Proof by Induction/Binomial Theorem Revision Series 2017

Stephen King

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Proof by Induction

- Verify that the proposed statemtent is true for the first value, typically $n = 1$
- Assume that the statement is true for$n = k$, where k is any positive integer.
- Prove the statement is true for $n = k + 1$, usually based on your assumption
- Combining the previous steps, we can conclude that; if the statement is true for $n = 1$, then it must be true for $n = 2$, $n = 3$, etc. It is true for all positive values of n.

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Step 2: Assume true for $n = k$

$$1^2 + 2^2 + 3^2 + ...k^2 = \frac{k}{6}(k+1)(2k+1)$$

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$$1^2 + 2^2 + 3^2 + ...k^2 + (k+1)^2 = \frac{k+1}{6}(k+2)(2k+3)$$
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$$1^2 + 2^2 + 3^2 + ...k^2 + (k+1)^2$$

$$= \frac{k}{6}(k+1)(2k+1) + (k+1)^2$$

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$$1^2+2^2+3^2+......k^2+(k+1)^2=\frac{k+1}{6}(k+2)(2k+3)$$

$$1^2+2^2+3^2+......k^2+(k+1)^2$$

$$=\frac{k}{6}(k+1)(2k+1)+(k+1)^2$$

$$=\frac{k}{6}(k+1)(2k+1)+\frac{6}{6}(k+1)^2$$

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$$= \frac{k(k+1)(2k+1) + 6(k+1)^2}{6}$$

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$$= \frac{(k+1)[2k^2 + k + 6k + 6]}{6}$$

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Therefore, true for $n = k + 1$

Therefore the proposition is true for $n = k + 1$
We showed the statement is true for $n = 1$, assumed it was true for $n = k$, and proved it was true for $n = k + 1$. Therefore, by induction, the statement is true for all $n \in N$

Binomial Theorem

The Binomial Theorem is used for expanding brackets. The formula is :

$$(x+y)^n = \binom{n}{0}x^n + \binom{n}{1}x^{n-1}y + \binom{n}{2}x^{n-2}y^2 + ... + \binom{n}{n-1}xy^{n-1} + \binom{n}{n}y^n$$

This formula is in page 20 of the log tables.

Example

Expand $(a + 2b)^4$:

$$(a+2b)^4 = \binom{4}{0} a^4 + \binom{4}{1} a^3 (2b) + \binom{4}{2} a^2 (2b)^2 + \binom{4}{3} a (2b)^3 + \binom{4}{4} (2b)^4$$

which can be tidied up to:

$$(a + 2b)^4 = a^4 + 8a^3 b + 24a^2 b^2 + 32ab^3 + 16b^4$$