Leaving Certificate Examination, 2014

Sample paper prepared by Leamy Maths Community

Mathematics

Project Maths - Phase 2

Paper 2

Higher Level

Sunday 26 April

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Leamy Maths Community

300 marks

Sample Instructions

There are three sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	2 questions

Answer **all eight** questions as follows:

In Section A, answer

- Questions 1 to 5 and
- **either** Question 6A **or** Question 6B

In Section B, answer both Question 7 and Question 8.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination.

You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A Concepts and Skills 150 Marks

Answer **all six** questions from this section.

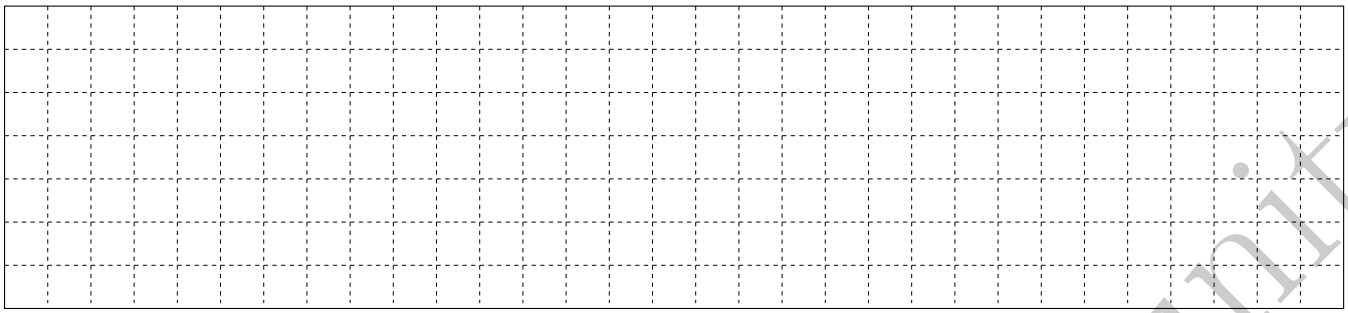
Question 1 **(25 Marks)**

(a) How many ways are there to choose 3 cards out of a pack of 52 cards?

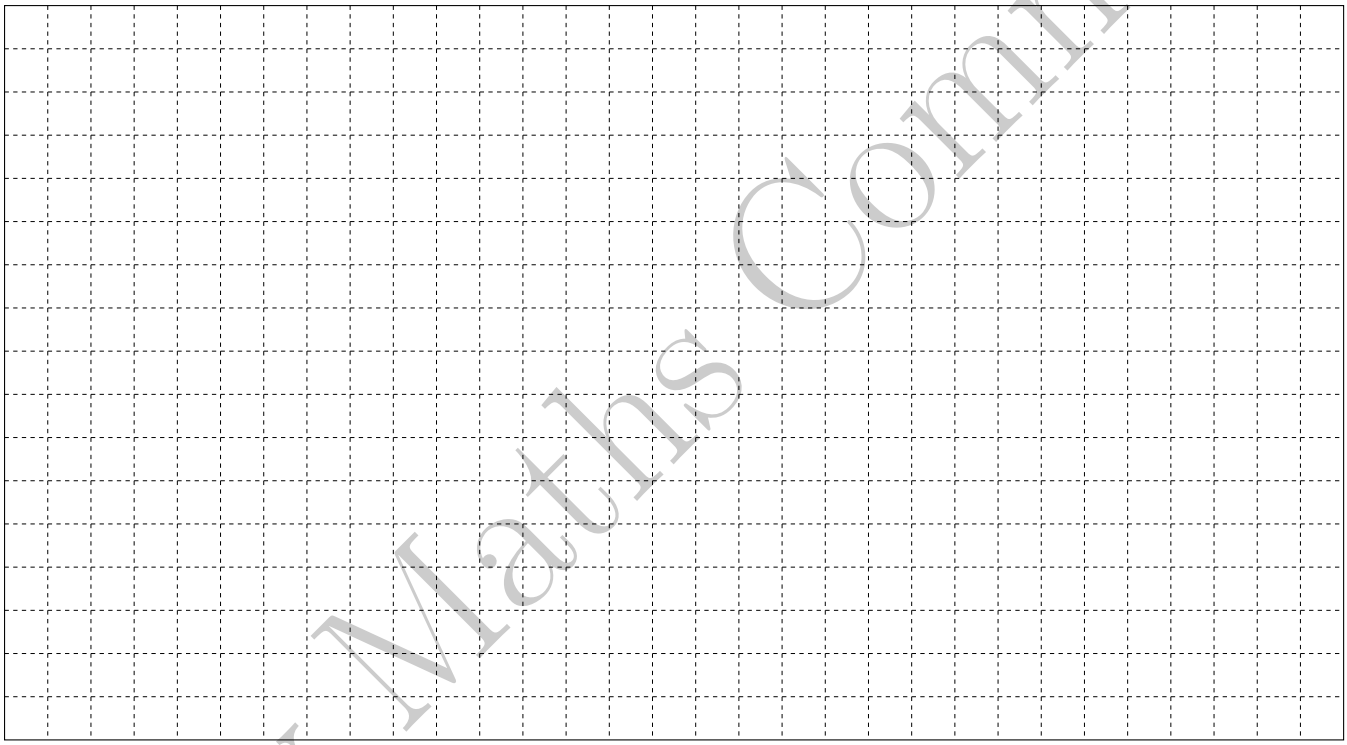
(b) How many ways are there to choose 3 Kings from the 4 Kings available in the deck of cards?

(c) If you pick 3 cards from a deck of 52 cars, what is the probability to get three Kings?

(d) What is the probability to pick no King?



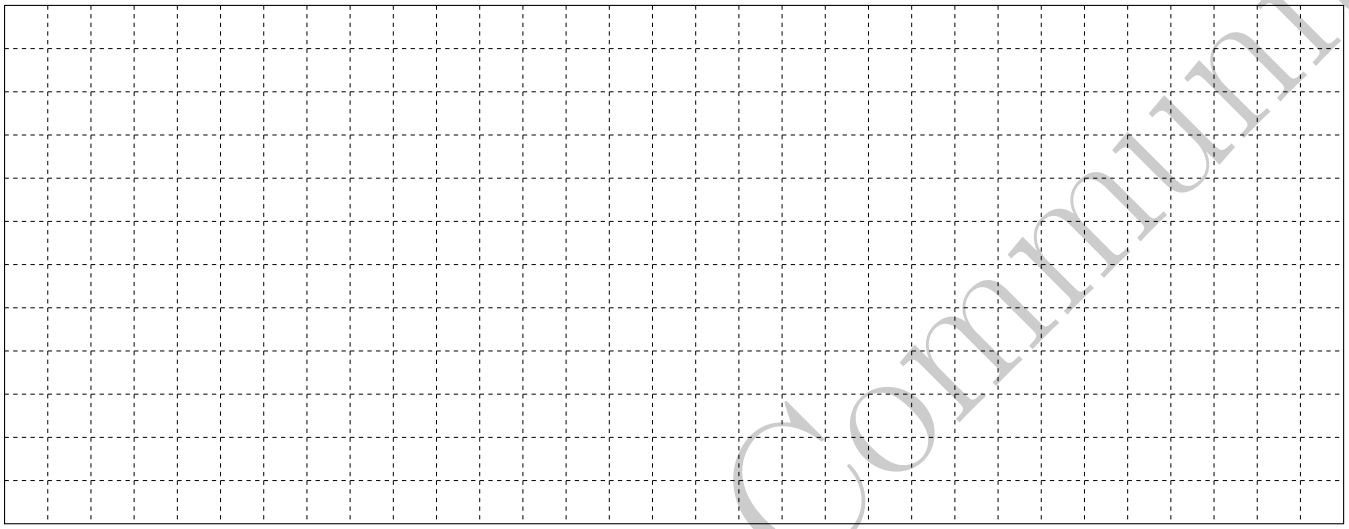
(e) What is the probability to pick 2 Kings or more?



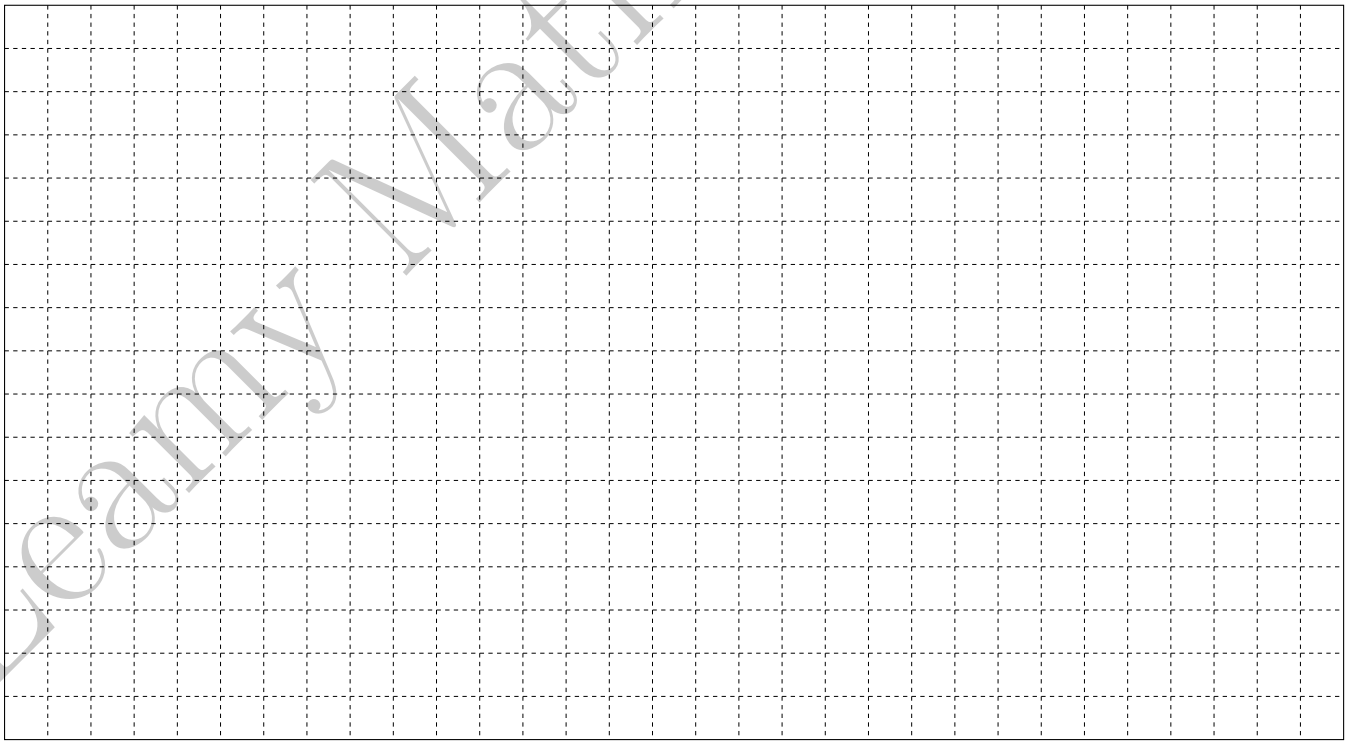
Question 2 **(25 Marks)**

There are 65 students in 6^{th} year. 30 play football, 30 play hurling and 30 play rugby. 6 don't play any of these 3 sports while 3 practice all three. 12 only play rugby, 10 only play football and 9 only play rugby.

(a) Draw a Venn diagram



(b) How many students play football and rugby, football and hurling and rugby and hurling? (Include the size of the class in one of your equations)



(c) If you pick a student randomly, what is the probability the student plays hurling?

(d) If you pick a student randomly, what is the probability the student does not play football?

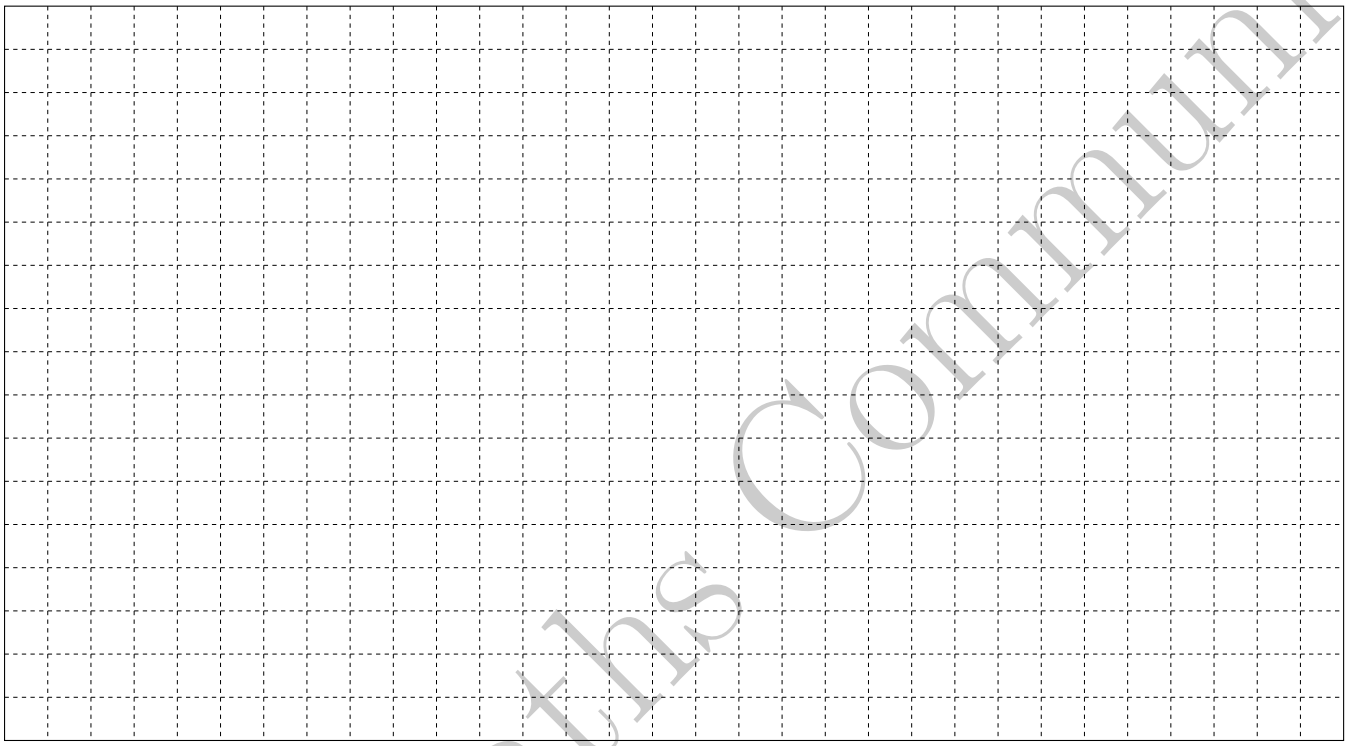
(e) Given that a student plays football, what is the probability that this student also plays at least one other sport

Question 3 **(25 Marks)**

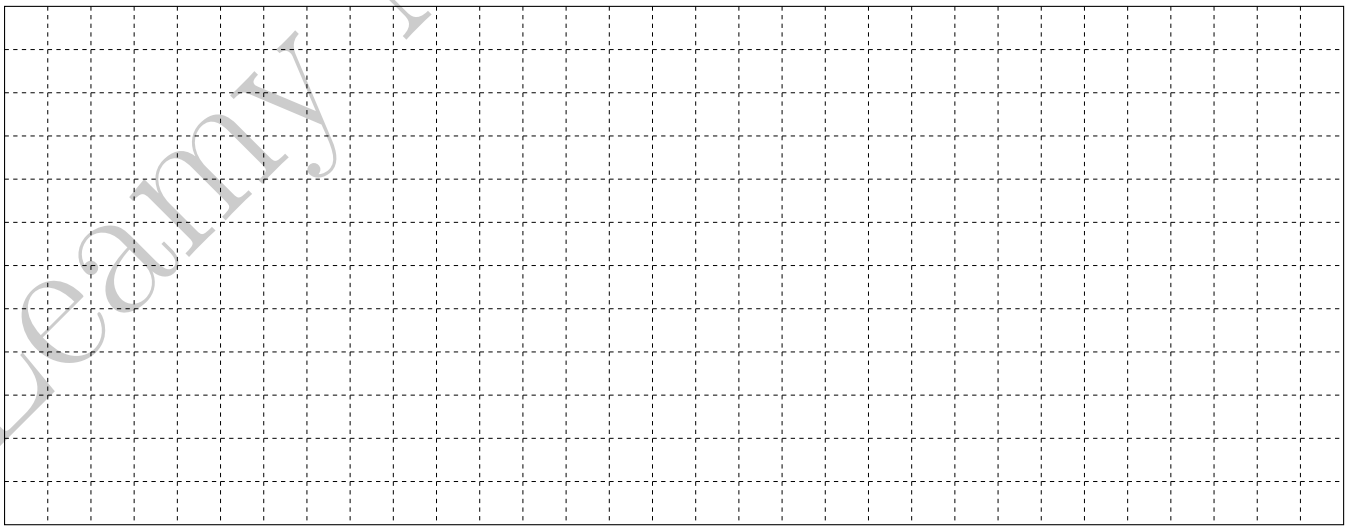
(a) Calculate the distance between the line (L)

$$y = \frac{3}{4}x - 1$$

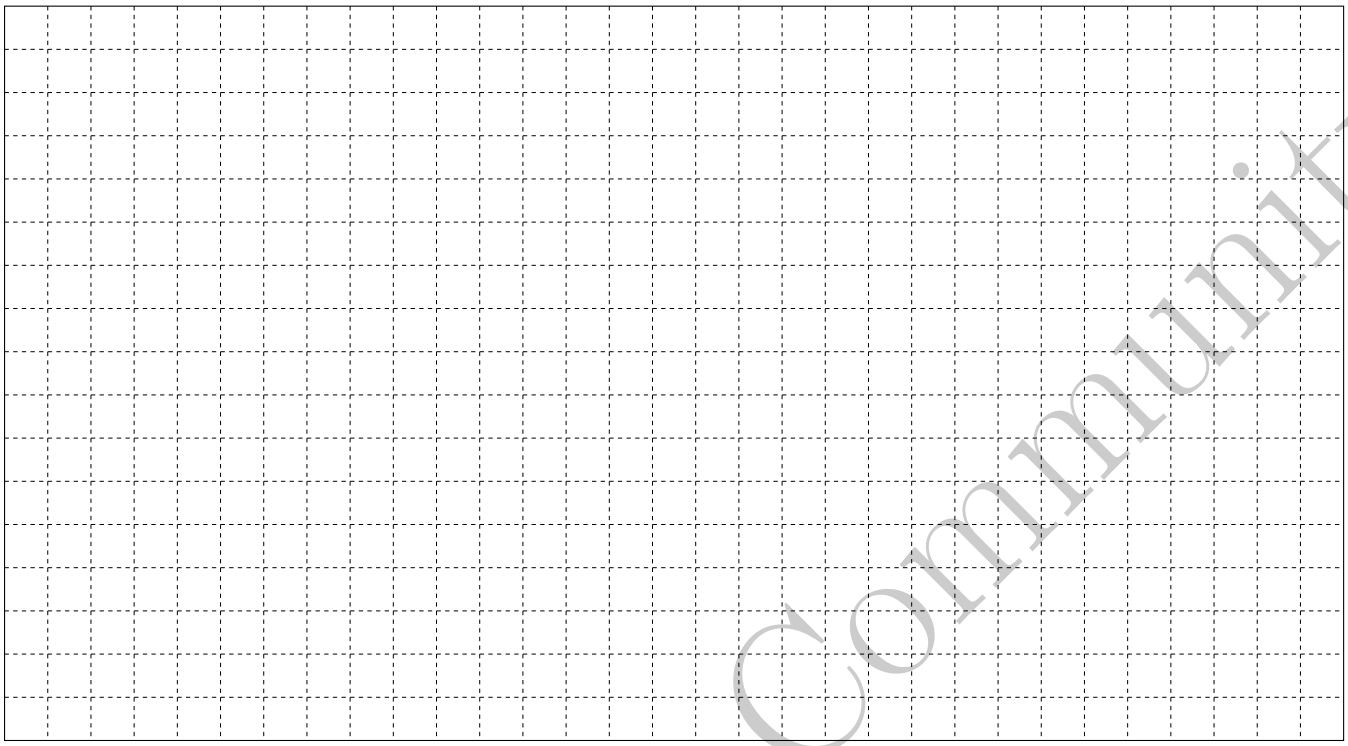
and the point P $(4, \ -1)$



(b) Calculate the equation of the circle (C) of centre $(4, \ -1)$ and tangent to which line (L) is a tangent.



(c) Calculate the equation of the tangents to the circle (C) perpendicular to (L)

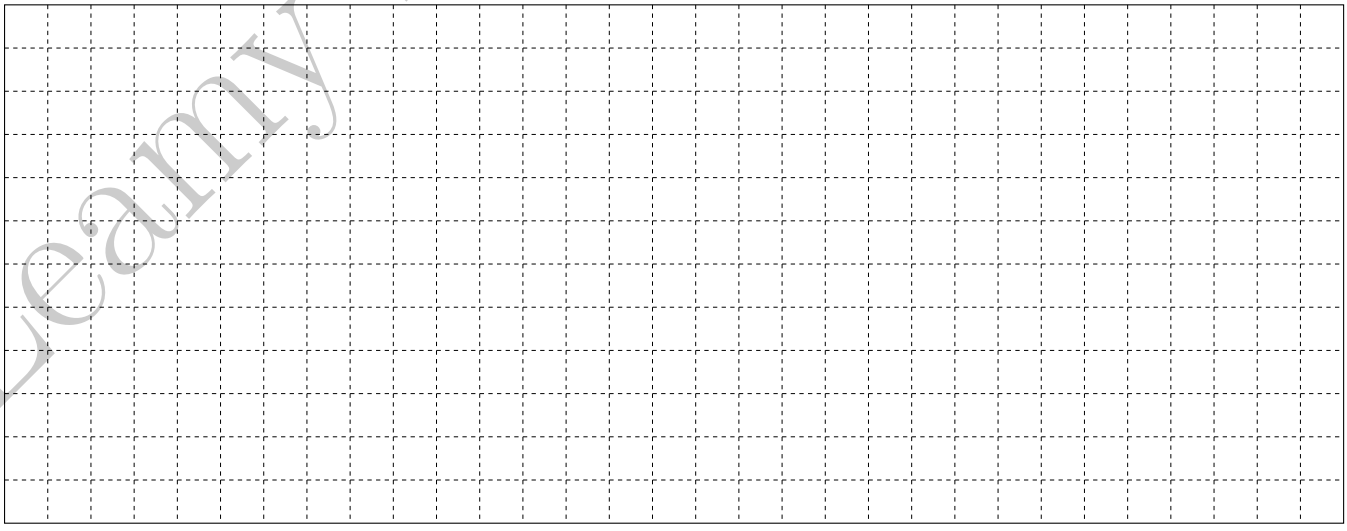


Question 4 **(25 Marks)**

(a) Solve the following equations where x is in radians

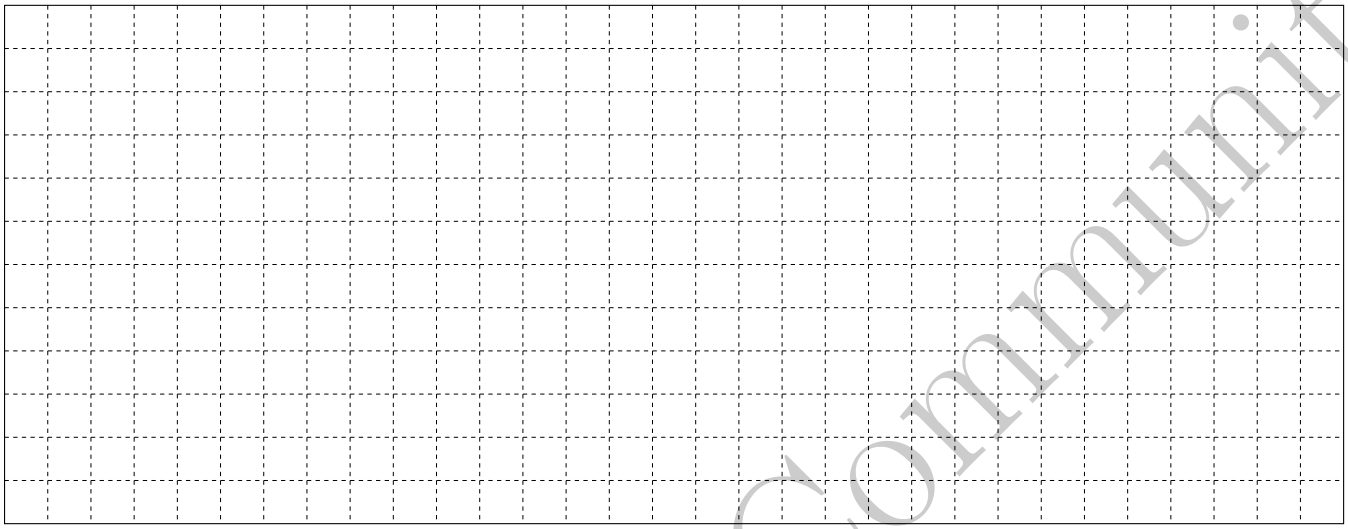
(i)

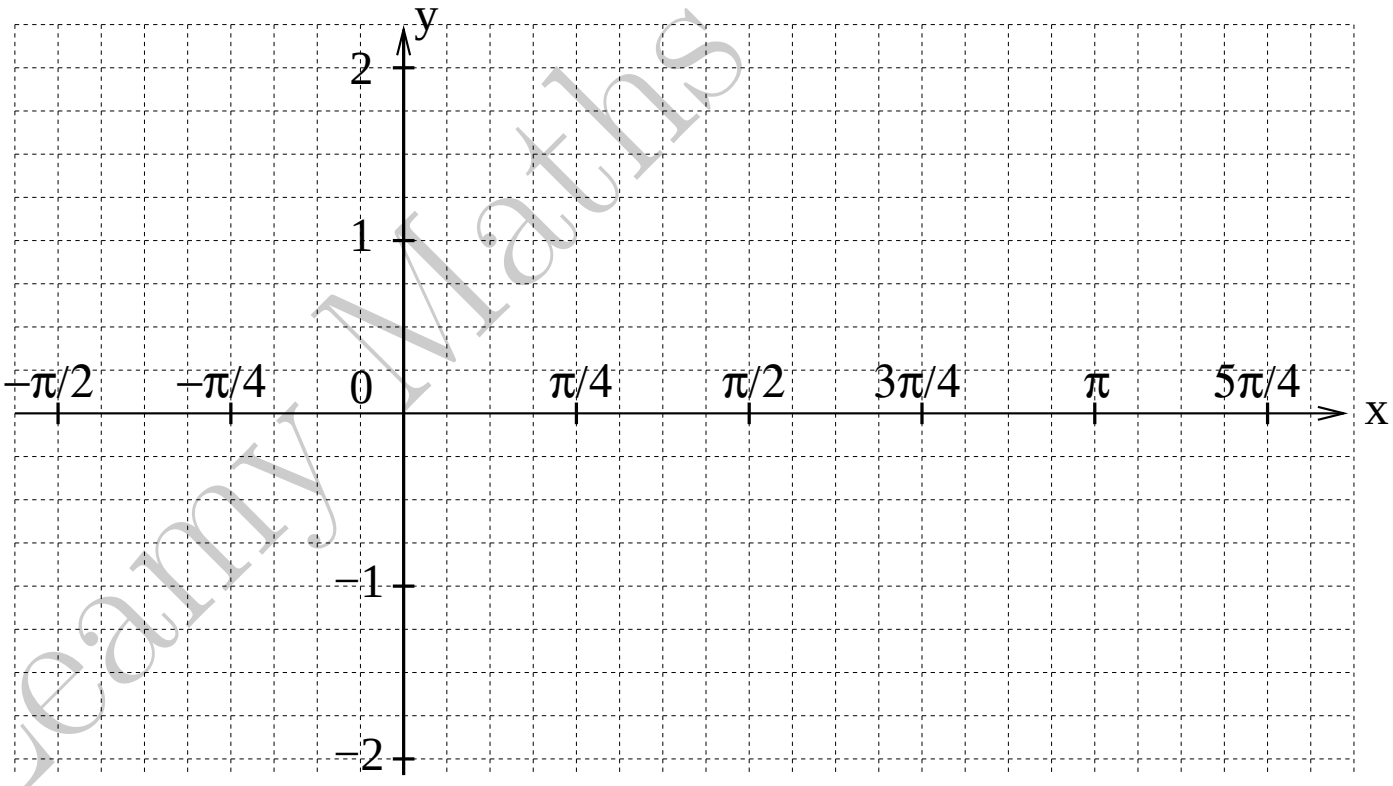
$$\sin x = -\frac{1}{2}\,, \qquad 0 \leq x \leq 2\pi$$



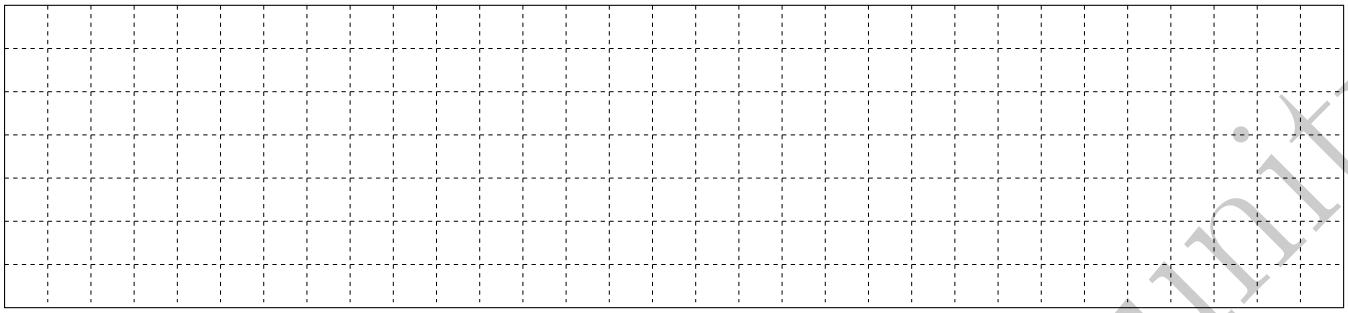
(ii)

$$\tan 2x = 1 \ , \qquad 0 \leq x \leq \frac{3}{2}\pi$$

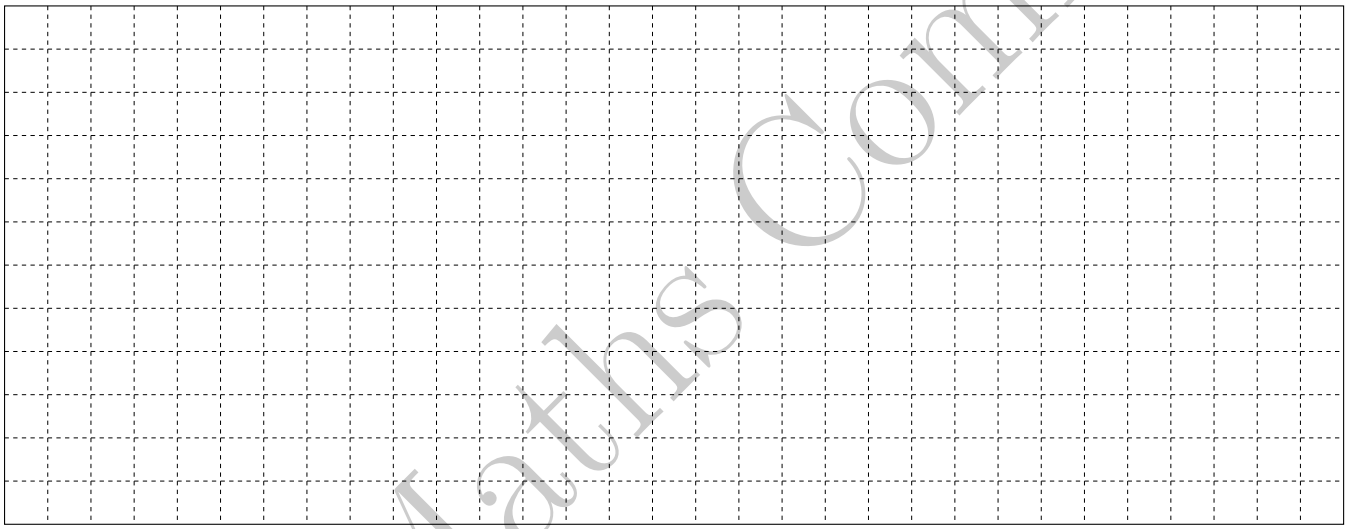




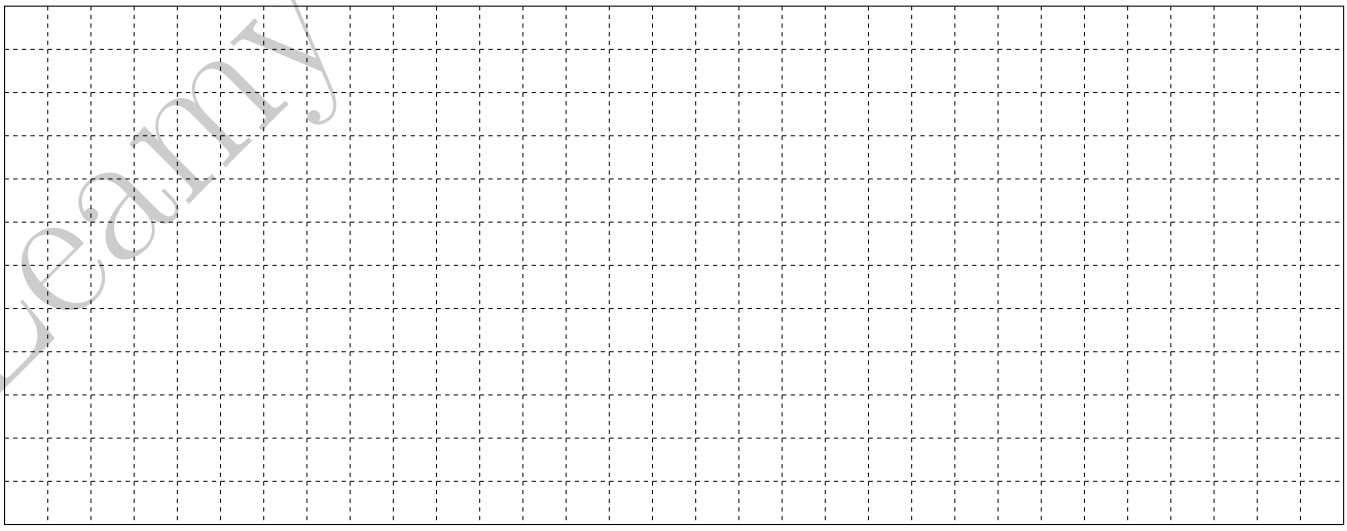
(b) Use the grid above to graph the function $f(x) = \sin(x)$ and illustrate how this can be used to verify the results of the equation of part (i).



(c) Plot the functions $g(x) = 2f(x)$ and $h(x) = f(2x)$. In each case, label the curve and specify the range and period of the function.

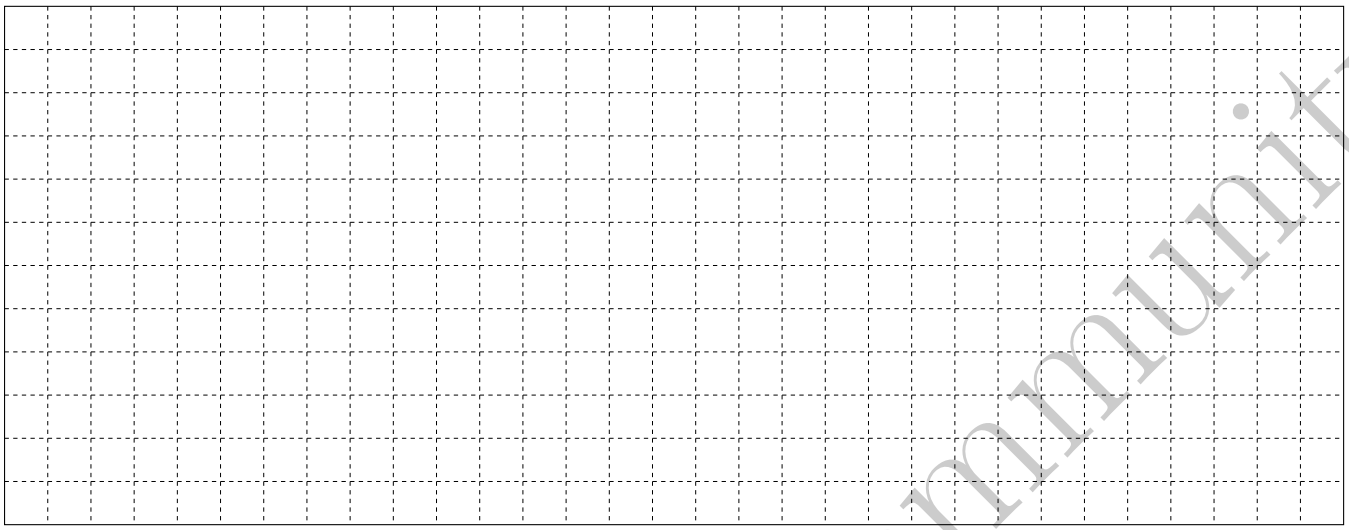


(d) Define and plot a function with a period $\pi/2$

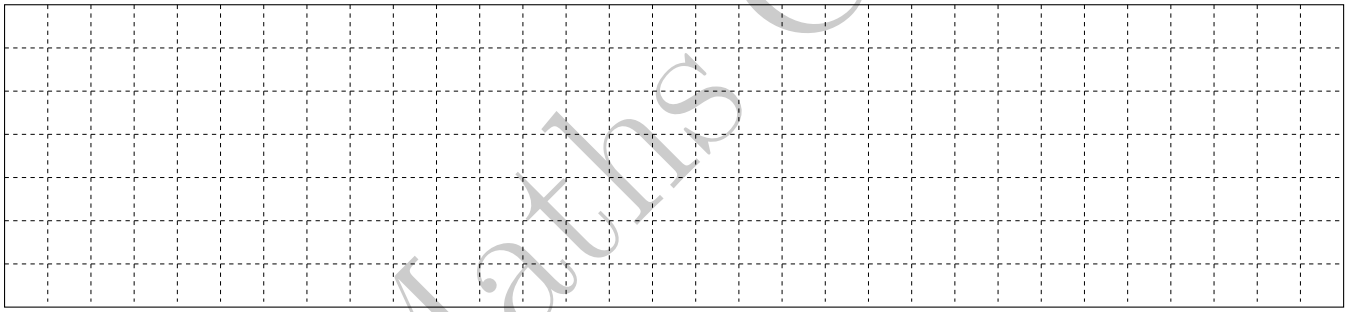


Question 5 **(25 Marks)**

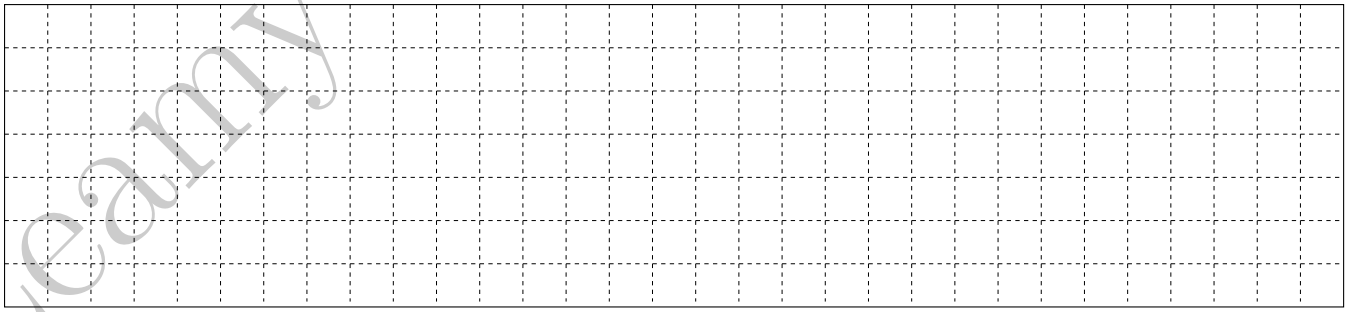
(a) Calculate the equation of the line parallel to $2x + y = 4$ which contains the point $(2,\ 5)$



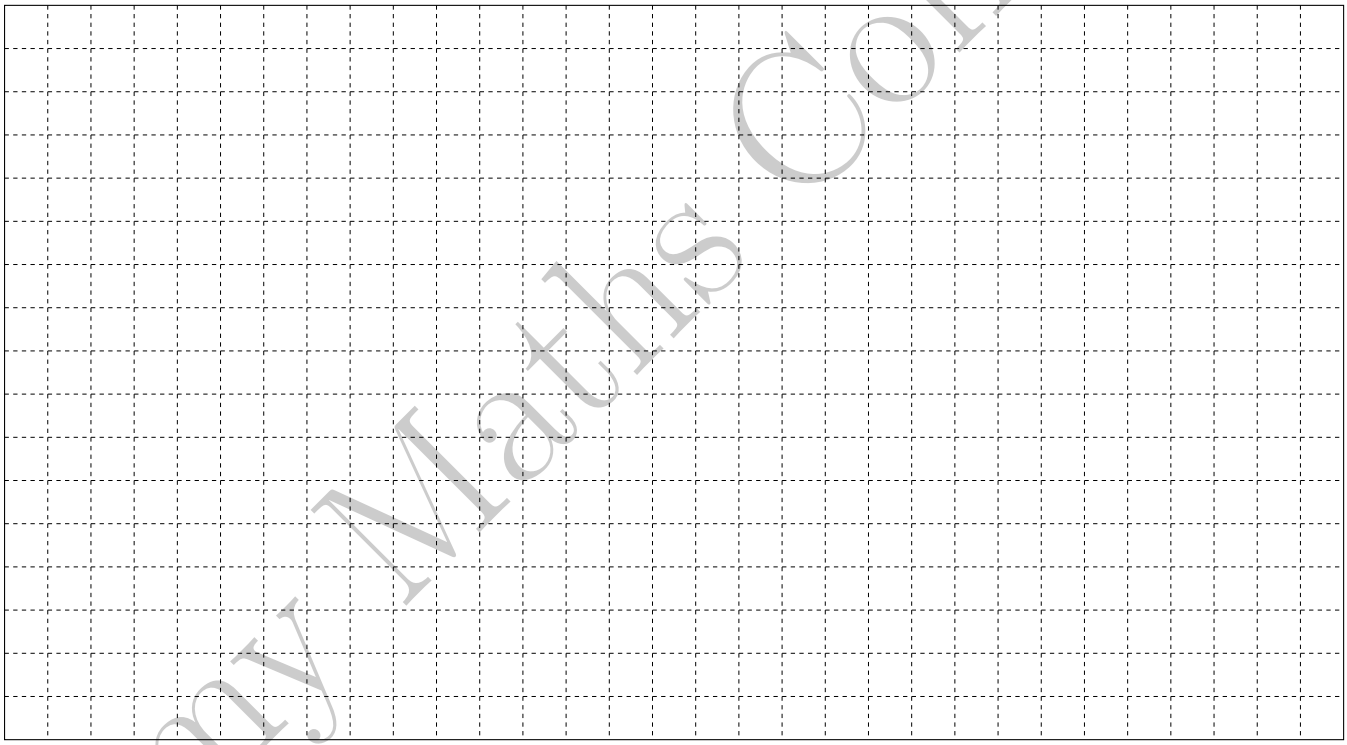
(b) What is the angle between the lines $y = 2x + 4$ and $y = x + 1$?



(c) Calculate the equation of the lines which make an angle of $45°$ with $2x + y = 4$ and contain the point $(4,\ 6)$.



(d) Calculate the intersection of these two lines with $y + 2x = 4$.

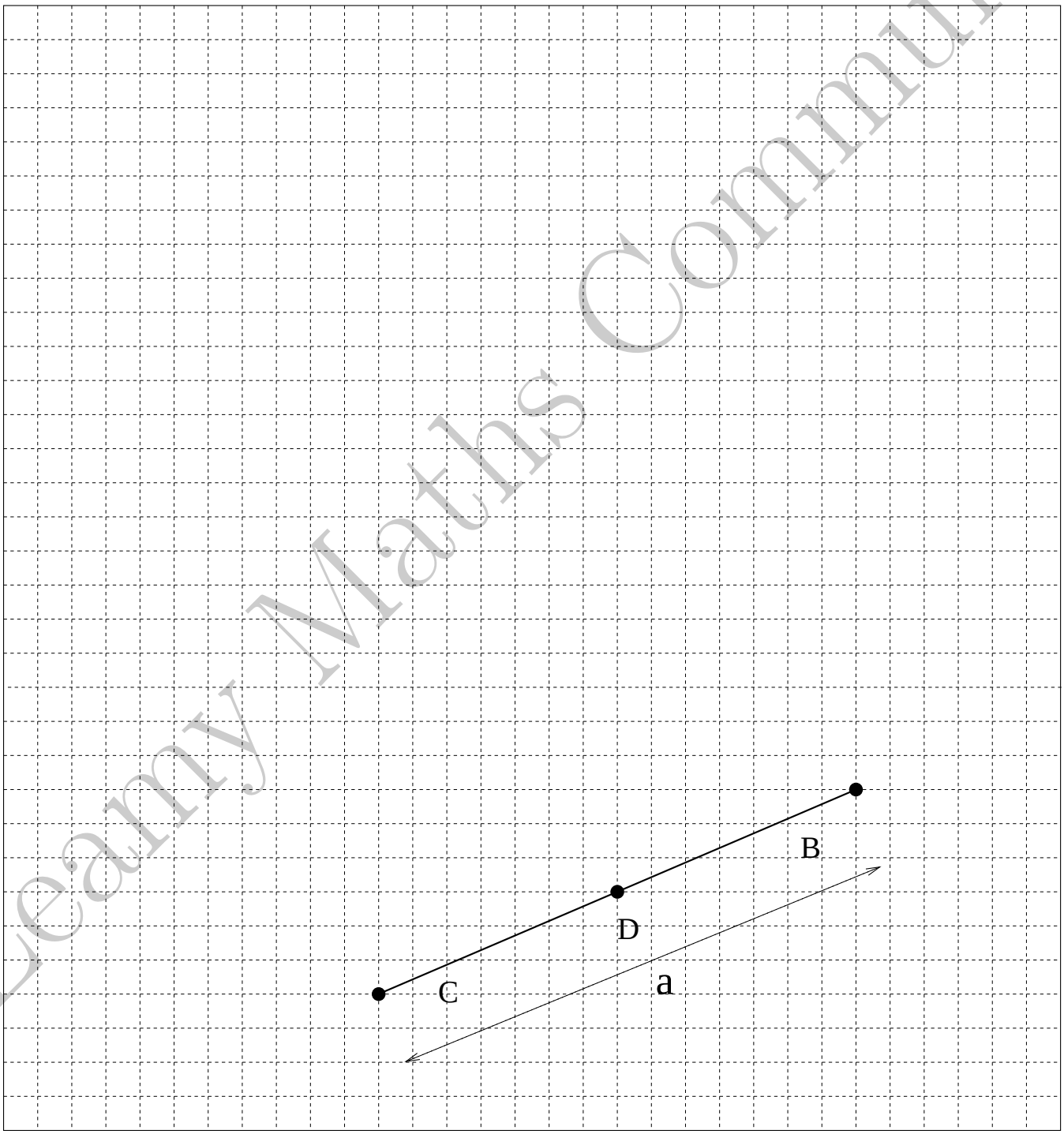


Question 6 **(25 Marks)**

Answer **either** 6A **or** 6B.

Question 6A

Without using a graduate ruler, construct the an isosceles triangle ABC with a a perimeter of $5a$ where a is the length of the segment below. Use BC as the base of your triangle. Construct the circumcircle and the incircle for this triangle and explain why the two centres are aligned with point A and D



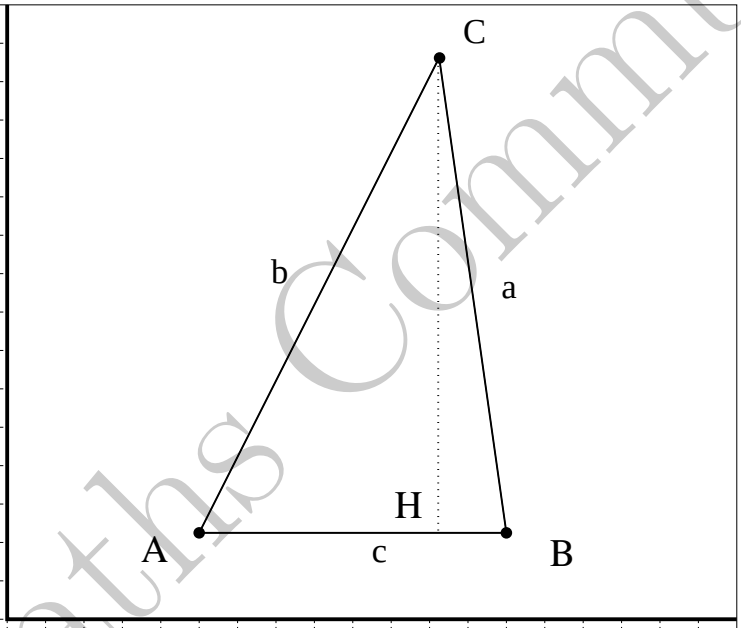
Question 6B

In the triangle below, by considering the length CH, prove that

$$\frac{a}{\sin \hat{A}} = \frac{b}{\sin \hat{B}}$$

and

$$a + b + c = (b + c)\cos \hat{A} + (a + c)\cos \hat{B} + (a + b)\cos \hat{C}$$



Section B Contexts and Applications 150 Marks

Answer Question 7 **and** Question 8.

Question 7 **(75 Marks)**

A casino thinks that one player is cheating. They want to check whether their suspicions are correct or not. When the player plays roulette, he plays ten times and always wins 8 times or more when he bets that the number is odd.

(a) In a roulette, there are 37 numbers, 0 to 36.

(i) If you play roulette, what is the probability to get an odd number

(ii) In what conditions can you use the Bernoulli trials? Are they verified here?

(iii) If you were to bet on odd number coming out in the roulette, what is the probability to win eight times out of ten?

(iv) If you were to bet on odd number coming out in the roulette, what is the probability to win eight times or more out of ten?

(v) Do you think the player is cheating?

(b) The expected value of gains of a player in this casino are normally distributed with a mean of -€100 and a standard deviation of €200.

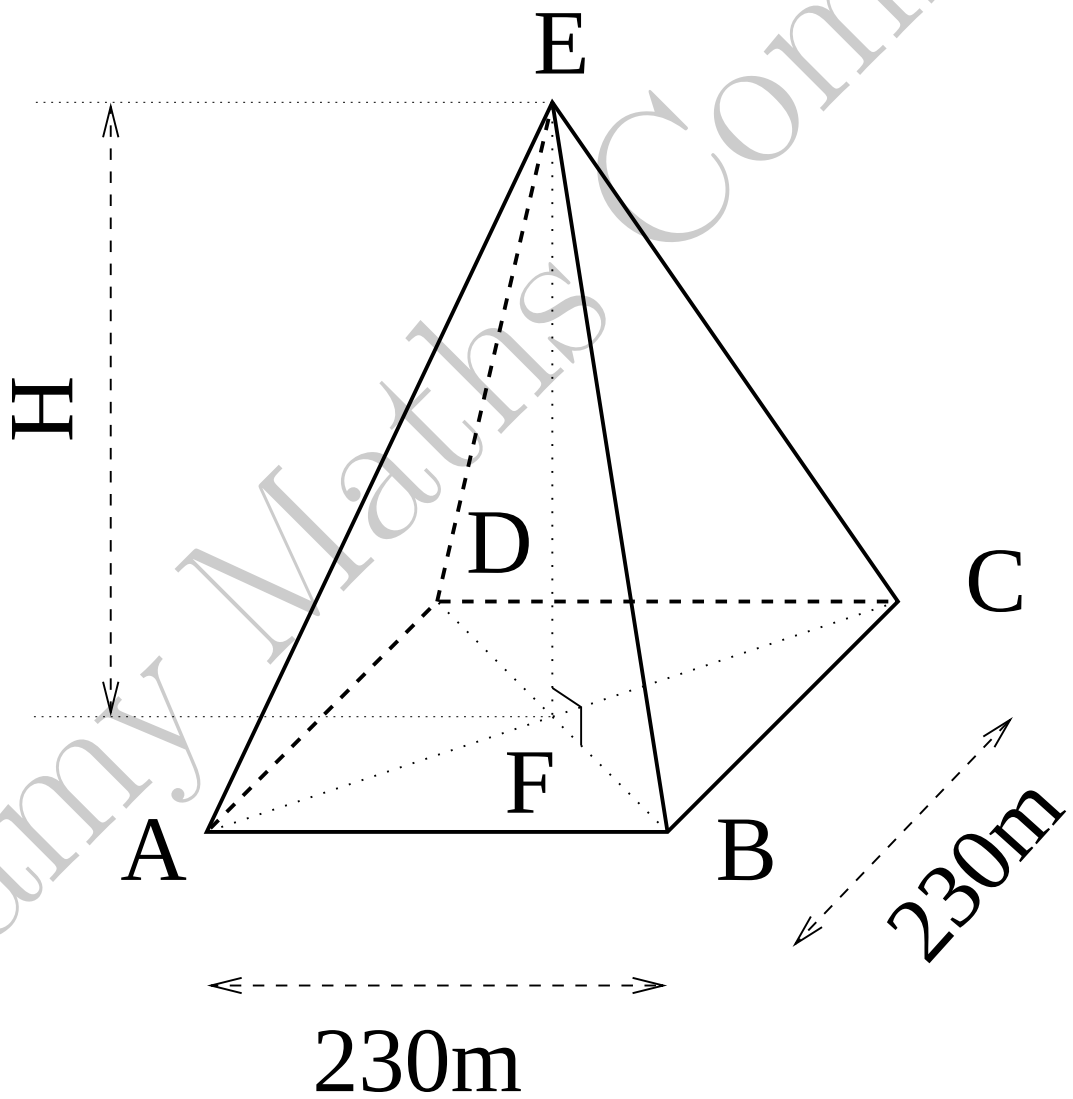
(a) Why is the mean negative?

(b) What is the probability to lose money when you go to the casino (*i.e.* have a win less than €0)?

(c) What is the probability to gain more than €400?

(d) One night, the player was part of a group of 100 people. 39 of them earned more than €400. Is there anything suspicious (conduct a hypothesis test)?

Question 8 **(75 Marks)**



The great pyramid of Giza is a perfect geometrical pyramid. Its base is a square with sides measuring 230m. Its height was unknown for a long time.

(a) In the antiquity, the Greek mathematician Thales wanted to measure the height of the pyramid but could only use limited technology. He decided to use a wooden stick. He stated that when the length of the stick's shadow would equal the length of the stick, the shadow of the pyramid would show its height. Using similar triangles, explain why Thales was right. What would be the angle of elevation of the sun?

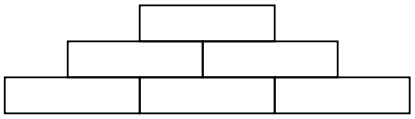
(b) The pyramid height may also be calculated using Pythagoras theorem.

(i) Using Pythagoras theorem in the square ABCD, calculate the length AC

(ii) Using the fact that (AC) and (EF) are perpendicular, estimate the height of the pyramid if the slant length is AE=218m.

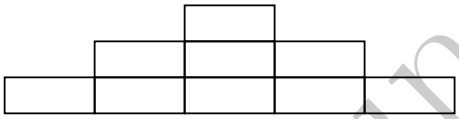
(c) You want to reproduce the pyramid using some PVC blocks. The blocks are rectangles of height 1cm. The length in unknown. You want your reproduced pyramid to be 145cm high. The two designs shown below are possible.

Design 1



Level 1
Level 2
Level 3

Design 2

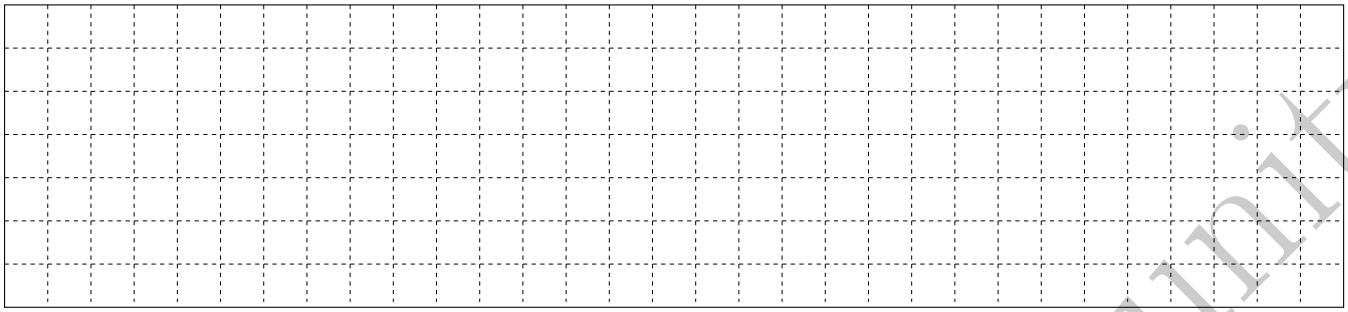


(i) How many layers are necessary?

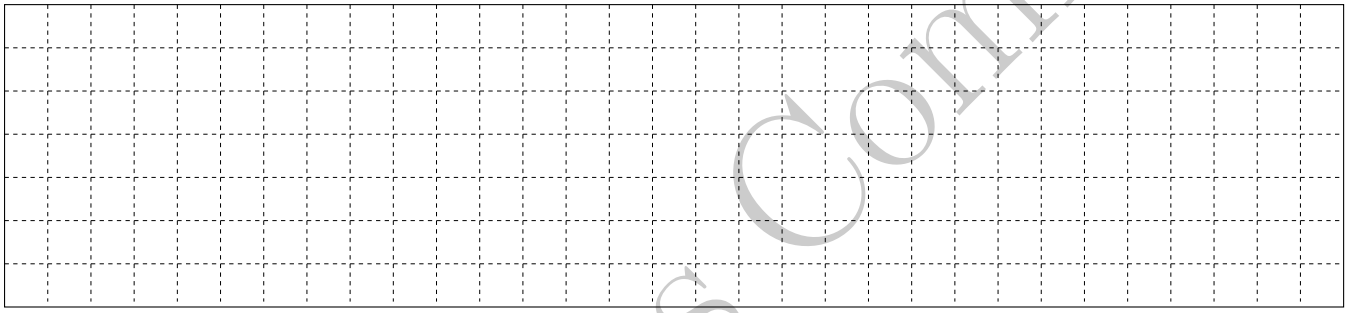
(ii) Calculate the size of the square side (in number of blocks) for each layer. Give a general formula at level n

Level	1	2	3	10	20
Blocks in design 1					
Blocks in design 2					

(iii) How many blocks do you have on the bottom level with design 1? What should be the length of the block?

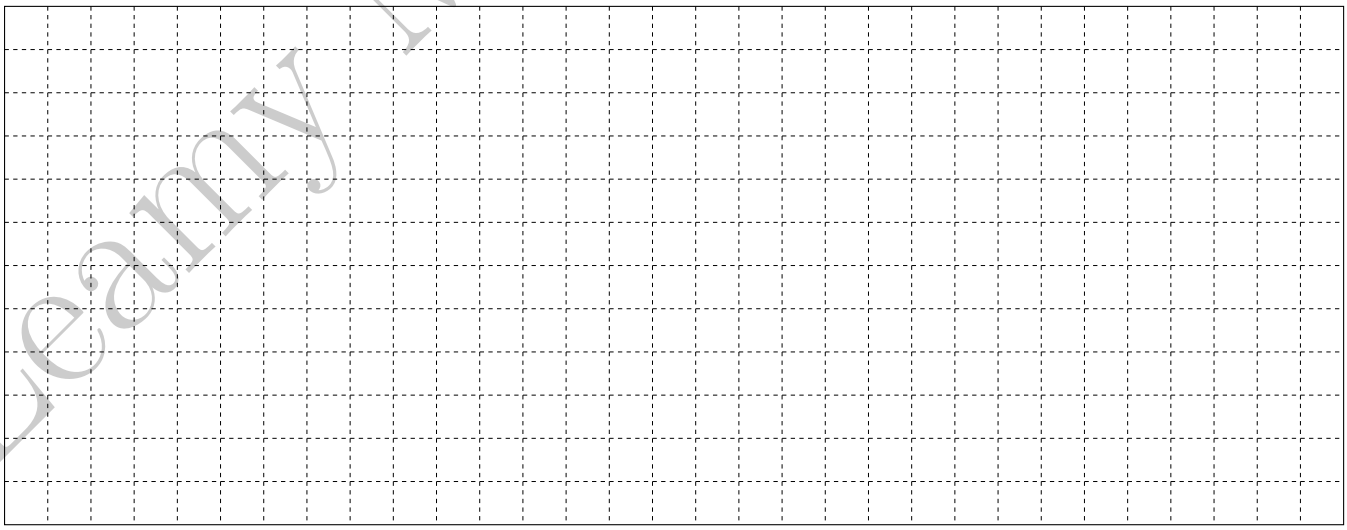


(iv) How many blocks do you have on the bottom level with design 2? What should be the length of the block?



(v) How many blocks will be visible on the surface of each side for the first design? You may use the formula

$$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}$$



Rough Work