Leaving Certificate Examination, 2014

Sample paper prepared by Leamy Maths Community

Mathematics

Project Maths - Phase 2

Paper 1

Higher Level

Saturday 25 April

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Leamy Maths Community

300 marks

Sample Instructions

There are two sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer questions as follows:

In Section A, answer all six questions.
In Section B, answer all three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination.

You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	100 Marks

Answer **all six** questions from this section.

Question 1 **(25 Marks)**

(a) If (x+1) is a factor in the cubic function $f : x \mapsto x^3 - 2x^2 + ax + 2$, find the value of a.

(b) Find the other two roots of the function.

(c) Using parts (a) and (b) or otherwise, solve the equation

$$y^6 - 2y^4 + ay^2 + 2 = 0$$

where a is the value calculated above(Hint substitute $x = y^2$)

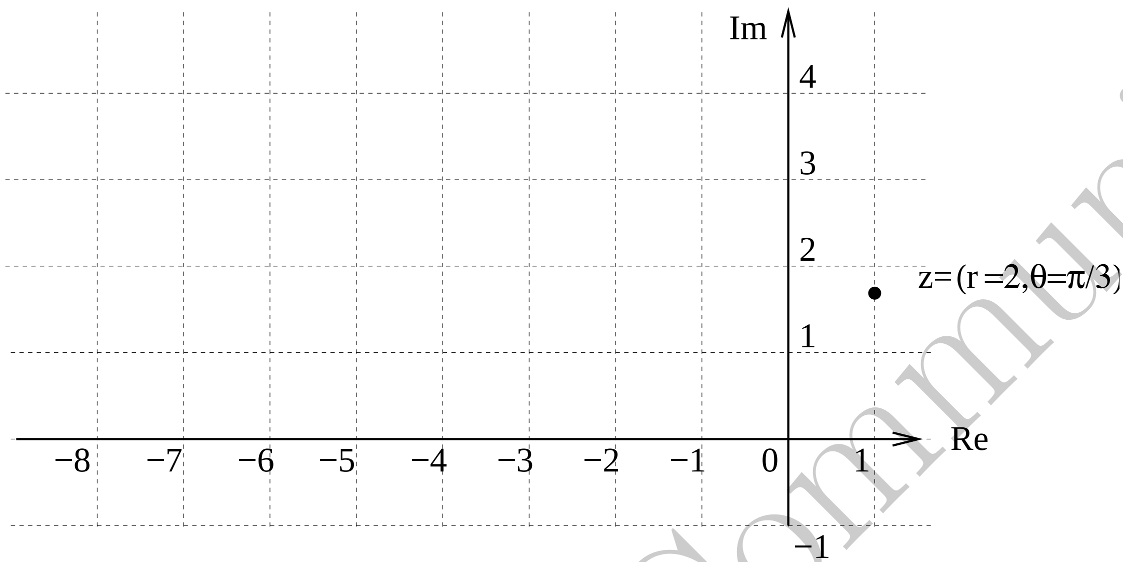
Question 2 **(25 Marks)**

(a) Expand $(\cos\theta + i\sin\theta)^2$.

(b) Using de Moivre's theorem, calculate $(\cos\theta + i\sin\theta)^2$ and show that

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ \sin(2\theta) &= 2\cos\theta\sin\theta\end{aligned}$$

(c) The complex number z is defined by its modulus $|z| = 2$ and its argument $Arg(z) = \pi/3$. Using de Moivre's theorem, place the points z^2 and z^3 on the graph. Label the points and give their coordinates in polar form



Question 3 **(25 Marks)**

(a) Solve the following inequalities

(i)

$$\frac{x-2}{x-3} < 2 \ .$$

(ii)

$$e^{2x+1} > 4 \ .$$

(b) Solve the simultaneous equations

$$\begin{cases} x + y + z = 4 \\ x + 2y + 3z = 9 \\ 2x + 4y - z = -3 \end{cases}$$

Question 4 **(25 Marks)**

(a) The population of the urban area in Limerick grows exponentially

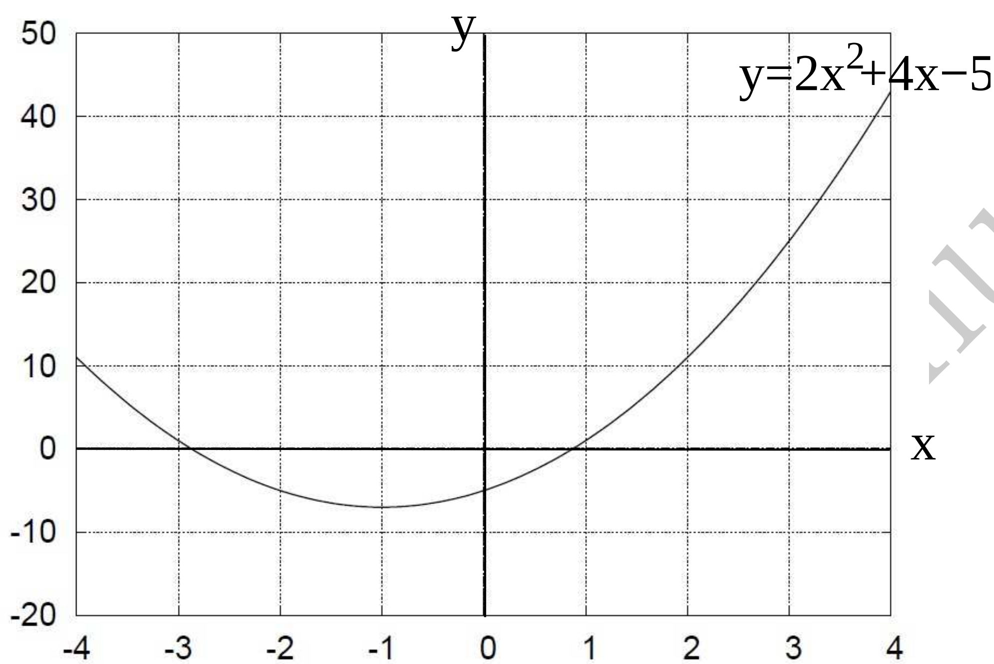
$$p = Ae^{Bt}$$

where p is the population and t is the time in years. In 2011, there were 91,000 people in the urban area of Limerick. Assuming there are 92,500 in 2014 (3 years later), calculate A and B.

(b) Using the values of A and B calculated above, when will the population of the urban area reach 100,000 people?

Question 5 **(25 Marks)**

(a) Find graphically for what values of x do the two curves $y = 2x^2 + 4x - 5$ and $y = x^2 + 6x - 2$ cross and verify your answers algebraically.



(b) Shade the area between the two curves and use calculus to evaluate the value of the area.

Question 6 **(25 Marks)**

(a) A dairy farmer has x cows. Each cow produces an income of 20 euros per day. It costs the farmer $x^2 - 120x + 2100$ euros everyday to house and feed the cows. Show that the daily profit is

$$\pi(x) = -x^2 + 140x - 2100$$

Justify your calculation.

(b) What number of cows maximises this profit?

(c) Calculate the maximum profit and the break even points.

Section B Contexts and Applications 150 Marks

Answer **all three** questions from this section.

Question 7 **(50 Marks)**

Séan takes out a three year loan at the bank to buy his new €20000 car.

(a) Every month, Séan pays an interest rate of 0.5%

(i) Calculate the annual equivalent interest rate

(ii) Another bank offers Séan an annual equivalent rate of 6.5%. What should he do?

(b) Séan will make his first payment of value A in a month time. Using the monthly interest rate r=0.5%

(i) Calculate the present value of the first payment (payment he will make in one month)

(ii) Using **induction**, show that the sum of the present values of monthly payments is

$$PV = A\frac{(1+r)^n - 1}{r(1+r)^n}$$

Calculate the value for 3 years (=36 months)

(c) What monthly payment should Séan pay back every month?

(d) How much will he pay back in total?

(e) Why are the two figures different?

Question 8 **(50 Marks)**

A producer of hot water cylinders wants to build his product with a brand new material with excellent thermal properties but quite expensive. His objective is to manufacture cylindrical hot water cylinders of radius r and height h.

(a) To start with, the producer wants to know the optimal shape.

(i) Draw a picture of the cylinder and calculate its volume V and surface A (including the top and bottom surfaces) as a function of the radius r and the height h.

(ii) Show that the volume may be expressed as

$$V = \frac{r}{2}\left(A - 2\pi r^2\right)$$

(iii) Calculate the value of the radius which maximises the volume of the cylinder as a function of A

(iv) Calculate the value of the corresponding values for the height.

(v) Calculate the corresponding value of the volume.

(b) The producer now wants to optimise its offer and determine the optimal area he should use.

(i) Plot the volume of the tank as a function of areas between 0 and 20m^2 using appropriate scales

$$V = \frac{A^{3/2}}{3\sqrt{6\pi}}$$

(ii) Read on the curve what area corresponds to a volume of 2.5m^3. Explain your method

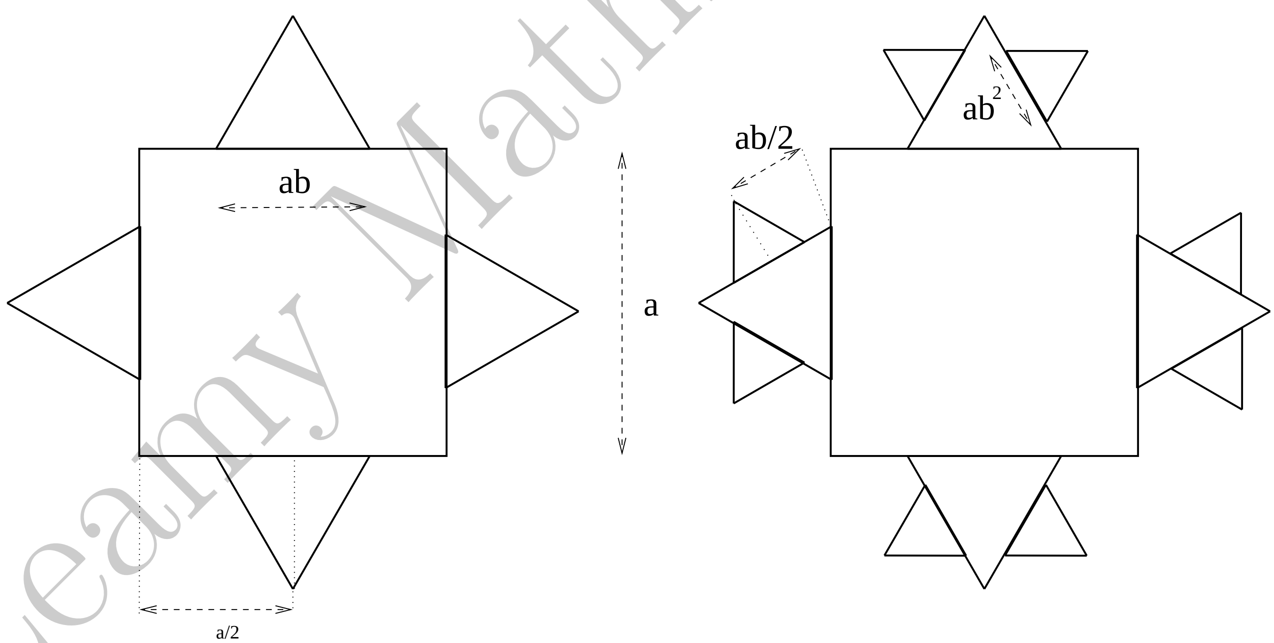
(iii) Retrieve the result by solving the appropriate equation.

Question 9 **(50 Marks)**

(a) Show that the area of an equilateral triangle with sides of dimension a is

$$Area = \frac{a^2\sqrt{3}}{4}$$

(b) A geometric figure is constructed as shown below



Level 1 Level 2

(i) Explain how you would construct the next level

(ii) Complete the table below

Level	1	2	3	4	5
Number of added triangles	4	8			
Measure of the side of an added triangle	ab	ab^2			

(iii) Show that the side of a triangle added at level n is ab^n. Calculate the area of an added triangle at this level.

(iv) Show that all the triangles added at level n have a total area of

$$Area_n = \frac{2^{n+1}\sqrt{3}a^2 \times b^{2n}}{4}$$

(v) Show that the series $Area_n$ is a geometric series of ratio $2b^2$

(vi) For what values of b does the sum of $Area_n$ to infinity exist? Give the the value of this infinite sum

Rough Work