

Leaving Certificate Examination, 2015

Sample paper prepared by Leamy Maths Community

Mathematics

Project Maths - Phase 3

Paper 1

Higher Level

Sunday 19 April

Paper written by J.P.F. Charpin



Leamy Maths Community

300 marks

Sample Instructions

There are two sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer questions as follows:

In Section A, answer all six questions.

In Section B, answer all three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination.

You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

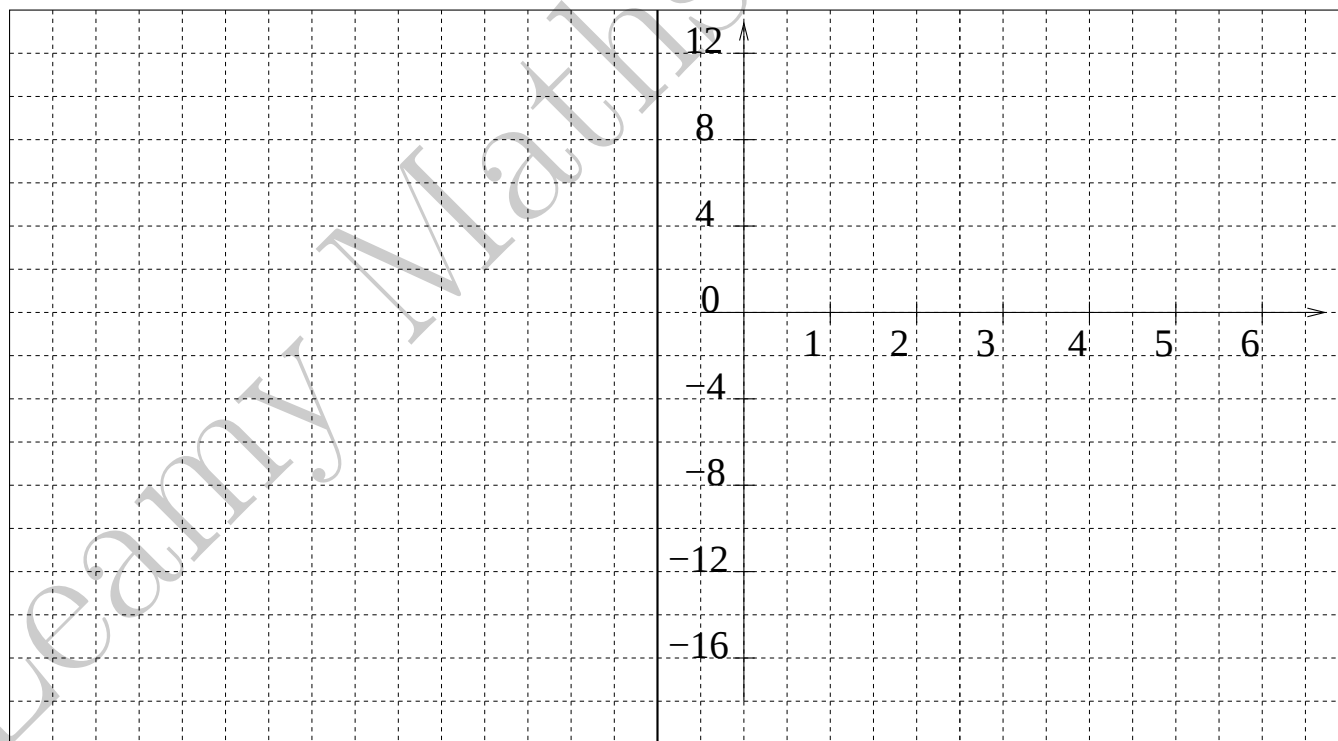
Answer **all six** questions from this section.

Question 1**(25 Marks)**

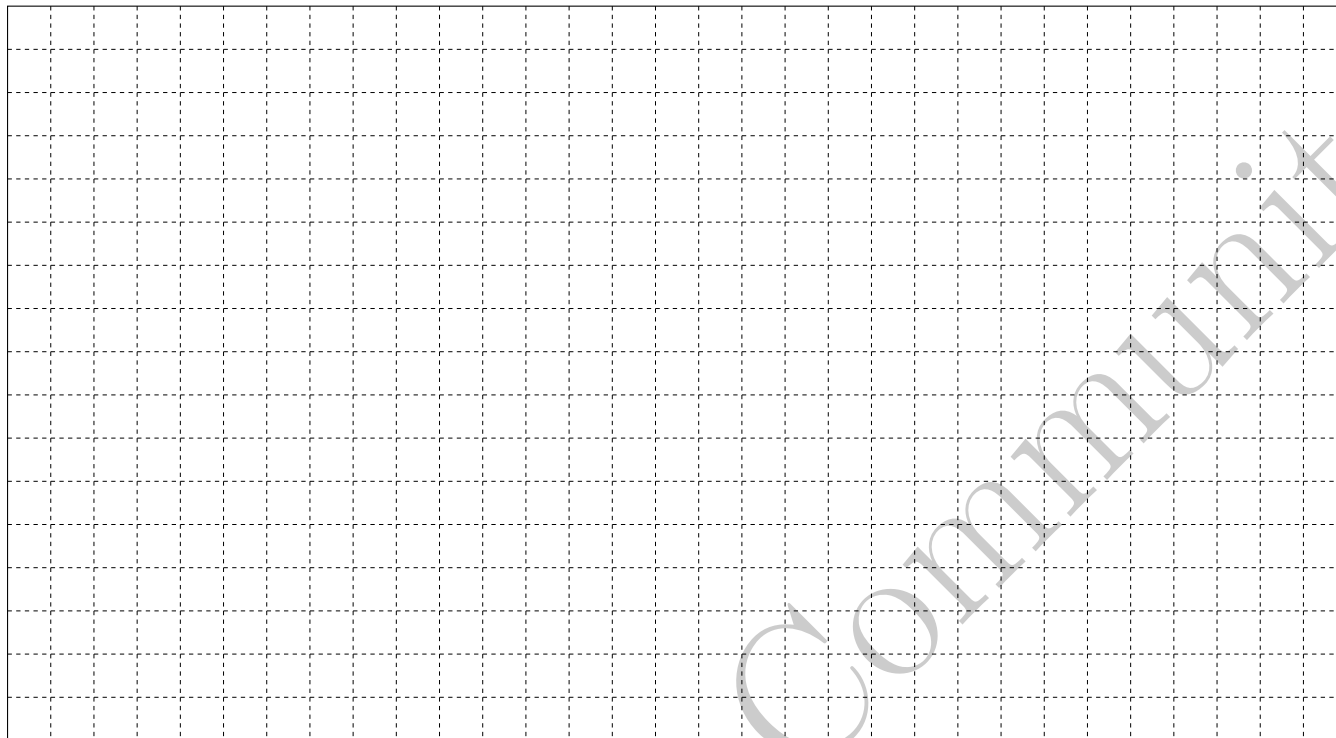
(a) Plot the curve

$$f(x) = \frac{1}{4}x^3 - 2x^2 - x + 8$$

x	0	1	2	3	4	5
f(x)						



(b) Explain why $(x-2)$ is a factor of the function f and find the other two roots of the function



(c) Calculate the equation of the tangent to the curve at $x = 1$ and plot it on the same graph as the function

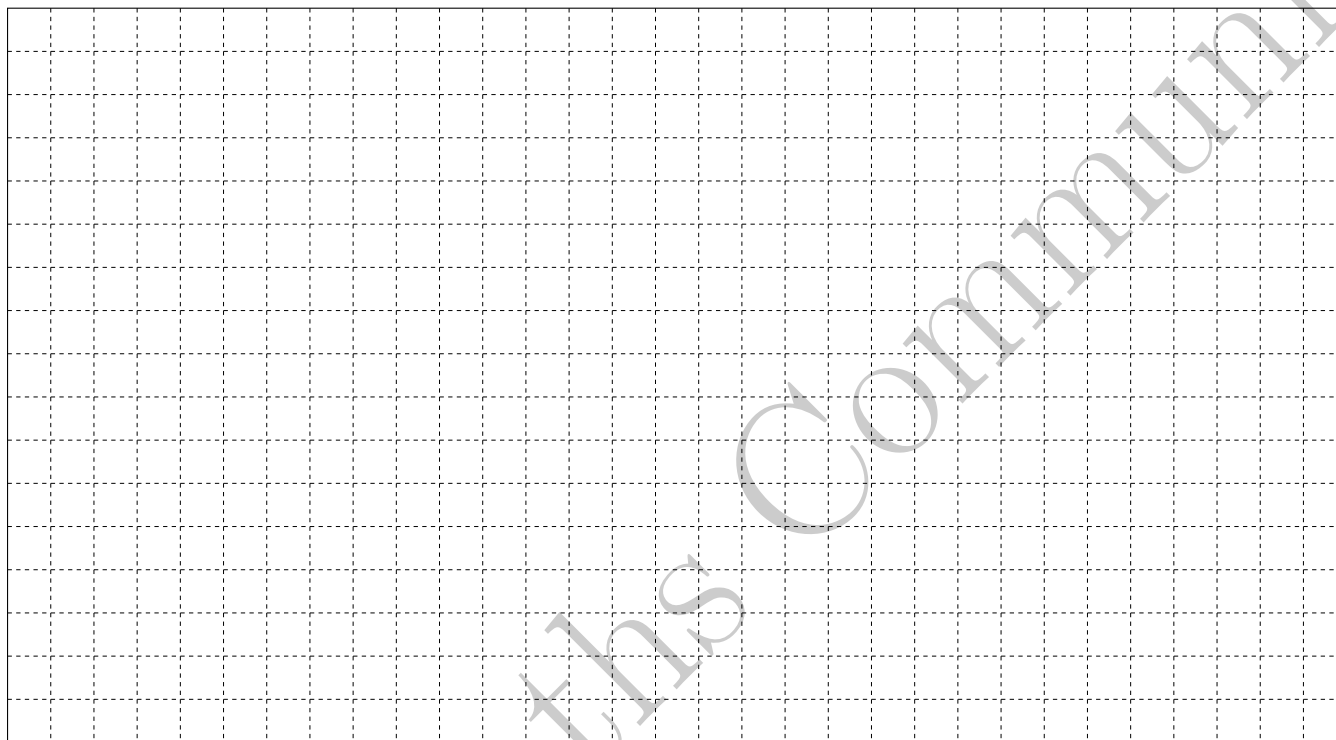


Question 2**(25 Marks)**

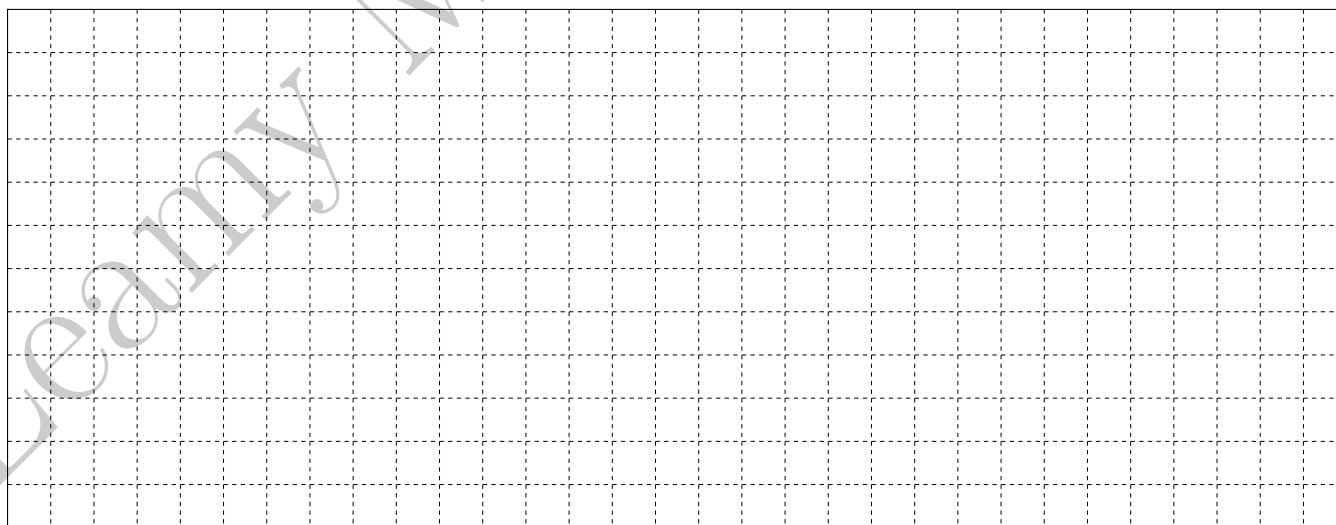
(a) Write the complex number

$$z_1 = \frac{-1}{1+i}$$

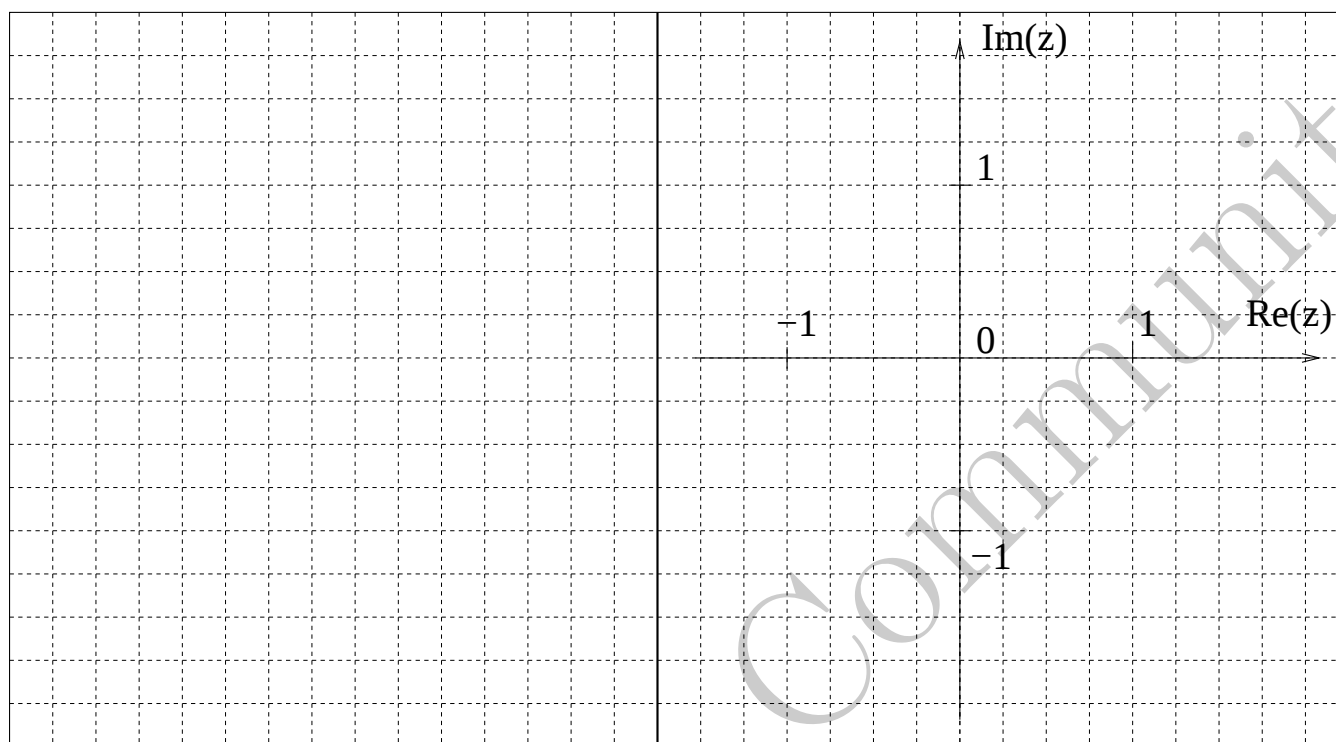
in the form $a + ib$ where a and b are real numbers and calculate its polar form.



(b) Calculate $\bar{z}_1$, $z_2 = 2iz_1$ and $z_3 = -\frac{i}{2}z_1$.



- (c) Plot z_1 , $\bar{z}_1$, z_2 and z_3 on the Argand diagram. What are the three geometrical transformations observed when transforming z_1 in $\bar{z}_1$, z_1 in z_2 and z_1 in z_3 ?

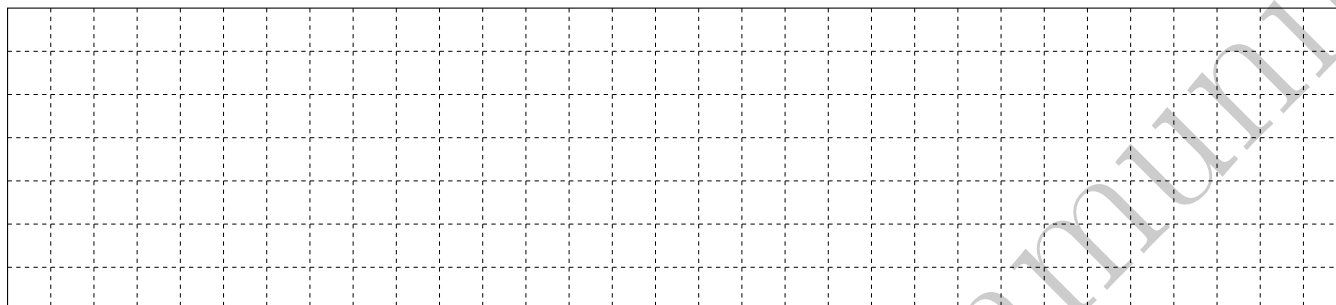


- (d) Calculate z_1^5 and $\bar{z}_1^5$ in Cartesian form.

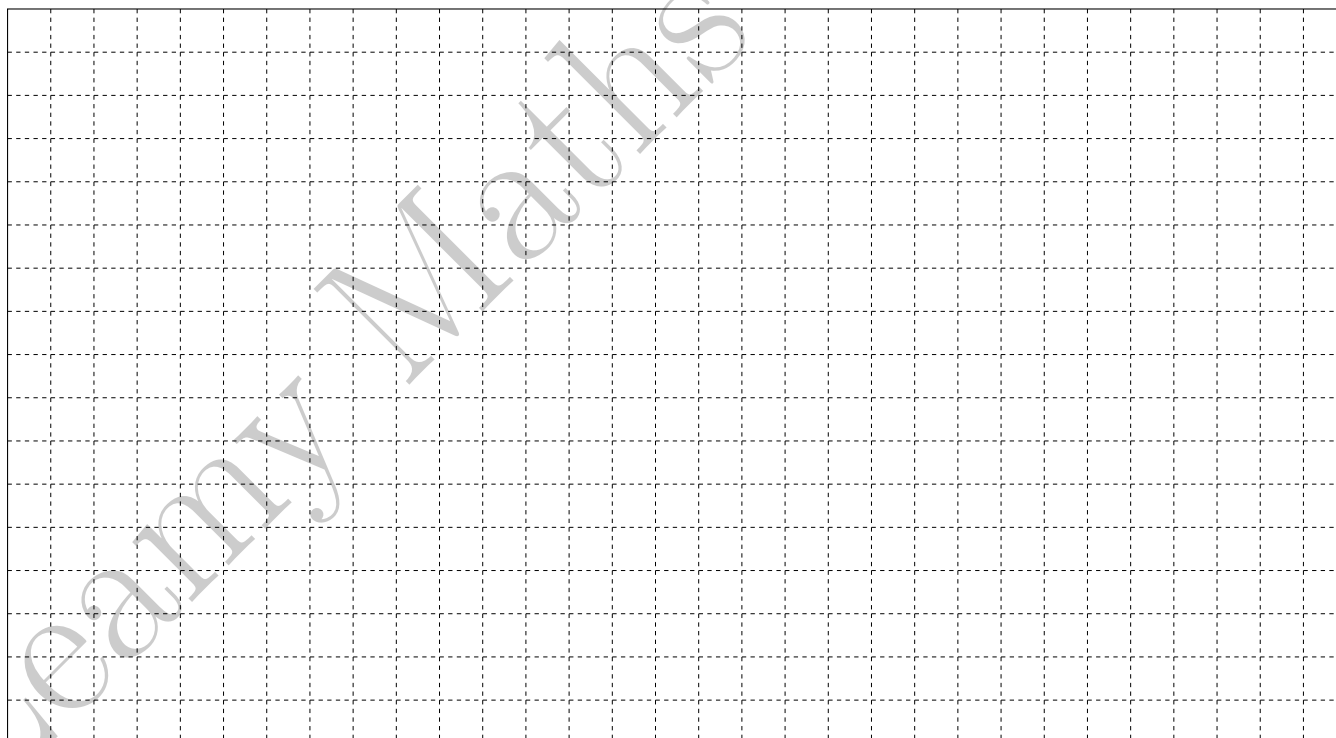


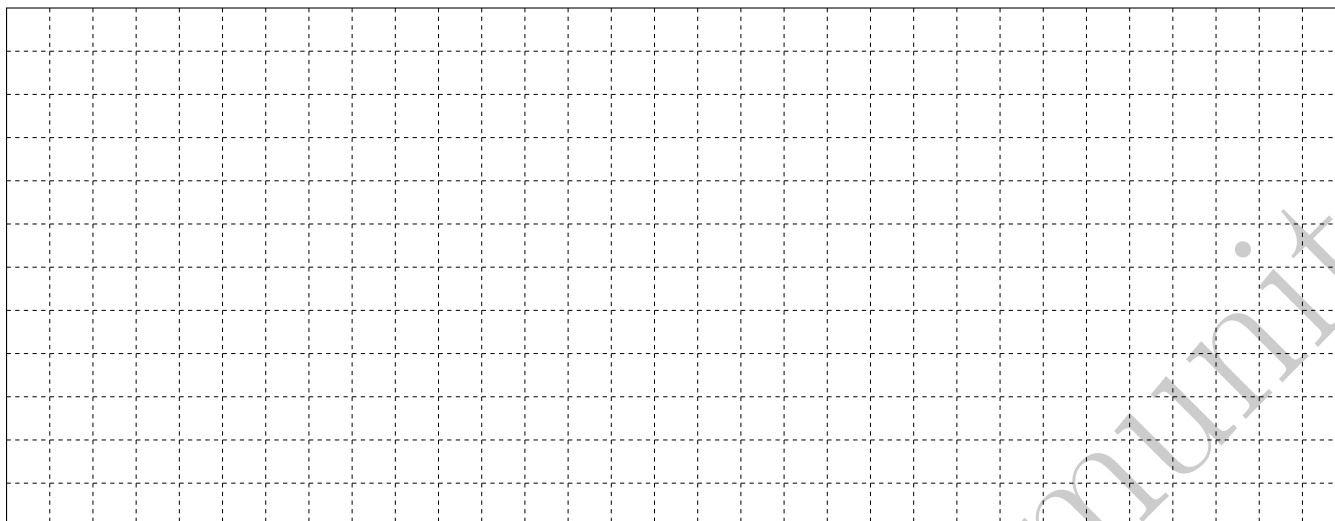
Question 3**(25 Marks)****(a)** Show that

$$n(n+1)(2n+1) + 6(n+1)^2 = (n+1)(n+2)(2n+3)$$

**(b)** Using this result, show by induction that for $n \in \mathbb{N}$,

$$1^2 + 2^2 + 3^2 + \dots + n^2 = \frac{n(n+1)(2n+1)}{6}$$





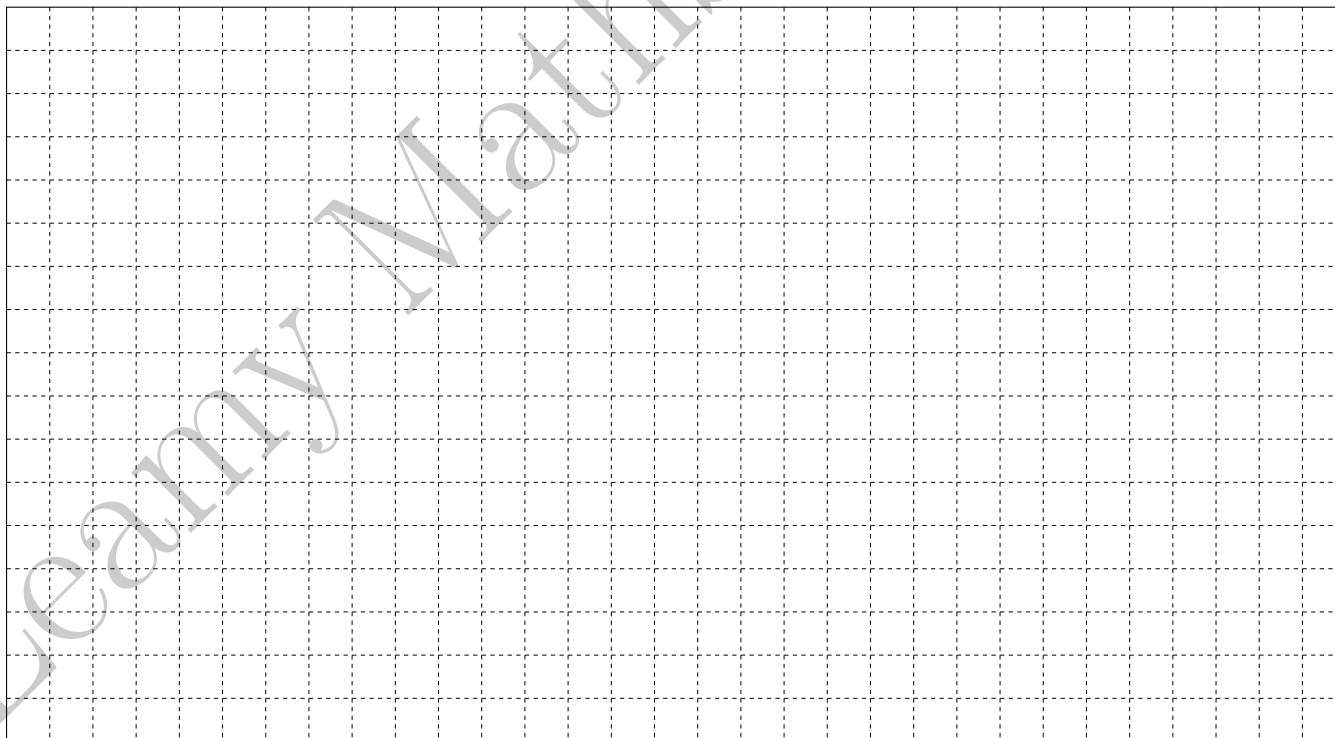
Explain why

$$2^2 + 4^2 + 6^2 + \dots + (2n)^2 = 4(1^2 + 2^2 + 3^2 + \dots + n^2)$$

and then show that

$$2^2 + 4^2 + 6^2 + \dots + (2n)^2 = \frac{2}{3}n(n+1)(2n+1)$$

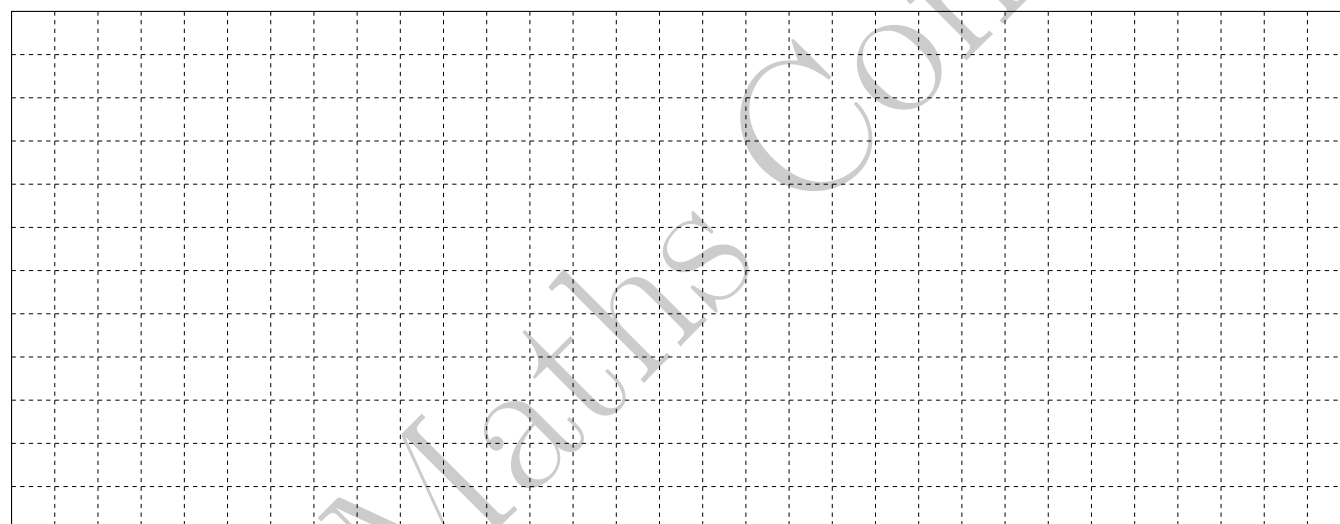
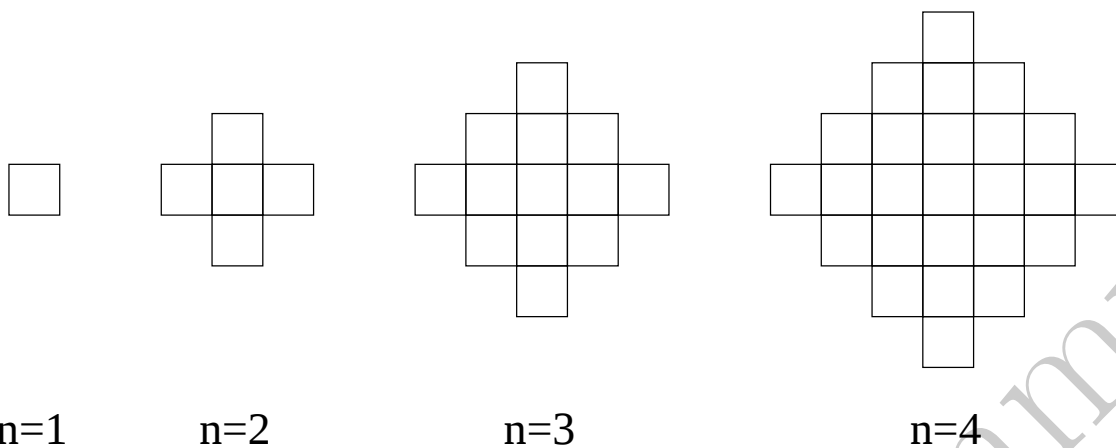
and calculate $2^2 + 4^2 + 6^2 + \dots + 100^2$. Detail your calculations.



Question 4

(25 Marks)

(a) Tiles are arranged in the pattern shown below Draw the next pattern and complete the table

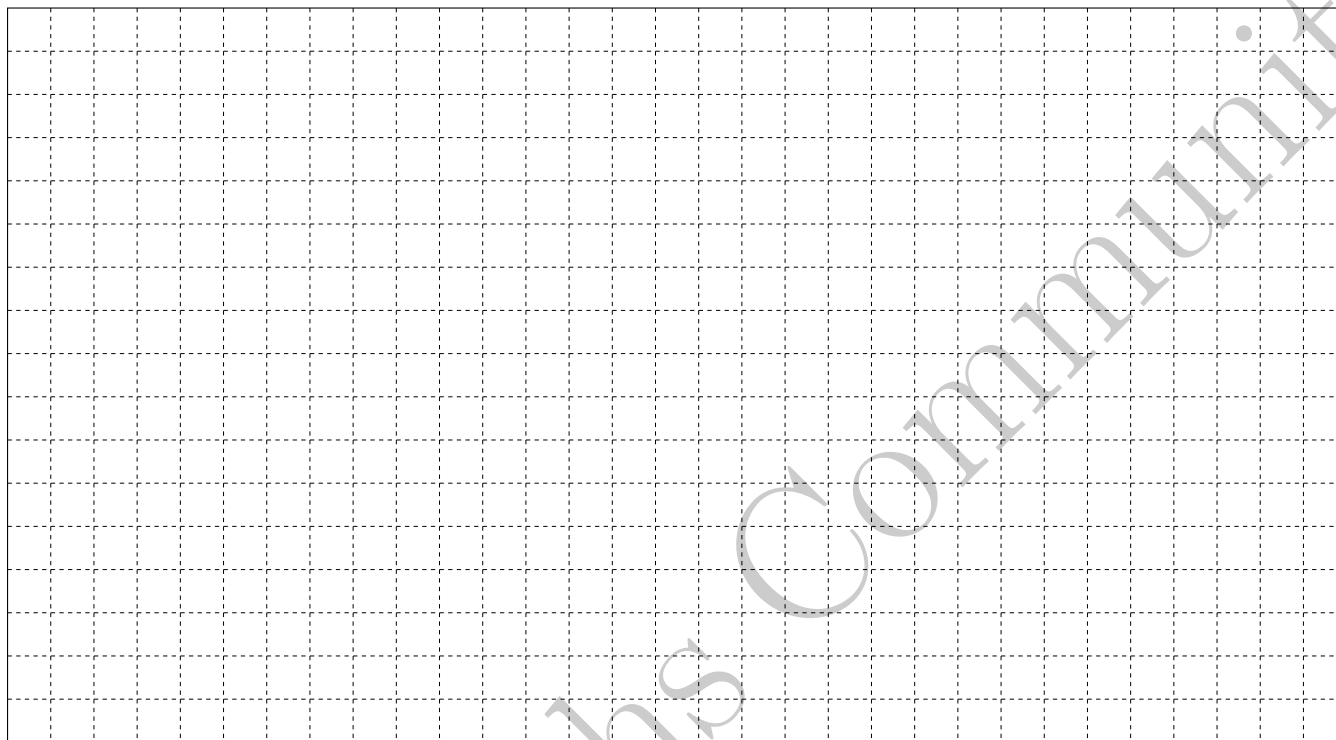


n	1	2	3	4	5
T_n					

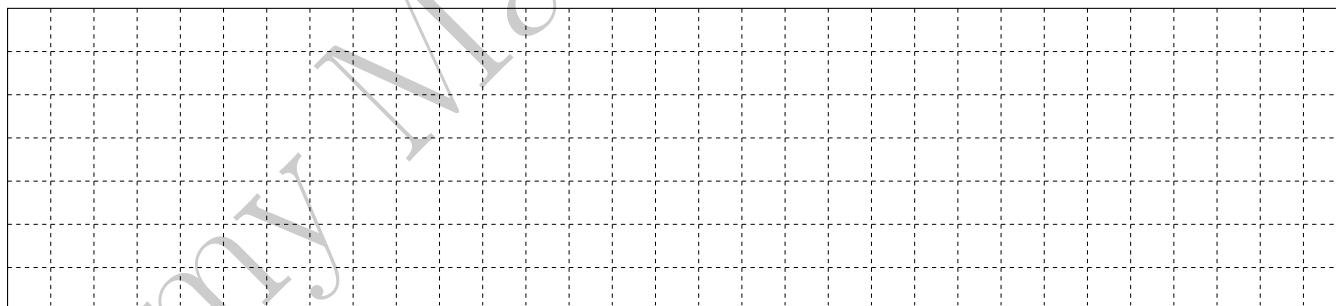
(b) Prove that the series may be described by

$$T_n = a n^2 + b n + c$$

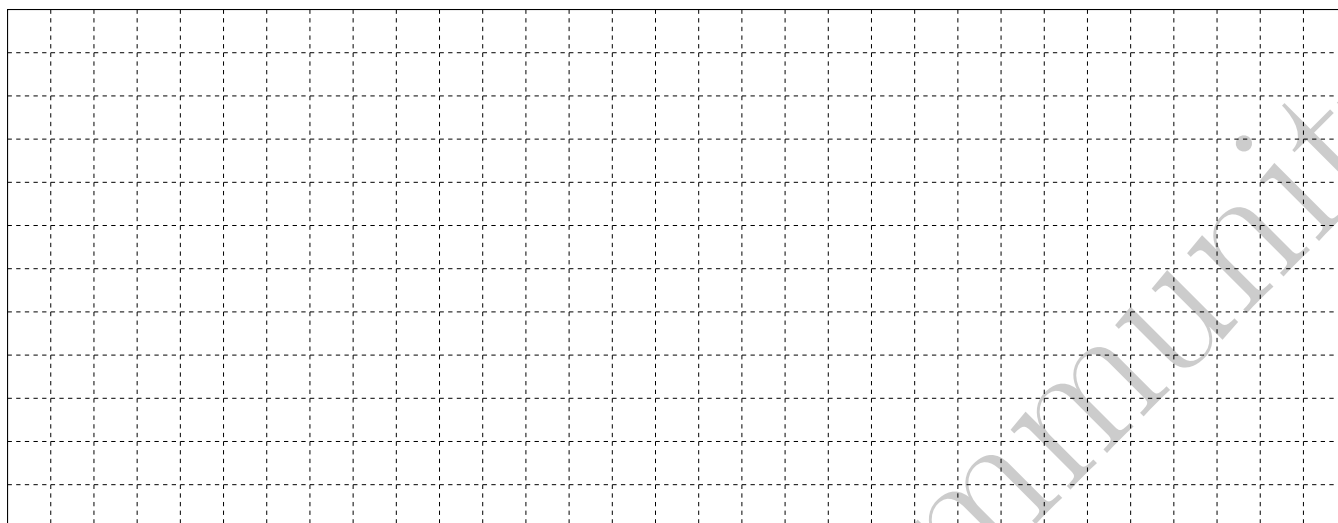
and calculate the values of a , b and c .



(c) How many tiles are necessary at level 10?



- (d) 1000 tiles are available. What will be the last level of the sequence completed? How many tiles will be left?



Question 5

(25 Marks)

- (a) Solve the following inequalities

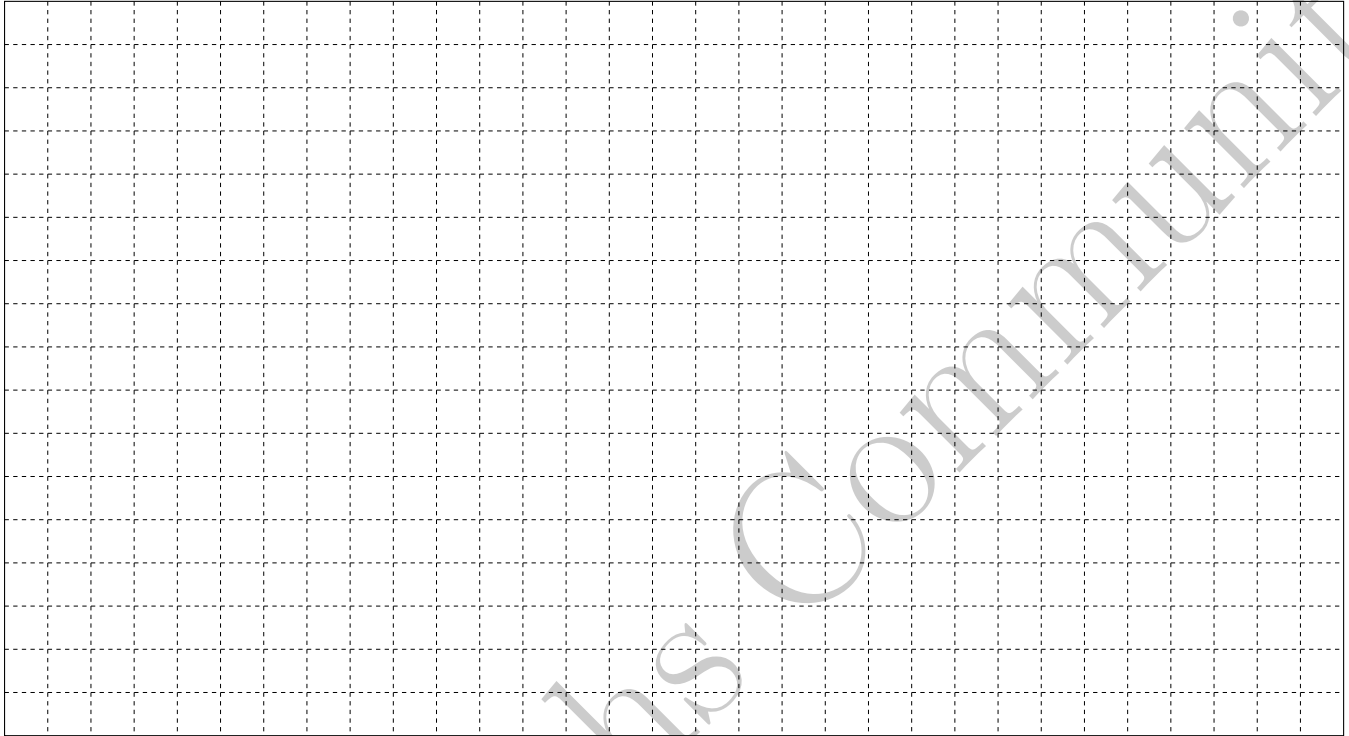
(i)

$$x^2 - 4x > -3$$



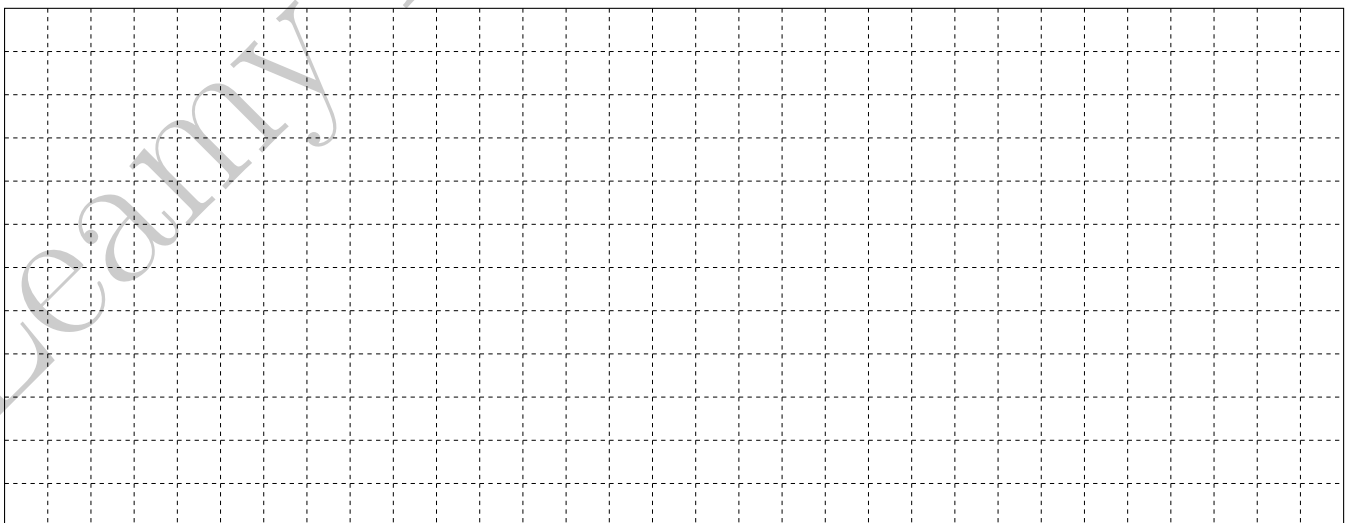
(ii)

$$\frac{x+2}{x+5} > -1$$



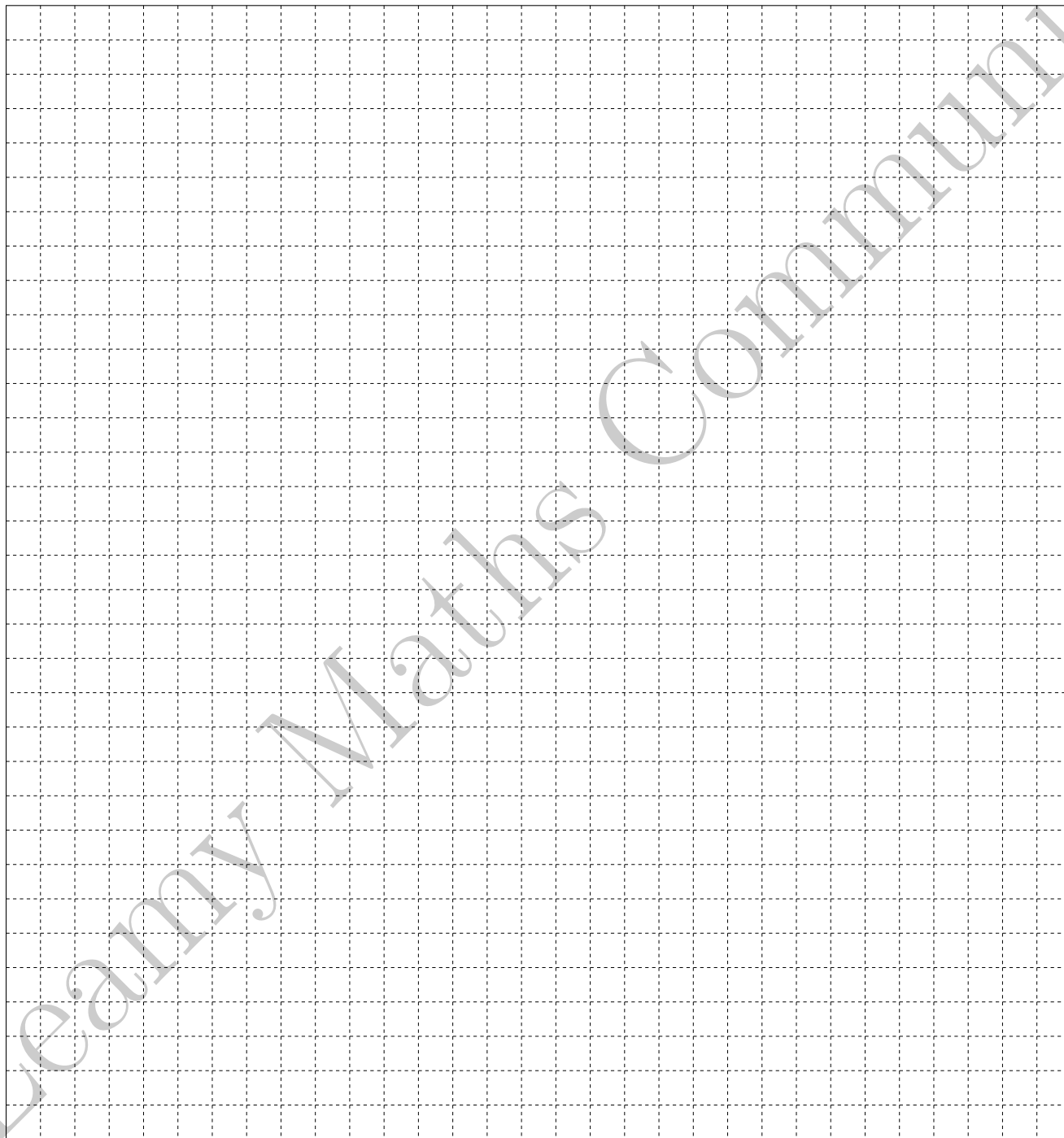
(iii)

$$\frac{1}{4} \geq \left(\frac{4}{5}\right)^x$$



(b) Solve the simultaneous equations

$$\begin{cases} 2x + 3y + 4z = -23 \\ x + 3y - 5z = 25 \\ -3x + 2y + 4z = -9 \end{cases}$$



Question 6

(25 Marks)

(a) Consider the function

$$f(x) = 4\cos(3x)$$

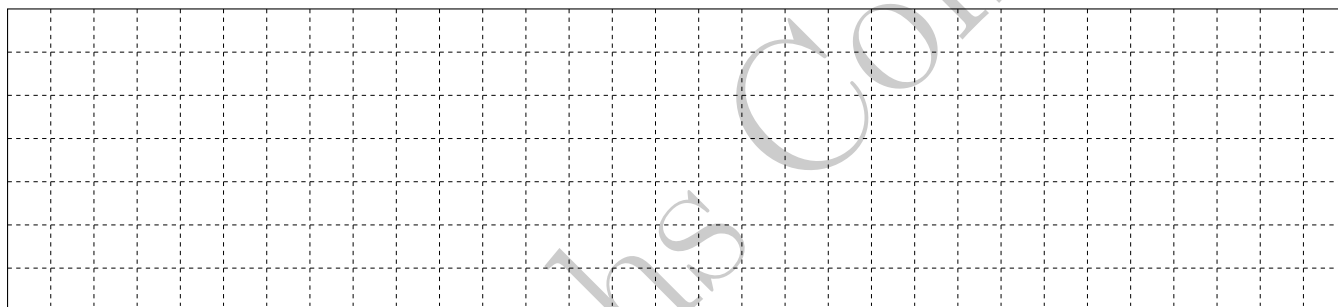
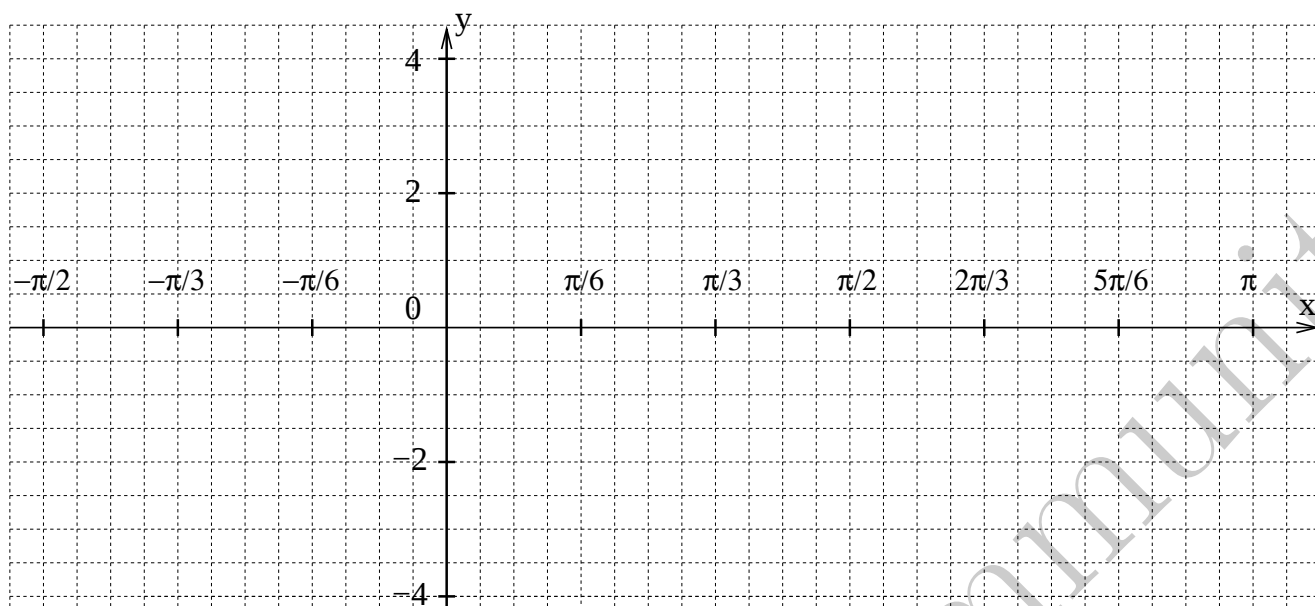
(i) What is the domain and range of this function?

(ii) Is the function periodic? If so give the period. Justify your answer.

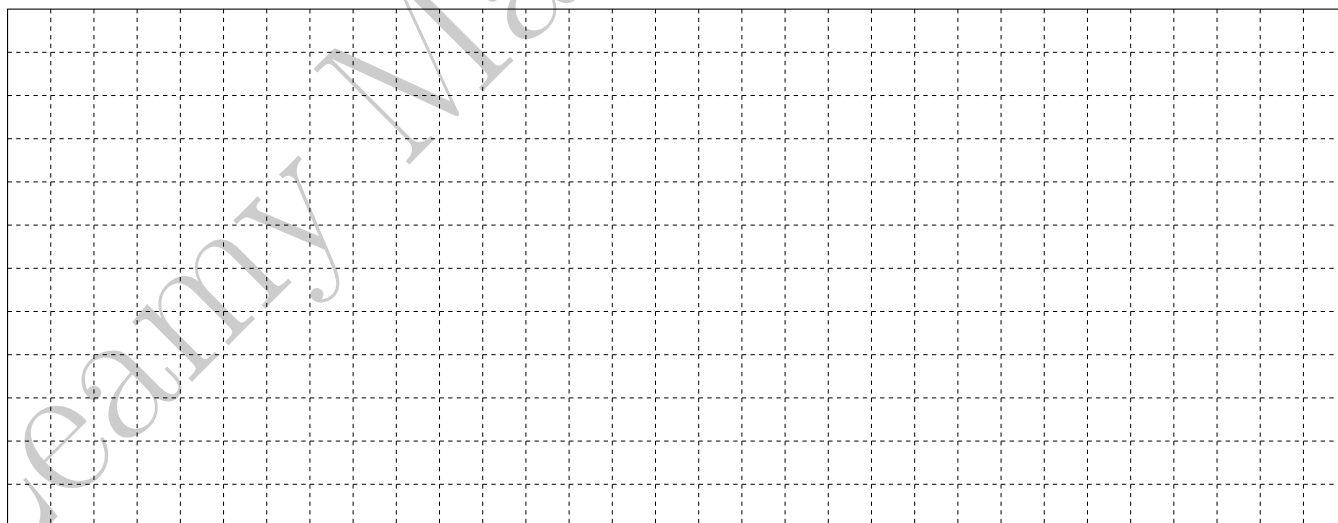
(iii) Is the function injective, surjective or bijective? If not provide the largest possible domain on which one of these properties is true. Justify your answers.

(b) Complete the table below and using the information above, draw the curve for $f(x)$. Explain why the information in the table is sufficient to draw the curve.

x	0	$\pi/12$	$\pi/6$	$3\pi/12$	$\pi/3$	$5\pi/12$	$\pi/2$	$7\pi/12$	$2\pi/3$
$f(x)$									



(c) Calculate the value of the function f on the interval $[0, \pi/6]$.



Answer **all three** questions from this section.

Question 7**(50 Marks)**

Aoife takes out a ten year loan at the bank to improve her home. She needs €50,000.

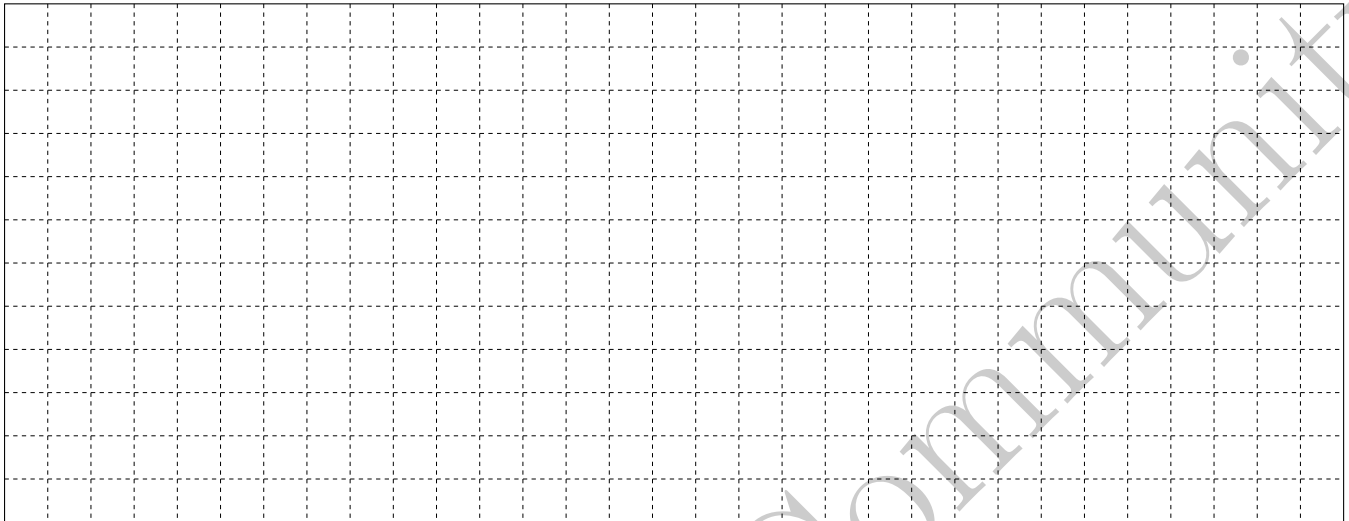
(a) Every month, Aoife pays an interest rate of 0.25%

(i) Calculate the annual equivalent interest rate

(ii) Another bank offers Aoife a 10 year rate of 35%. What should she do?

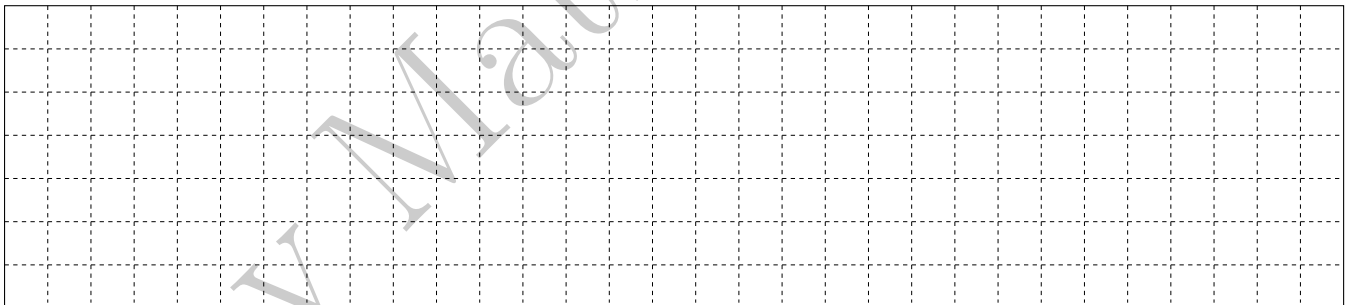
(iii) Aoife takes her loan at the annual rate of $r=3\%$. Calculate how much she will pay every year

- (iv) Assuming inflation is 2%, calculate the present value at the beginning of the ninth year of the payment to be made at the end of year 9 and the payment at the end of year 10. To the nearest €, how much should Aoife be charged to repay her loan at the beginning of year 9?

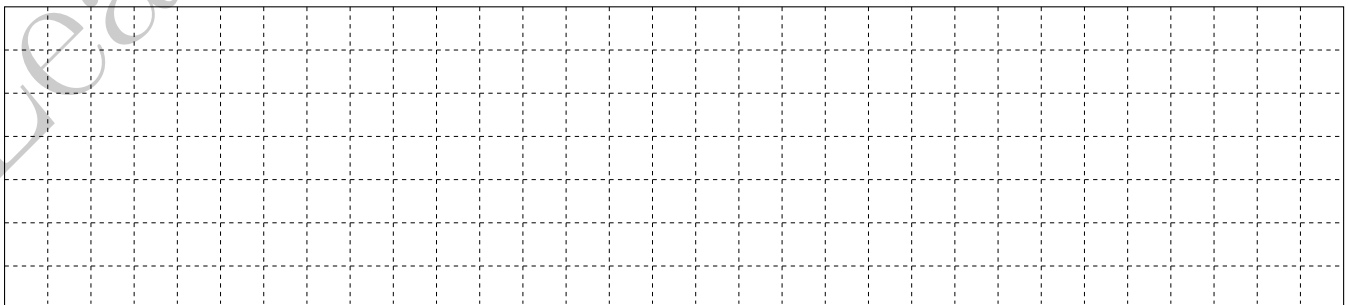


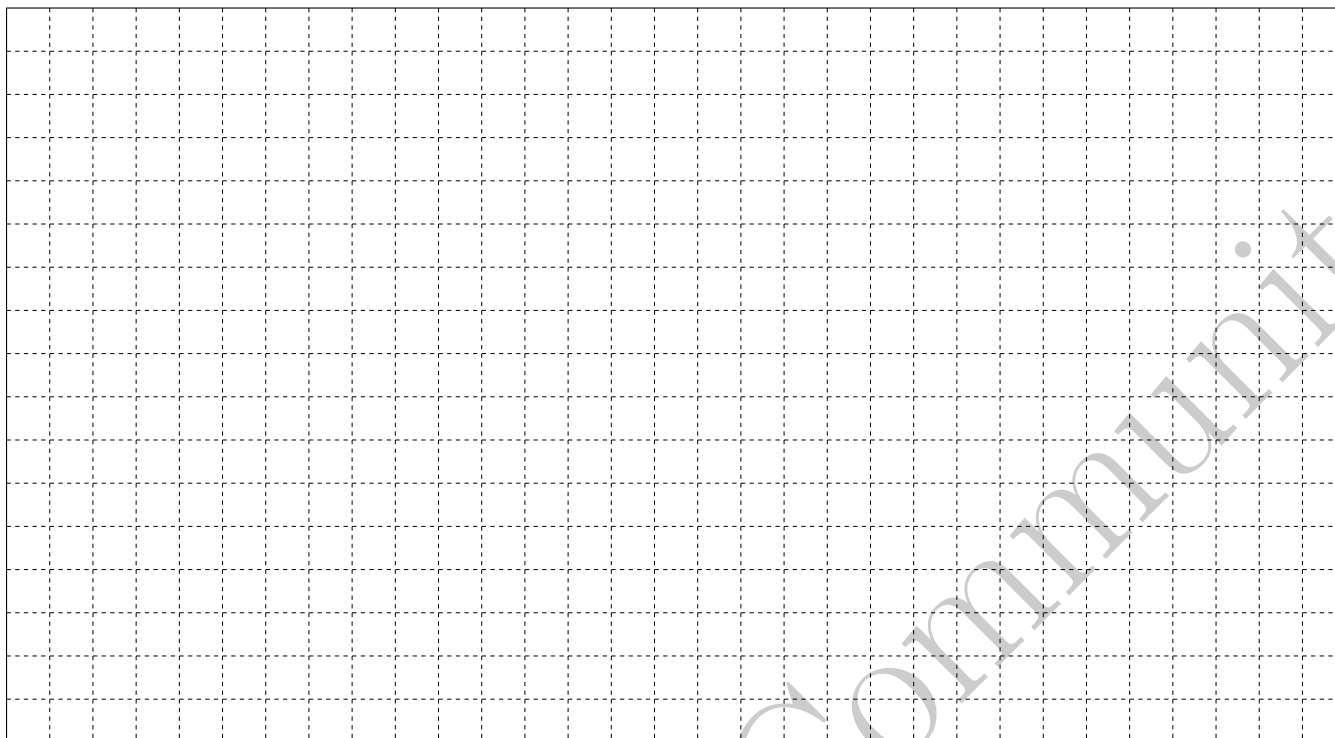
- (b) If you payback a loan early, banks charge penalties. If Aoife wants to clear her loan at the beginning of year 9, she will have to pay back one lump payment of €13,000. At the beginning of every year she can save €2,500 and she puts this money on an account earning 2.5% a year.

- (i) Calculate the amount on the account at the end of the first and second year year.

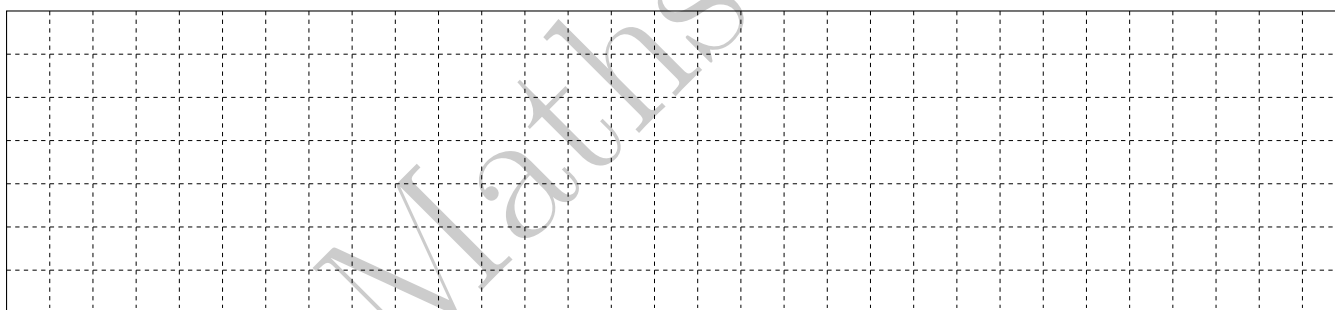


- (ii) Show how to use the sum of a geometric series to calculate the value of Aoife's investment after n years.

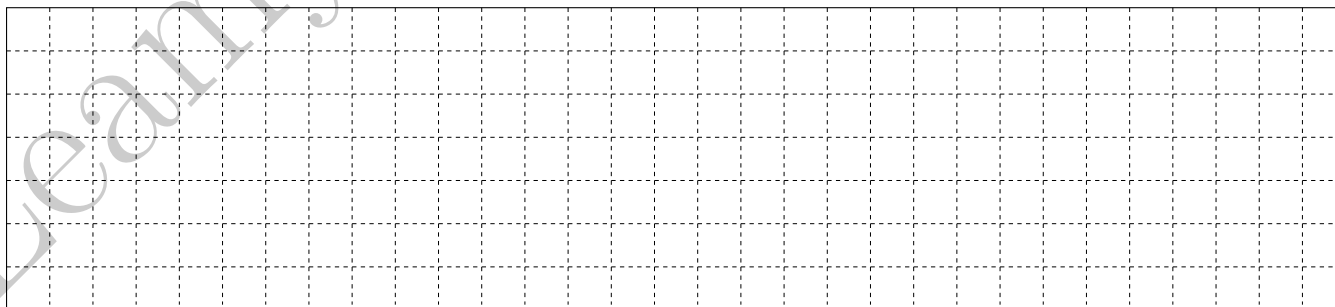


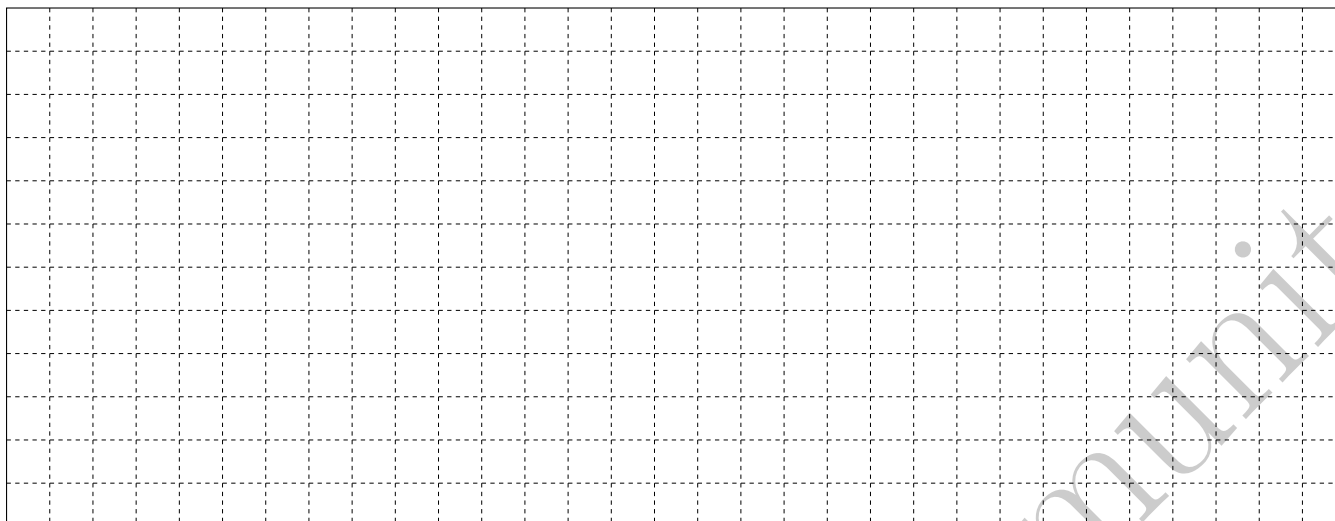


- (iii) Neglecting interest rates, estimate after how many years Aoife should start saving to have the €13,000 at the beginning of year 9.

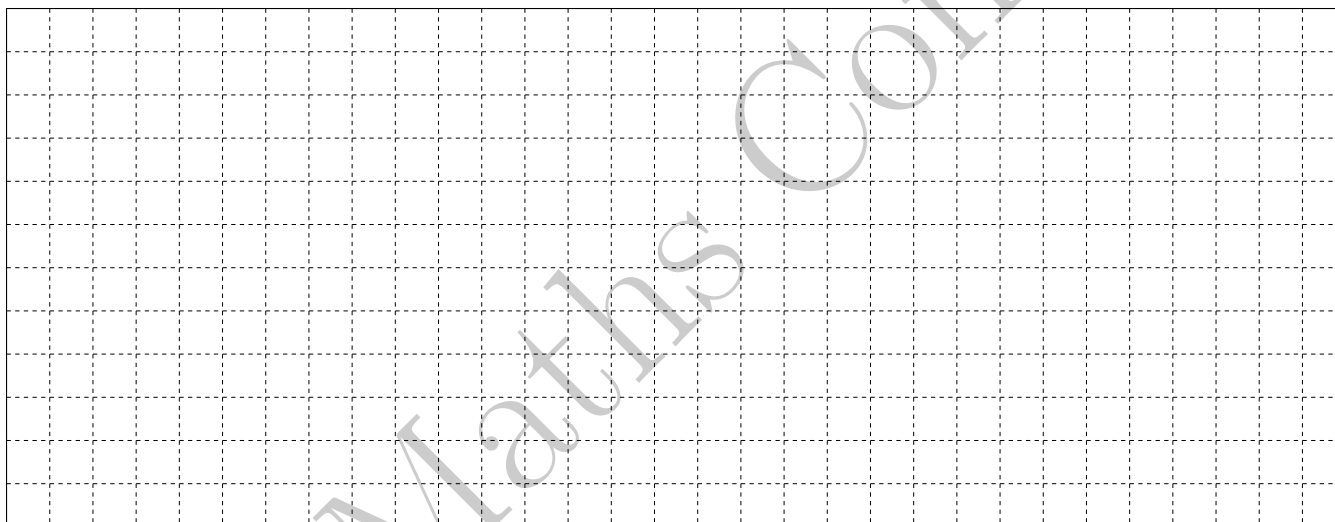


- (iv) Using trial and error, calculate the minimum number of years she should be saving to reach €13,000 now including interest rates.

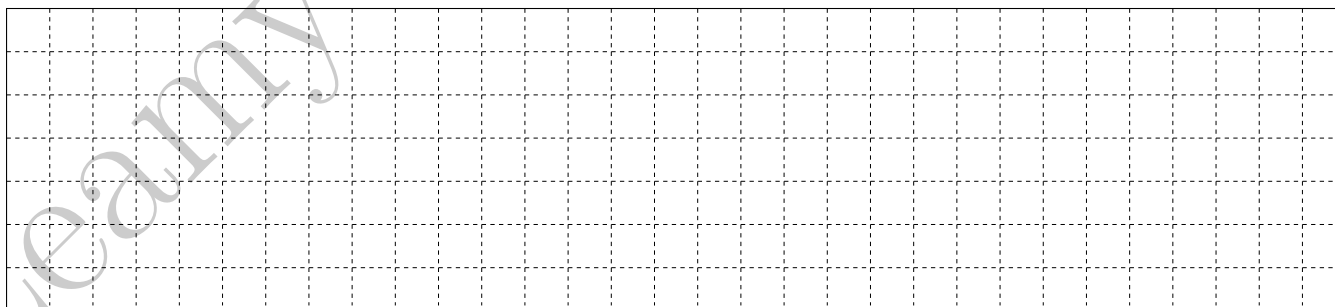




(v) Verify your answer by using the formula you found in part b(ii)?

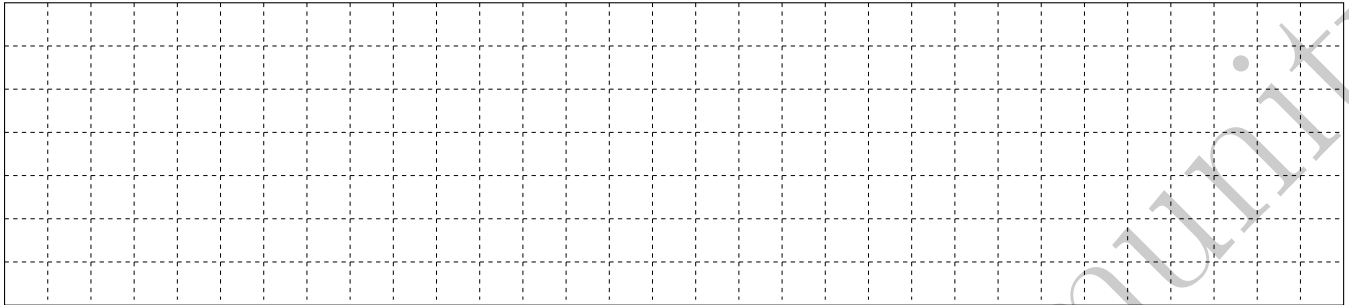


(vi) Why are the two numbers different?

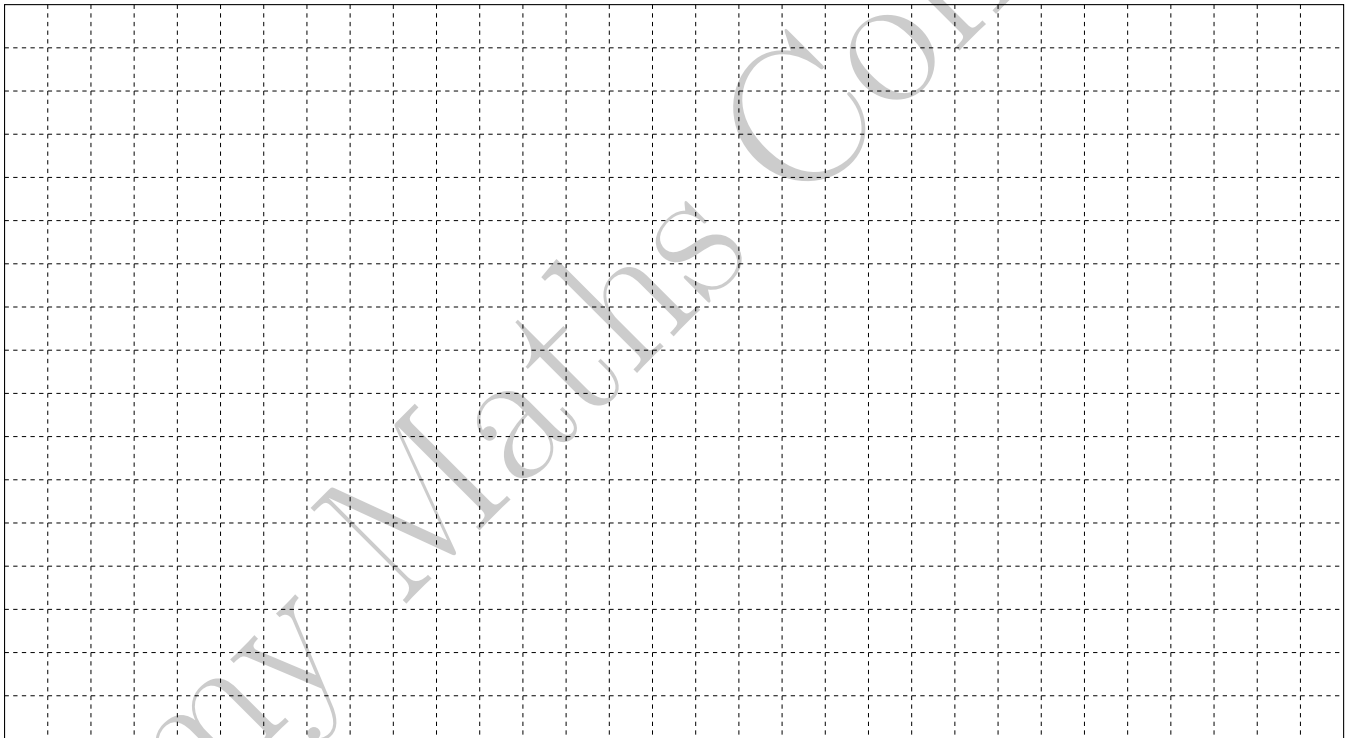


(c) Aoife will actually deposit a fixed amount every month on her savings account at 2.5% instead of a lump sum at the beginning of every month.

(i) Find the monthly interest rate with the rate in % correct to 3 decimal places.



(ii) Correct to 2 decimal places, how much should Aoife pay every month reach €2500 at the end of the first year?



Question 8**(50 Marks)**

A chef prepares a dish which needs to be served at a temperature higher than 30°C . The temperature of this food is modelled using the formula

$$T = T_a + ae^{bt}$$

where

- T_a represents the ambient temperature,
- T is the temperature of the dish,
- t is the time in minutes.

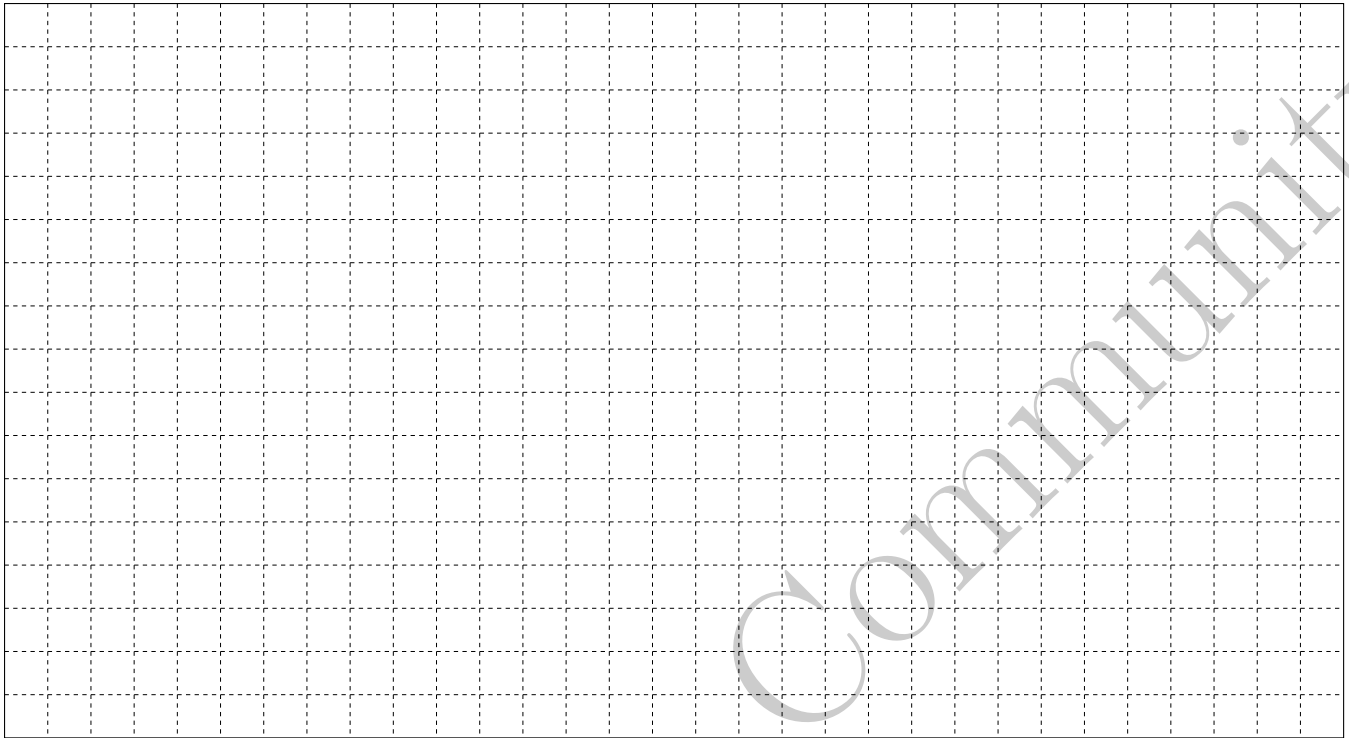
The ambient temperature in the kitchen is 20°C . When the chef finishes the dish, its temperature is 50°C . After 1 minute, the temperature has fallen to 45°C

(a) When the chef finishes the dish, its temperature is 50°C . After 1 minute, the temperature has fallen to 45°C

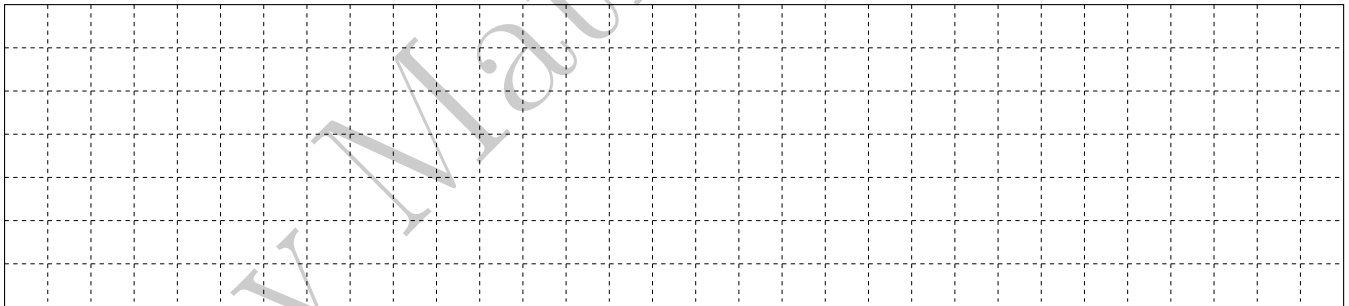
(i) What is the value of T_a ? What can you say about the sign of b (justify your answer)?

(ii) Using the information provided, calculate the values for a and b and give the expression for the temperature of the dish

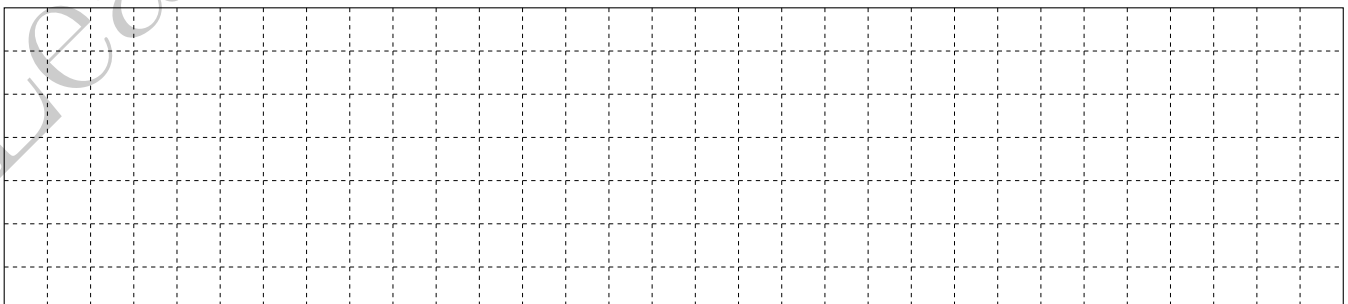
- (iii) Plot the temperature for $t \in [0 : 10]$ using points corresponding to $t=0, 2.5, 5, 7.5$ and 10 minutes



- (iv) Using the graphic, estimate at what time the temperature of the dish will reach 30°C . Explain how you read the result on the graph.



- (v) Calculate the exact result using the formula you found for the temperature of the dish.



- (b) The chef uses a heating device to make sure that the temperature of the dish does not fall below 30°C . The temperature of the dish is now provided by

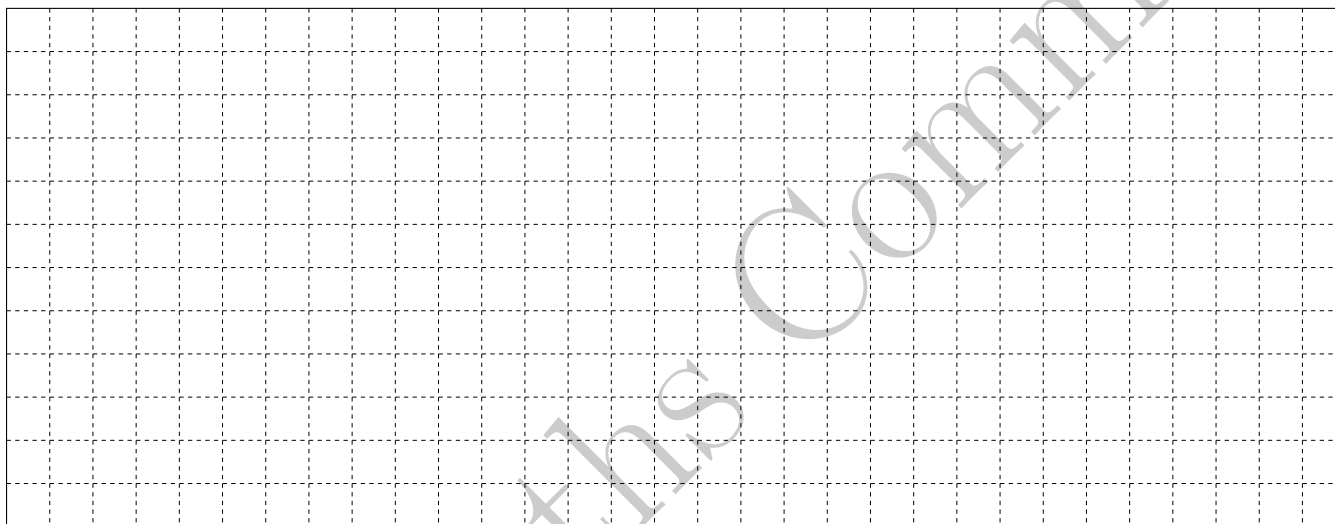
$$T = 20 + 30e^{-0.182t} + ct$$

where c represent the intensity of the heating device.

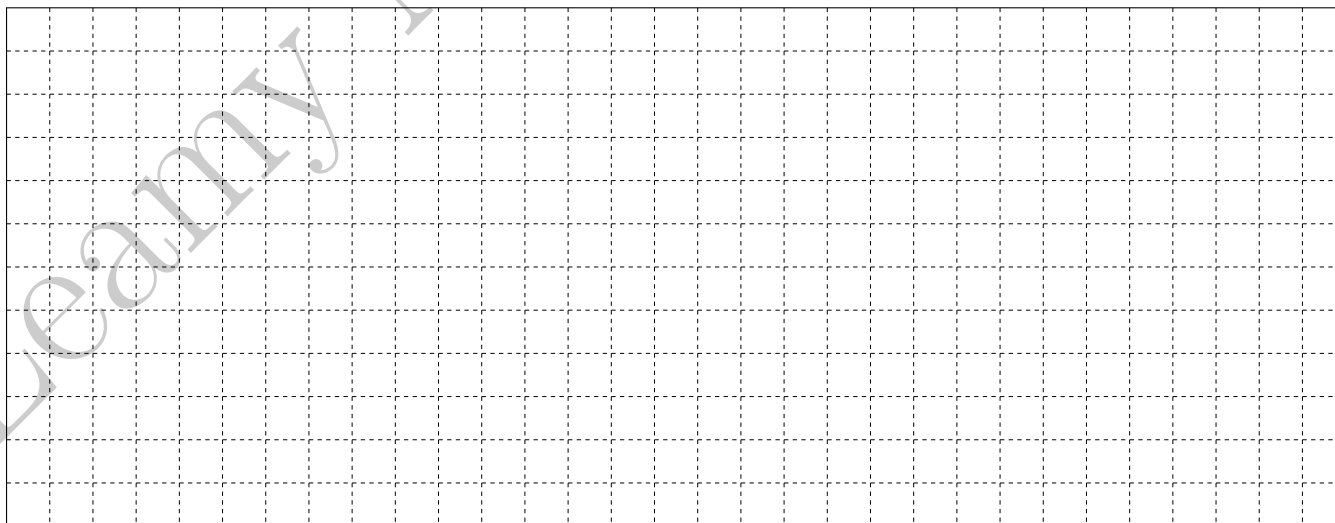
- (i) Show the the minimum temperature is reached at time

$$t_m = \frac{1}{0.182} \ln \left(\frac{5.46}{c} \right)$$

and justify that this is a minimum.



- (ii) The chef can choose between two settings, corresponding to $c = 0.5$ and $c = 1$. Using the result of the previous question to justify your answer, what value should he choose and what would be the corresponding minimum temperature?



Question 9**(50 Marks)**

The number of vehicles per hour between 05.00 am and 14.00 pm at a given spot on busy stretch of road is given by the function

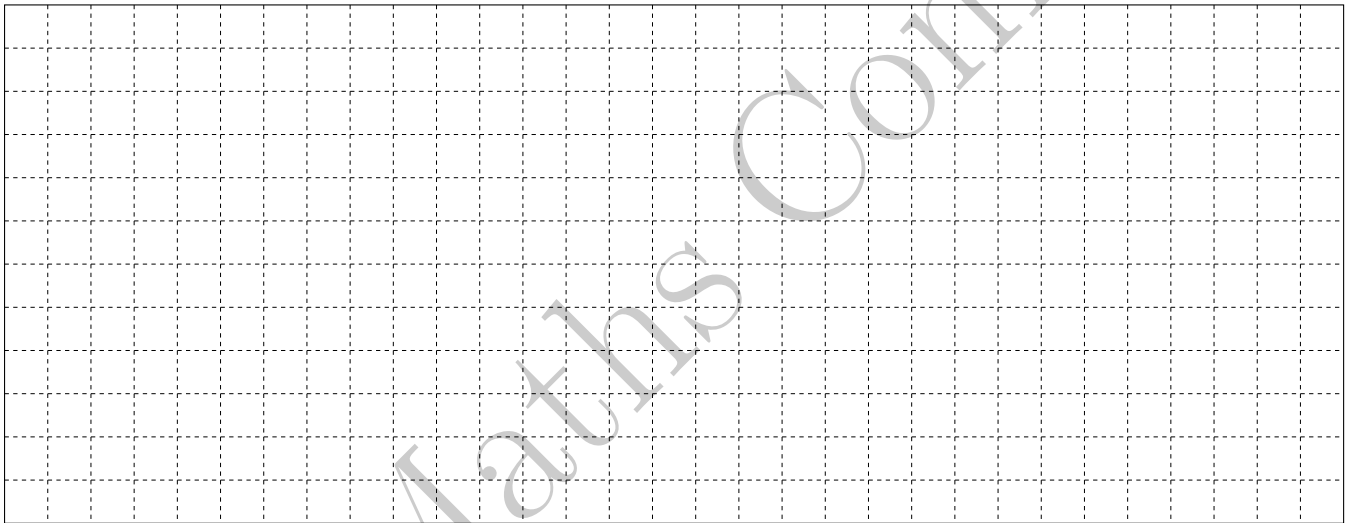
$$f(t) = \begin{cases} 20(t-5)^2(3t^2 - 62t + 325) & \text{if } 5 \leq t \leq 10 \\ 2500 & \text{if } 10 \leq t \leq 14 \end{cases}$$

Times are in hours. $t = 6$ correspond to 06.00am, $t = 8.5$ corresponds to 8.30am. In the following, you may write all times in the decimal form, using 8.5 for 08.30am.

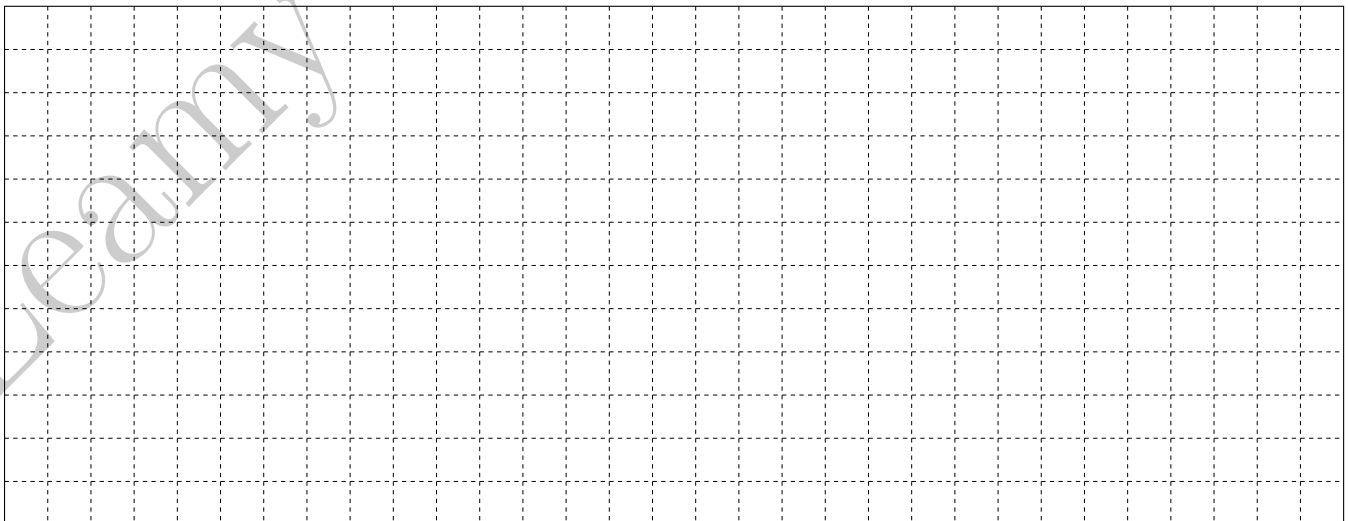
(a) Morning rush

(i) Show that the derivative of the function for $5 \leq t \leq 10$ may be written as

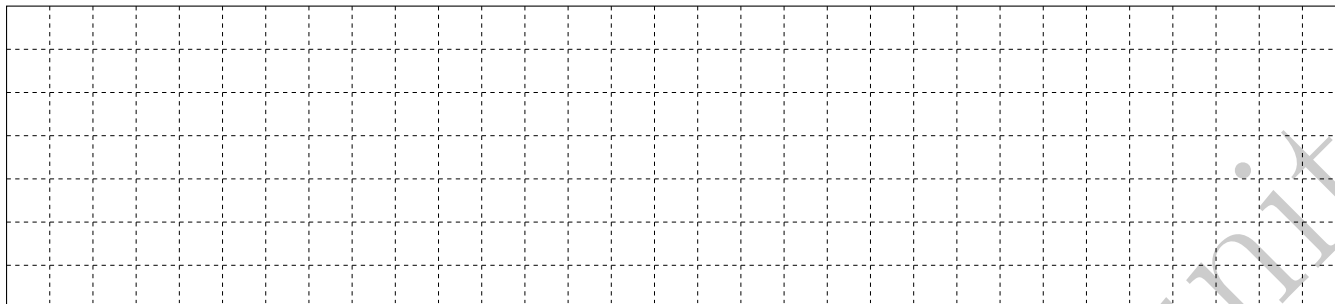
$$f'(t) = (t-5)(at^2 + bt + c)$$



(ii) Identify the times at which the traffic reaches minimum and maximum values.



(iii) Calculate the corresponding value of the number of vehicles per hour.

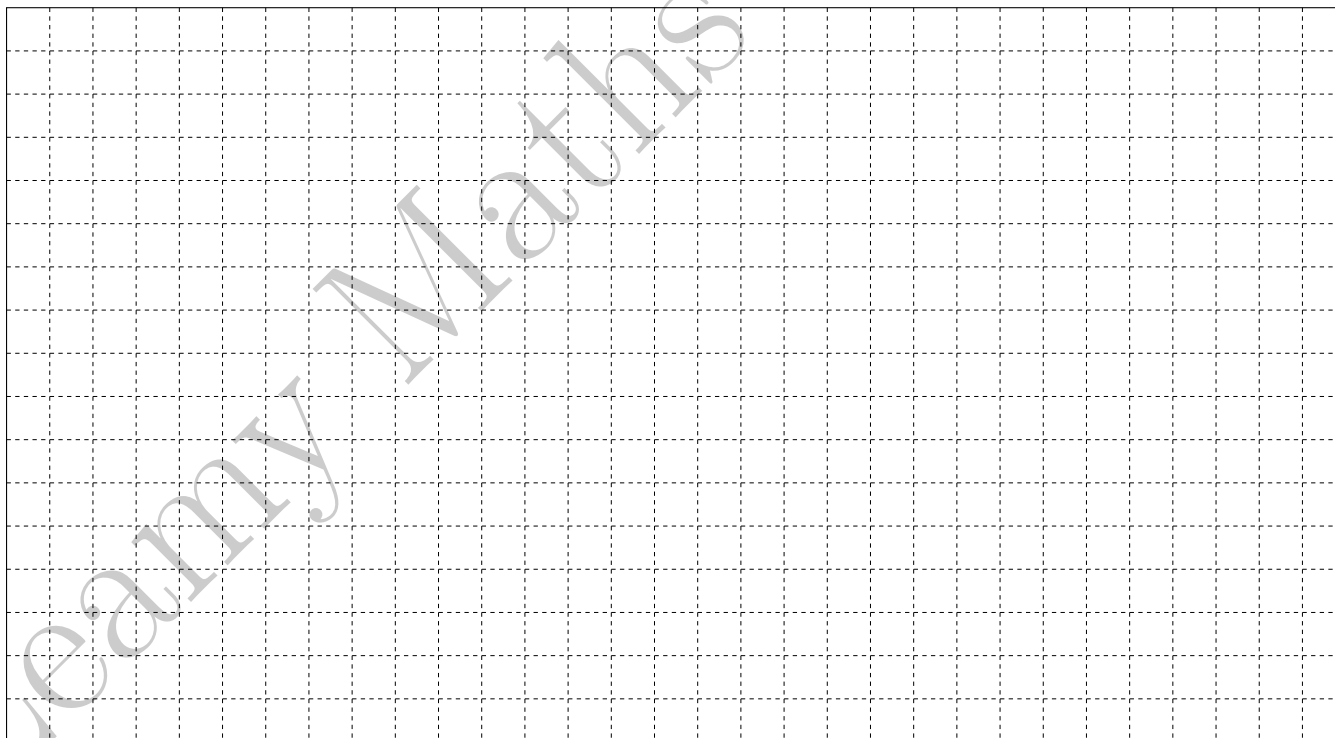


(b) The number of vehicles using the road between the times $t = a$ and $t = b$ is given by the integral

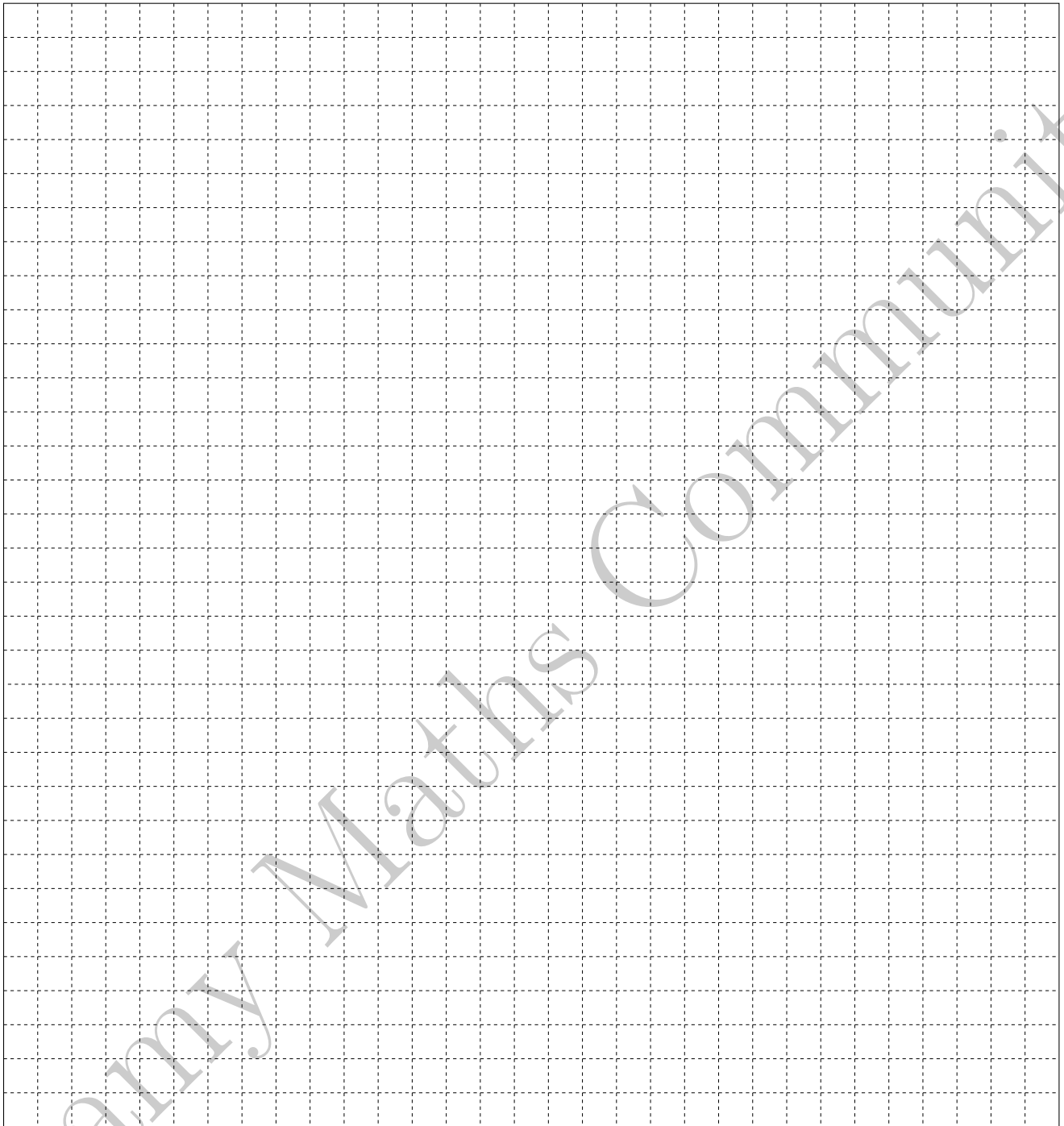
$$N = \int_a^b f(t) dt$$

(i) Show that

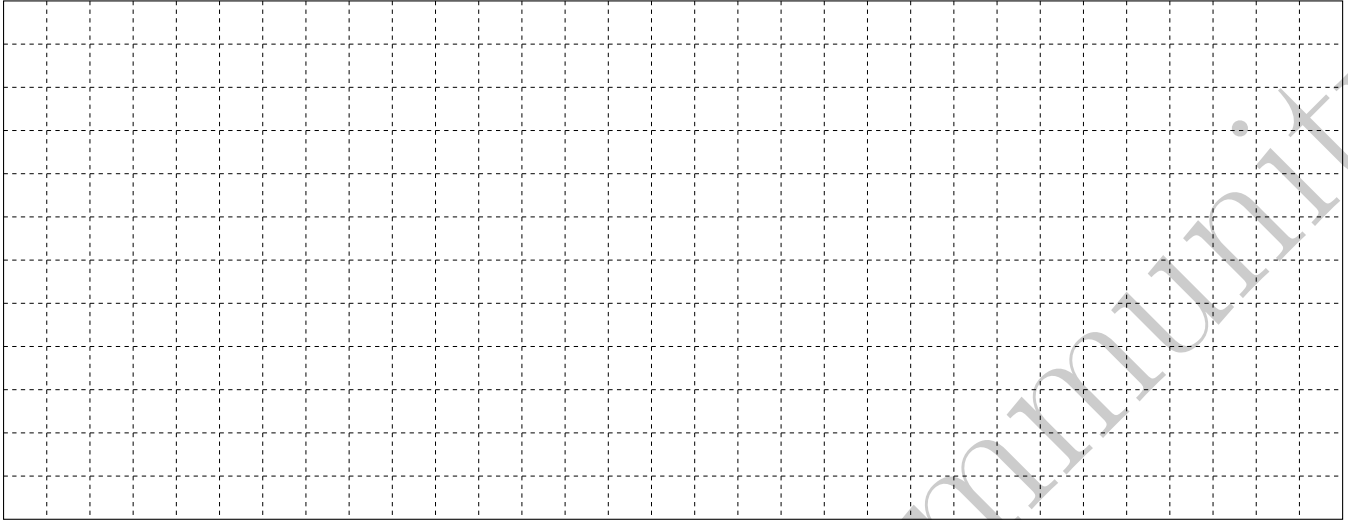
$$20(t - 5)^2(3t^2 - 62t + 325) = 60t^4 - 1840t^3 + 20400t^2 - 96000t + 162500$$



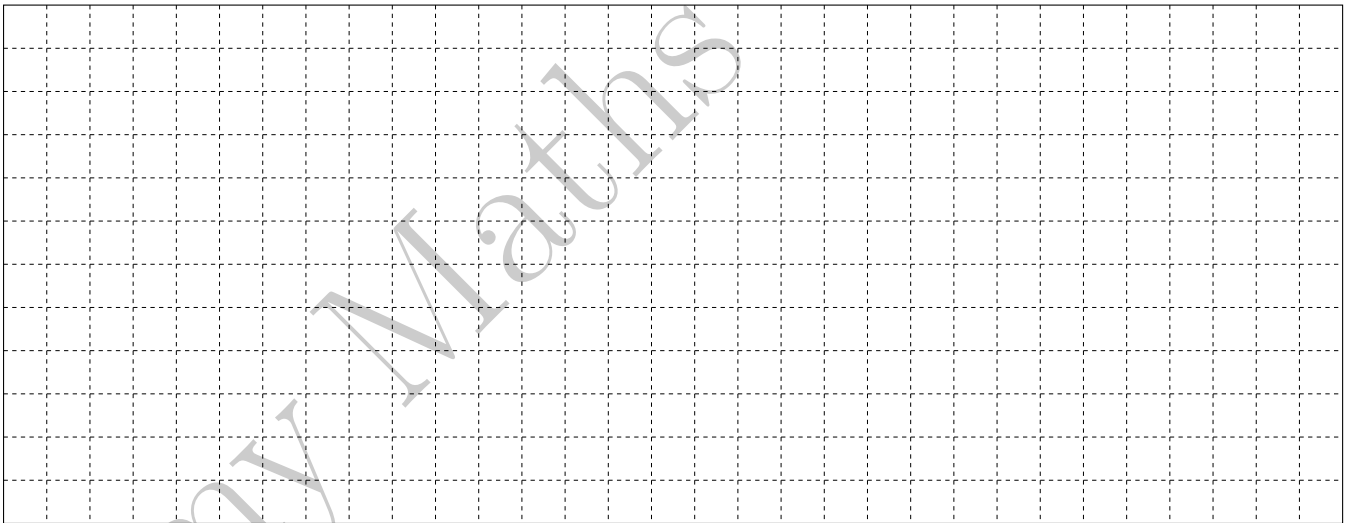
(ii) Show that 12,500 vehicles use the road between 5.00am and 10.00am.



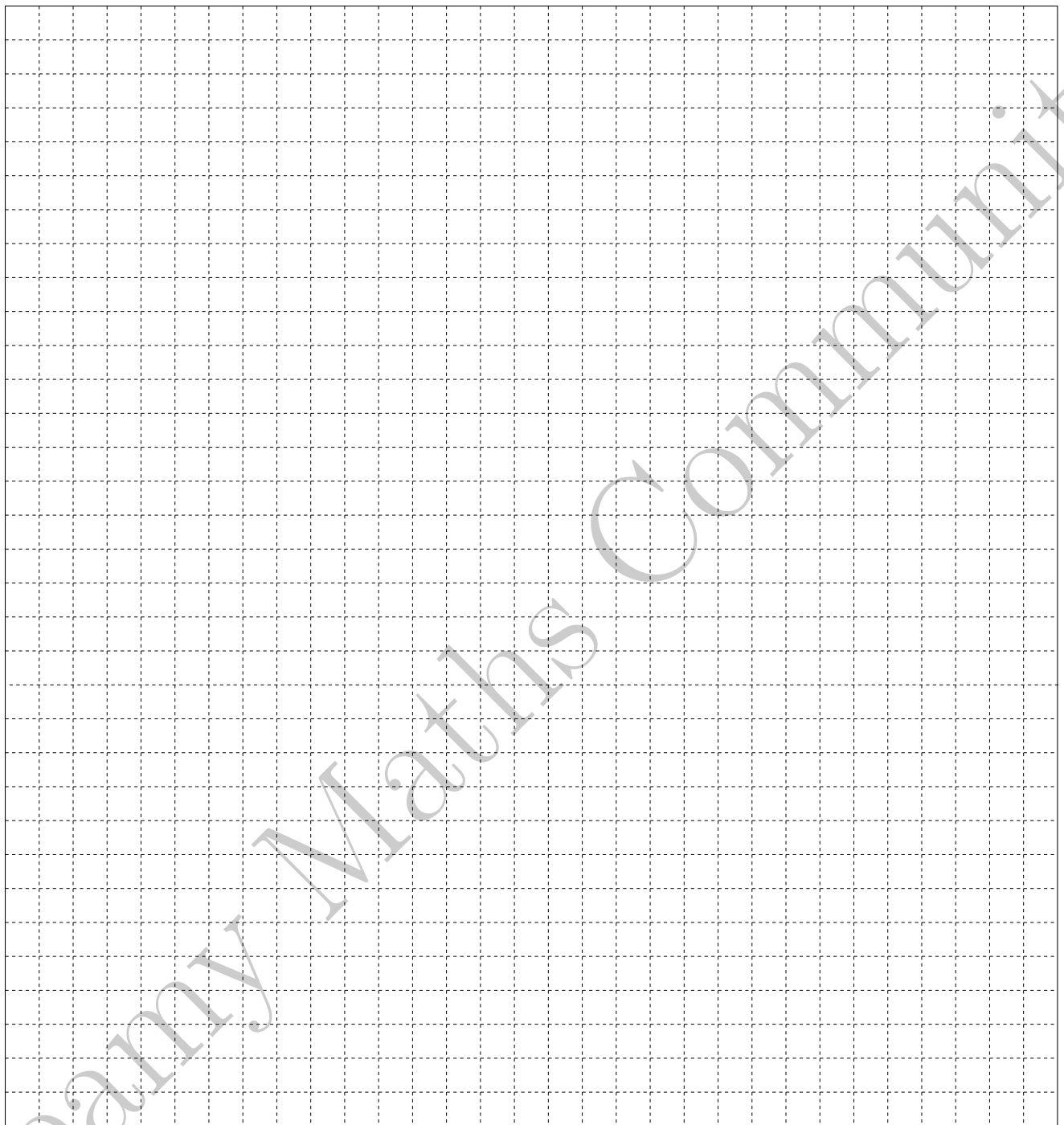
- (iii) Due to road works, only 18,000 vehicles will be able to use the road between 5.00am and 14.00pm. At what time will this number be reached?



- (c) Vehicles will now be charged when using this portion of the road. Drivers will need to pay €8 before 10.00am and €4 between 10.00am and 14.00pm. Calculate the revenue generated if there are no road works.



Rough Work



Rough Work

