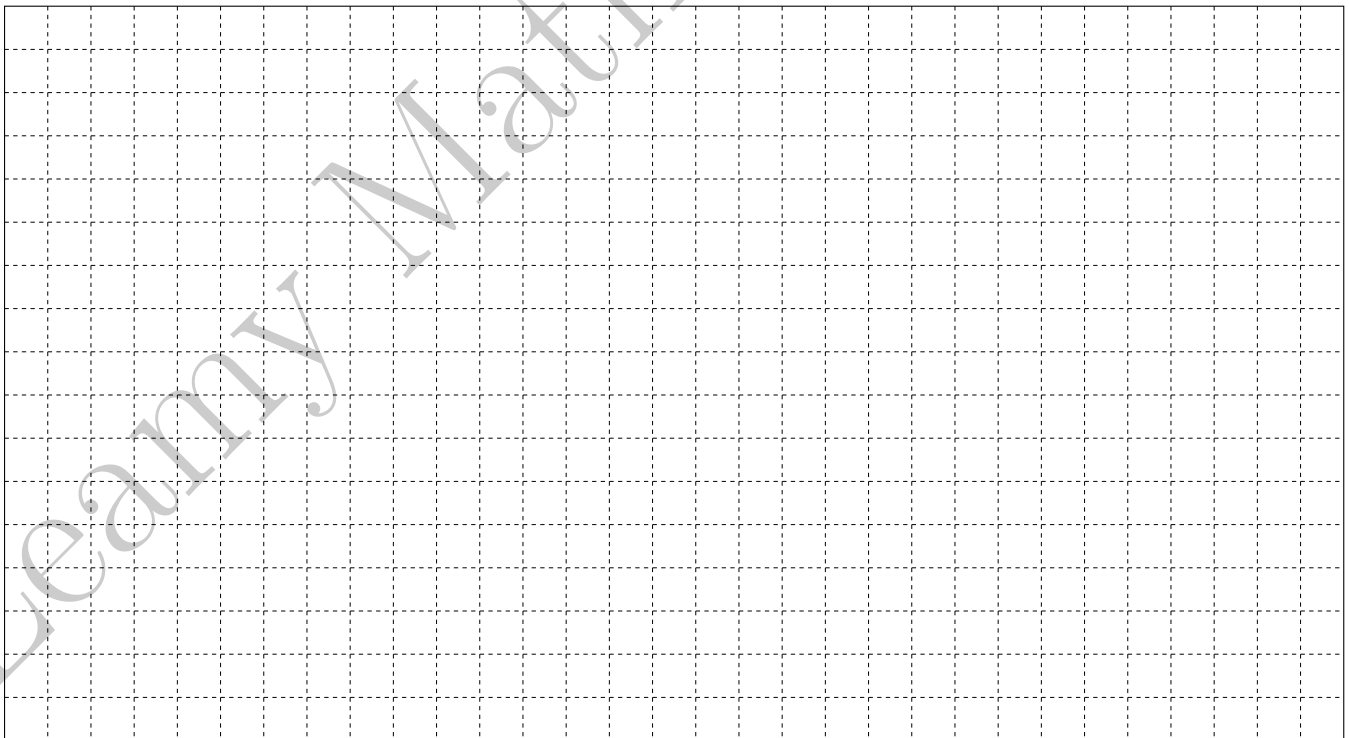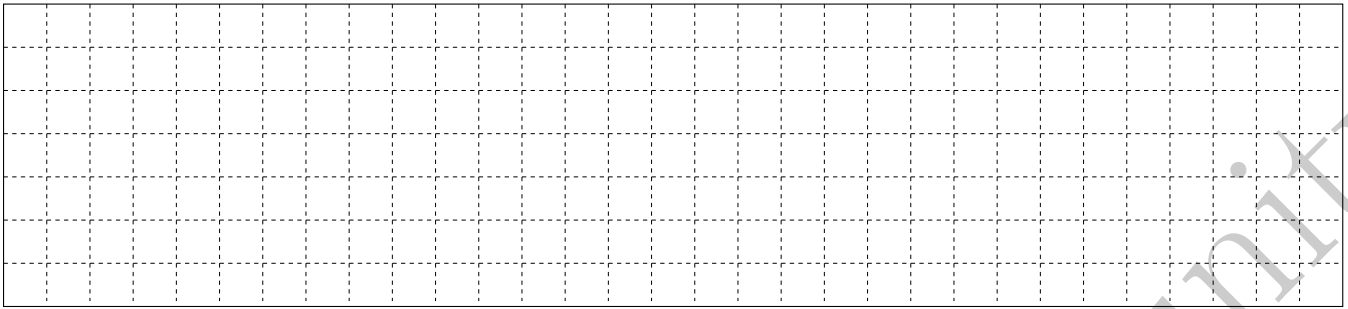
Question 14   (Suggested maximum time 15 minutes)

Below is a graph of the function $f(x) = x^2 + px + q$. The point $A(2, 5)$ and the point $B(0, -5)$ are on the function.
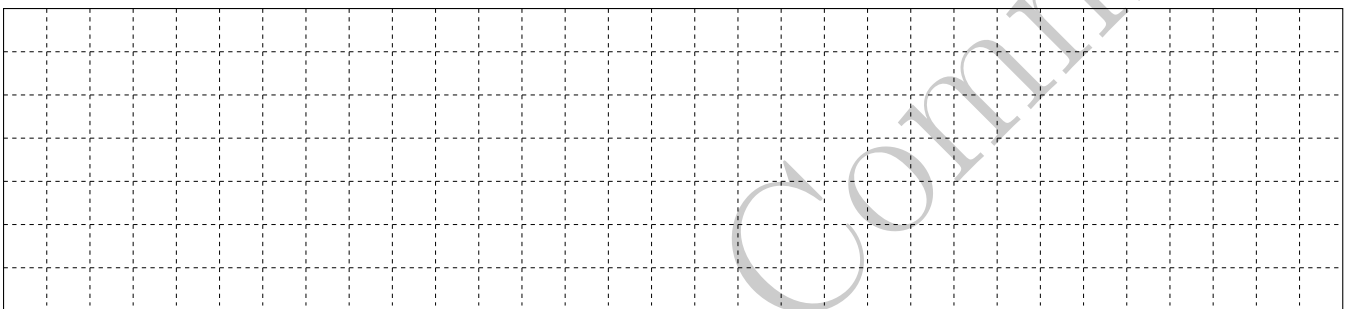

(i) Use the points A and B to find equations in p and q. Hence solve the equations to find the values of p and q

(ii) Hence, find the roots of $f(x) = 0$, giving your answer correct to one decimal place.

(iii) What are the co-ordinates of the points C and D?

Rough Work