Leaving Certificate Examination, 2015

Sample paper prepared by Leamy Maths Community

Mathematics

Project Maths - Phase 3

Paper 2

Higher Level

Sunday 26 April

Paper written by J.P.F. Charpin

Leamy Maths Community

300 marks

Sample Instructions

There are three sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	2 questions

Answer **all eight** questions as follows:

In Section A, answer

- Questions 1 to 5 and
- **either** Question 6A **or** Question 6B

In Section B, answer both Question 7 and Question 8.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination.

You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A Concepts and Skills 150 Marks

Answer **all six** questions from this section.

Question 1 **(25 Marks)**

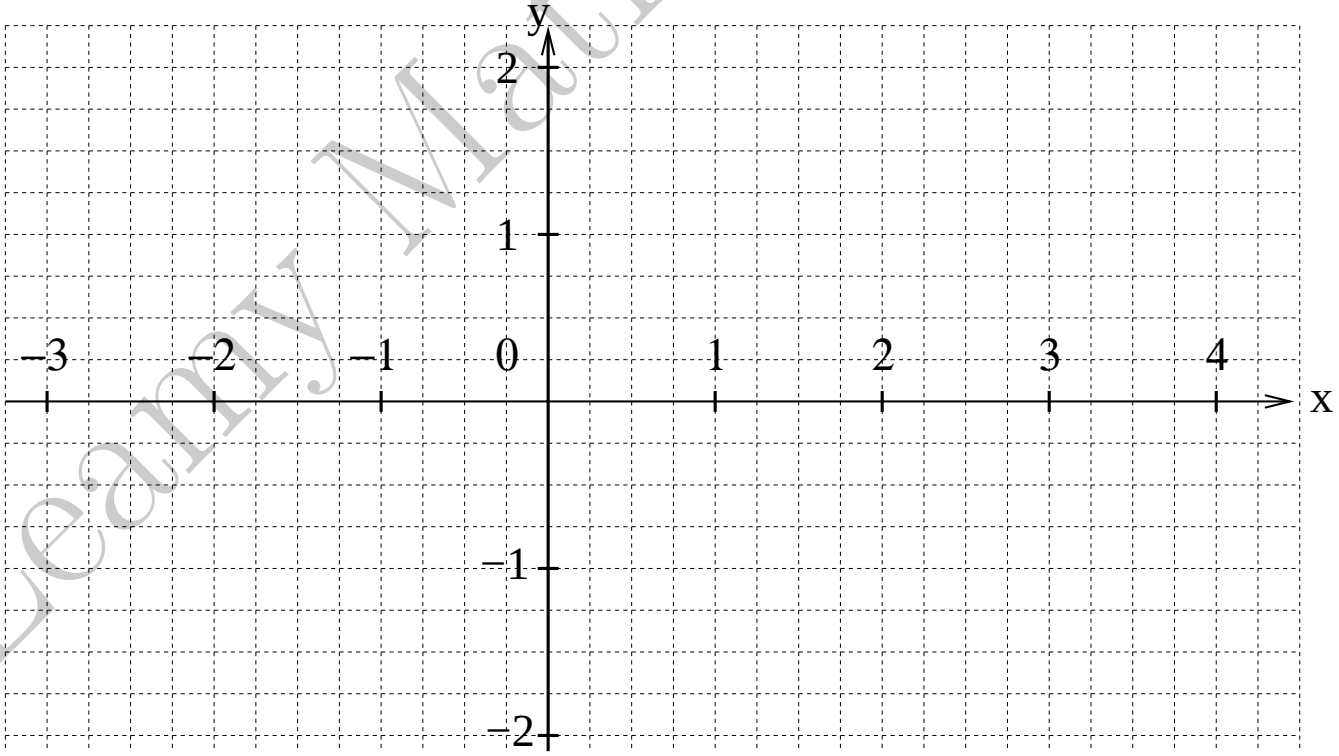
Line 1	$y = 2x + 1$
Line 2	$y = 3x - 2$
Line 3	$x + 3y = 4$
Line 4	$-2x + 4y = 4$
Line 5	$4x = 2y + 1$
Line 6	$x - y = 0$

(a) Which lines are parallel? Justify your answer.

(b) Which two lines are perpendicular? Justify your answer.

(c) Calculate the equation of the lines which make a 45° angle with Line (1) and has the same y intercept as Line (4).

(d) Plot Lines (3) and (6) on the diagram below and read on the graph where they cross.



(e) Calculate the intersection point between Line (3) and Line (6) analytically.

Question 2 **(25 Marks)**

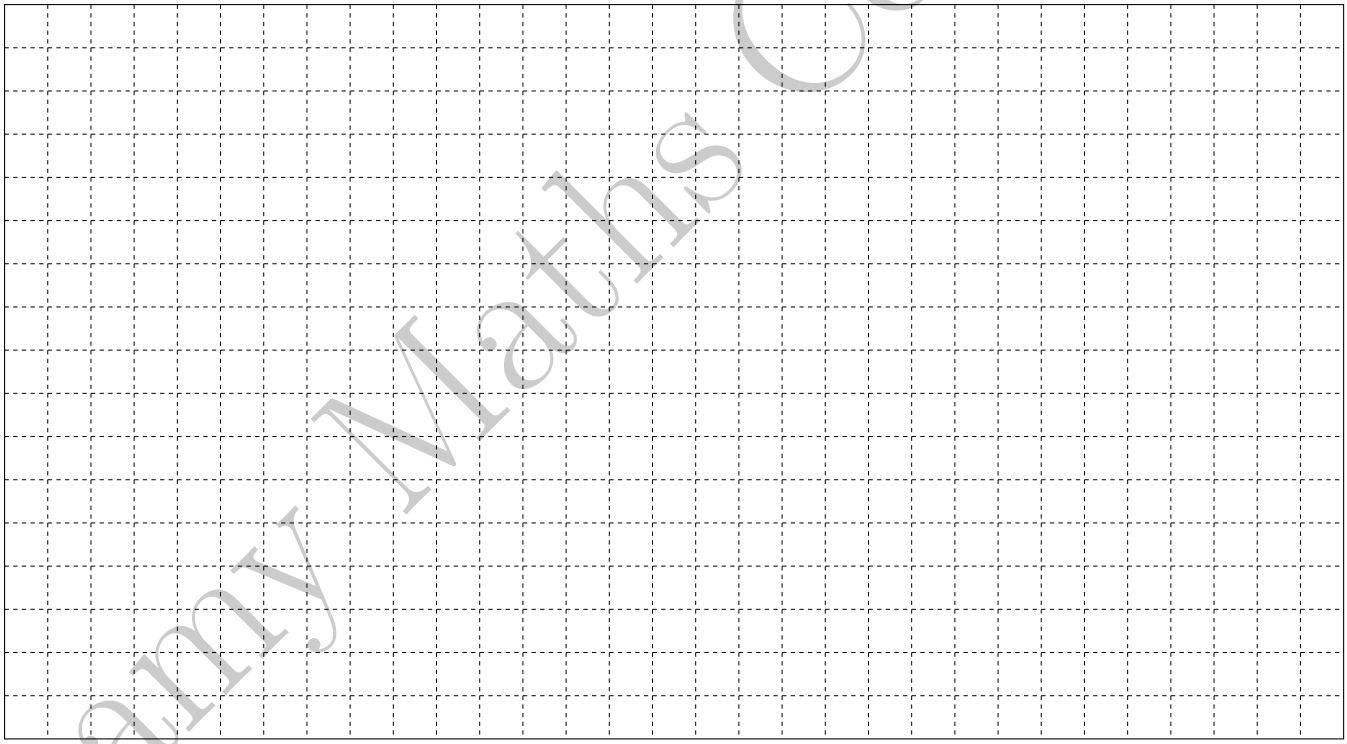
(a) Calculate the equation of the circle $(\mathcal{C}_1)$ of radius $R = 5$ and centre $(-3, -2)$.

(b) Determine the centre and radius of the circle $(\mathcal{C}_2)$ described by the following equation:

$$x^2 + y^2 - 18x - 14y + 30 = 0$$

(c) Show that the two circles $(\mathcal{C}_1)$ and $(\mathcal{C}_2)$ are externally touching.

(d) Calculate the coordinates of the point common to both circles $(\mathcal{C}_1)$ and $(\mathcal{C}_2)$



Question 3 **(25 Marks)**

In a school, a sample of 50 randomly selected students found that on average, they live 24 minutes drive away from the school with a 4 minute standard deviation.

(a) Construct a 95% confidence interval for the driving time from home to school the students of this school.

(b) The school principal believes that on average, students live 20 minute drive away from the school. Test this hypothesis using a 5% level of significance. Detail all steps of the method

(c) 50 sixth year students were asked if they had found a summer job. 20 students answered yes. Construct a 95% confidence interval for the proportion of students who will have a summer job.

(d) The school principal believes that on average, half the sixth year students will be working during the summer. Test this hypothesis using a 5% level of significance. Detail all steps of the method

Question 4 **(25 Marks)**

A screw manufacturer sells its products plastic bags. The number of screws per bag follows a normal distribution. On average, there are 150 screws per bag with a standard deviation of 9.

(a) A bag is chosen randomly

(i) Calculate the probability that the bag contains less than 155 screws

(ii) What is the probability to find between 140 and 160 screws

(iii) Can you find more than 180 screws in a bag? Justify your answer.

(b) 2% of the screws are defective. 10 screws are chosen randomly

(i) Calculate the probability that a single screw is defective

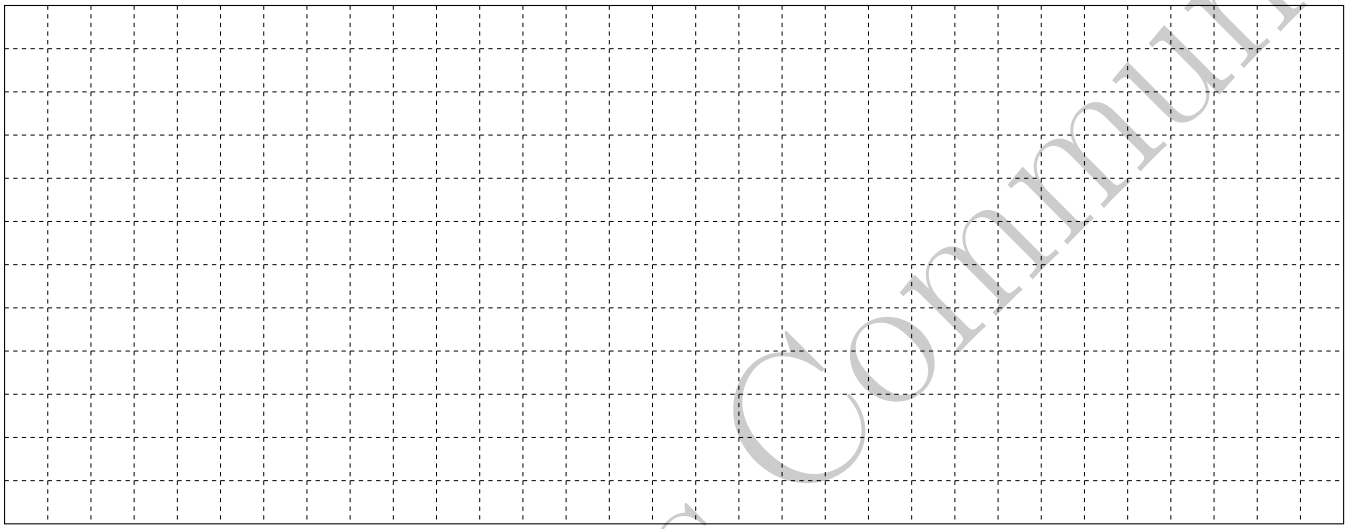
(ii) What is the probability that 2 screws or more are defective?

Question 5 **(25 Marks)**

(a) Calculate the solutions of the following equations

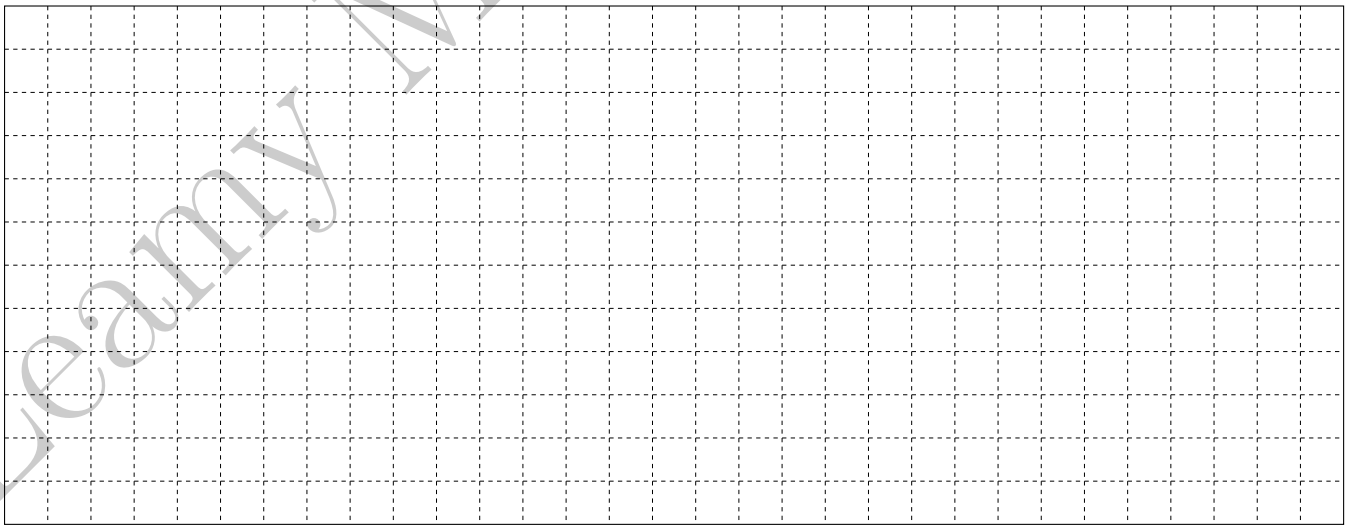
(i) $x \in [0,\ 2\pi]$

$$cosx = -\frac{1}{2}$$



(ii) $x \in [-2\pi,\ 2\pi]$

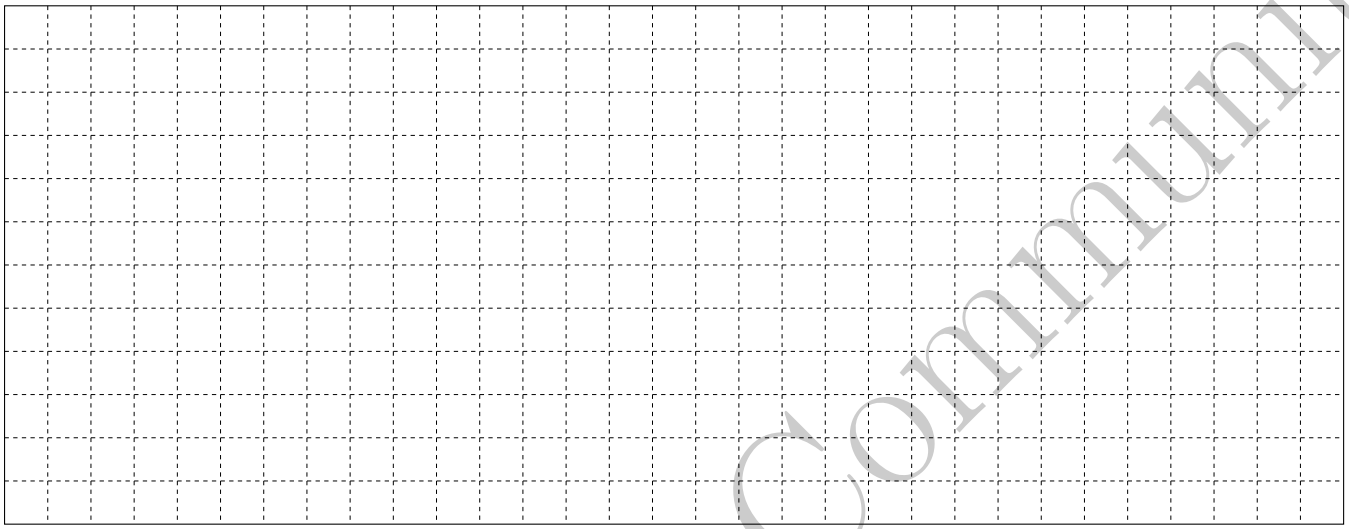
$$\sin(3x) = \frac{\sqrt{3}}{2}$$



(b) Trigonometry equation

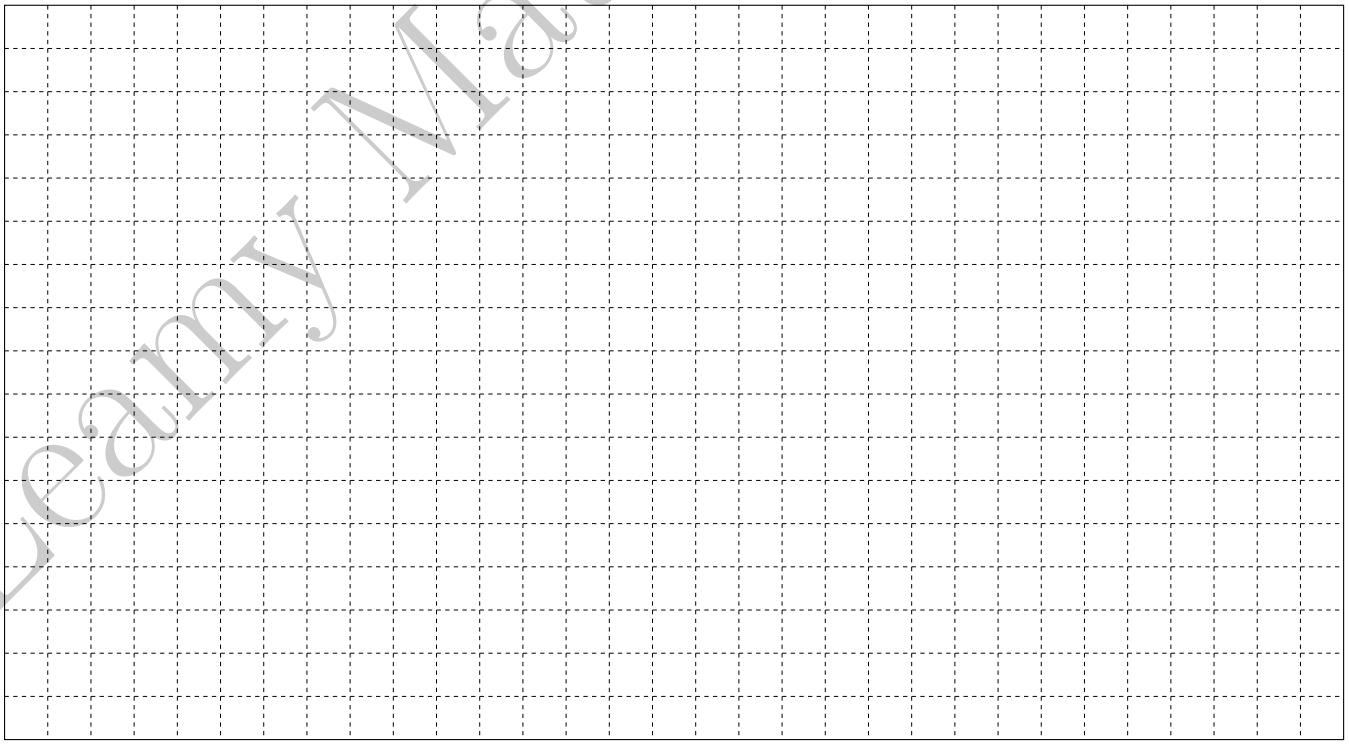
(i) Show that

$$2\sin(2x)\sin x - \cos x = \cos x\left(3 - 4\cos^2 x\right)$$



(ii) Solve the equation

$$2\sin(2x)\sin x = \cos x$$

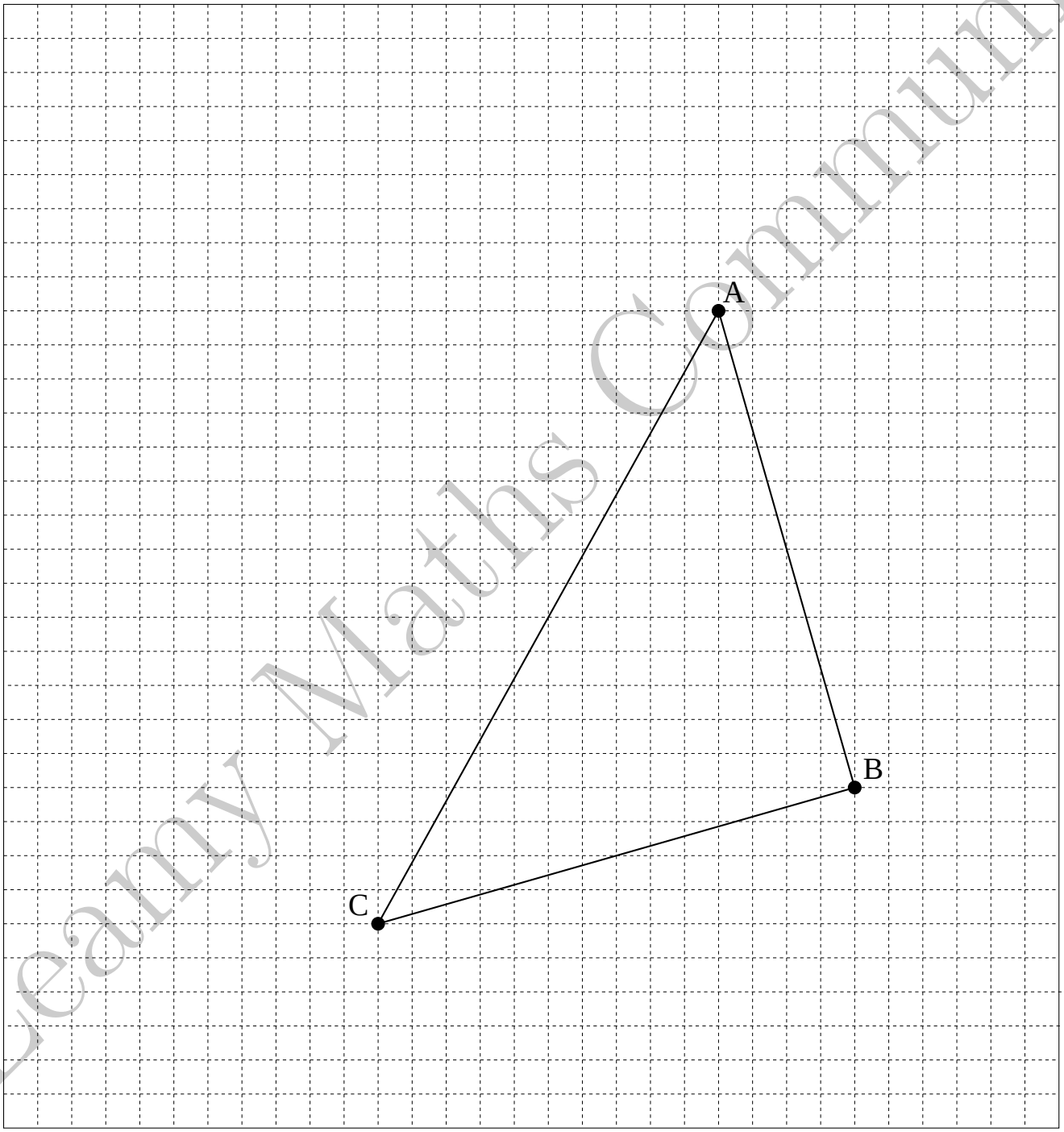


Question 6 **(25 Marks)**

Answer **either** 6A **or** 6B.

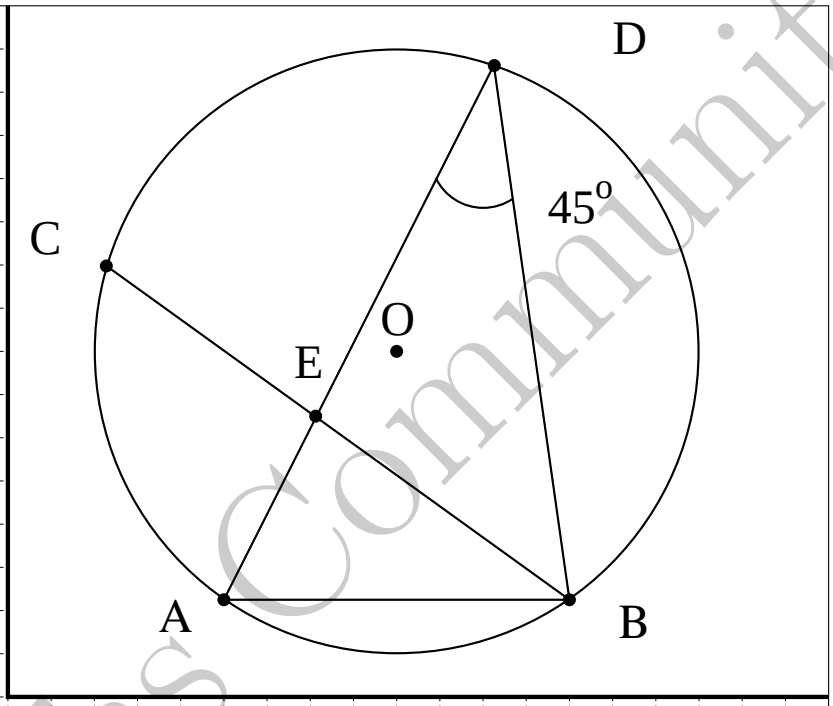
Question 6A

Define the circumcentre, the incentre and the centroid of a triangle. Construct these 3 points for the triangle below without using a graduate ruler or a set square.



Question 6B

Points A, B, C, and D are on a circle, see figure below, and $< ADB = 45°$. The lines (AD) and (BC) cross at point E. Show that if (AD) and (BC) are perpendicular and $|AB| = |AC|$, E is the centre of the circle. (Hint: compare the angles $< ACB$ and $< ADB$ and calculate other angles.)



Section B Contexts and Applications 150 Marks

Answer Question 7 **and** Question 8.

Question 7 **(75 Marks)**

The table below shows running times at the 2014 Dingle marathon.

Running time (hours)	$2.5-3$	$3-3.5$	$3.5-4$	$4-4.5$	$4.5-5$
Number of runners	6	35	78	87	50
Running time (hours)	$5-5.5$	$5.5-6$	$6-6.5$	$6.5-7$	
Number of runners	36	17	15	6	

(a) Characteristics of the distribution

(i) Create a suitable representation of the running time for the 2014 marathon.

(ii) Describe the distribution. This should include information such as shape, range, estimation for the median value. What is the position of the mean relative to the median? Can you define a mode?

(iii) Is it possible to use a standard normal distribution to represent the data? Justify your answer

(iv) Using mid interval values, estimate the average running time.

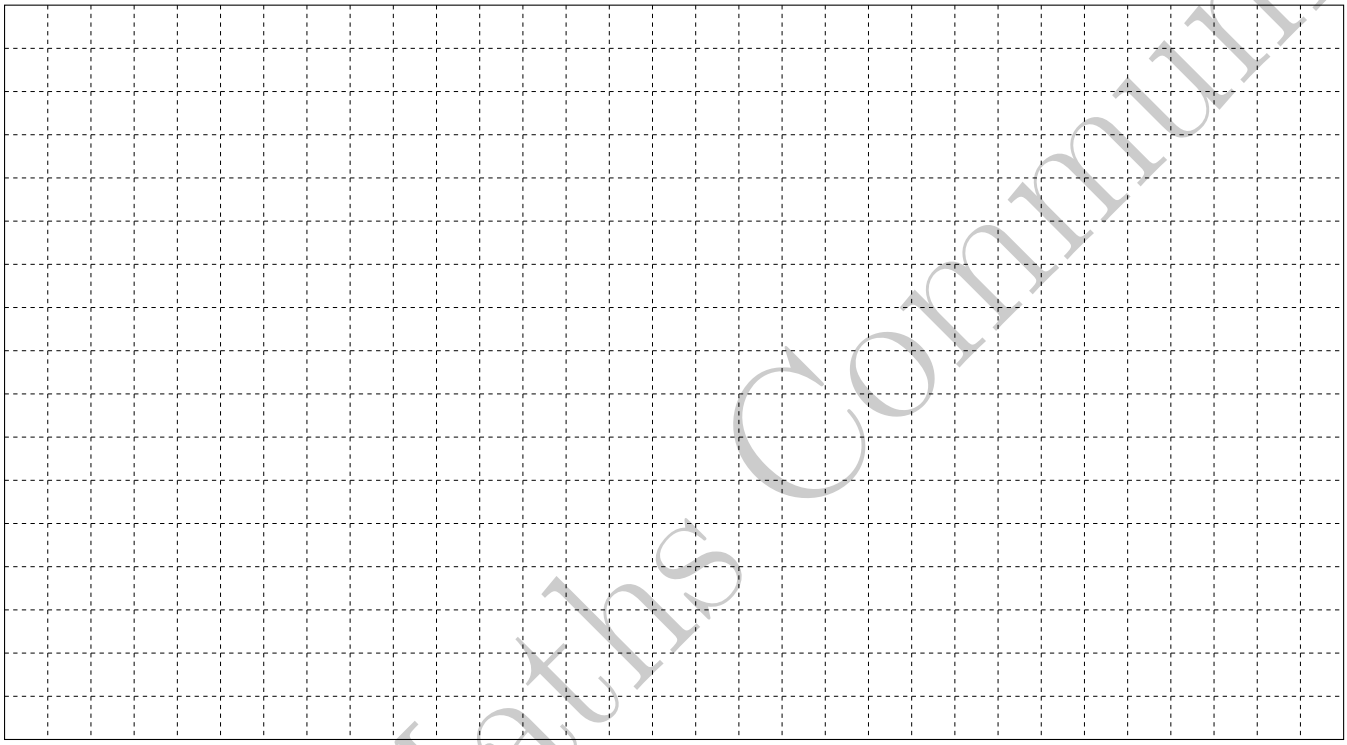
(b) The organisers of the race choose 10 random runners to quickly evaluate the average running time of participants. They find an average value of 4.75 hours and a standard deviation of 0.7 hours.

(i) Construct a 95% confidence interval for the average running time of this race according to this sample.

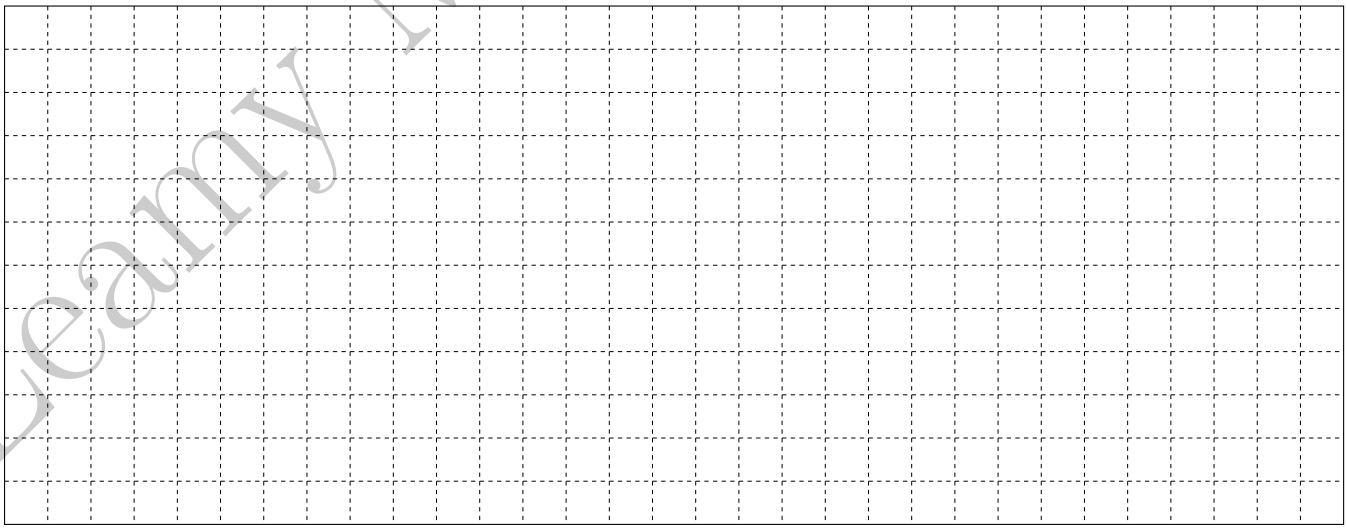
(ii) The actual average running time is actually 4.4 hours. The organisers say that their sample was representative of all runners in the race as far as the running time is concerned. Test this hypothesis using a 5% level of significance. Detail all steps of the method.

(c) The organisers want 10 values to estimate the race running time. 5% of runners abandon the race and do not complete the marathon. The organisers therefore want to choose more than ten people to make sure they have a sufficient number of data to estimate the average running time. They decide to take a sample of 12 people.

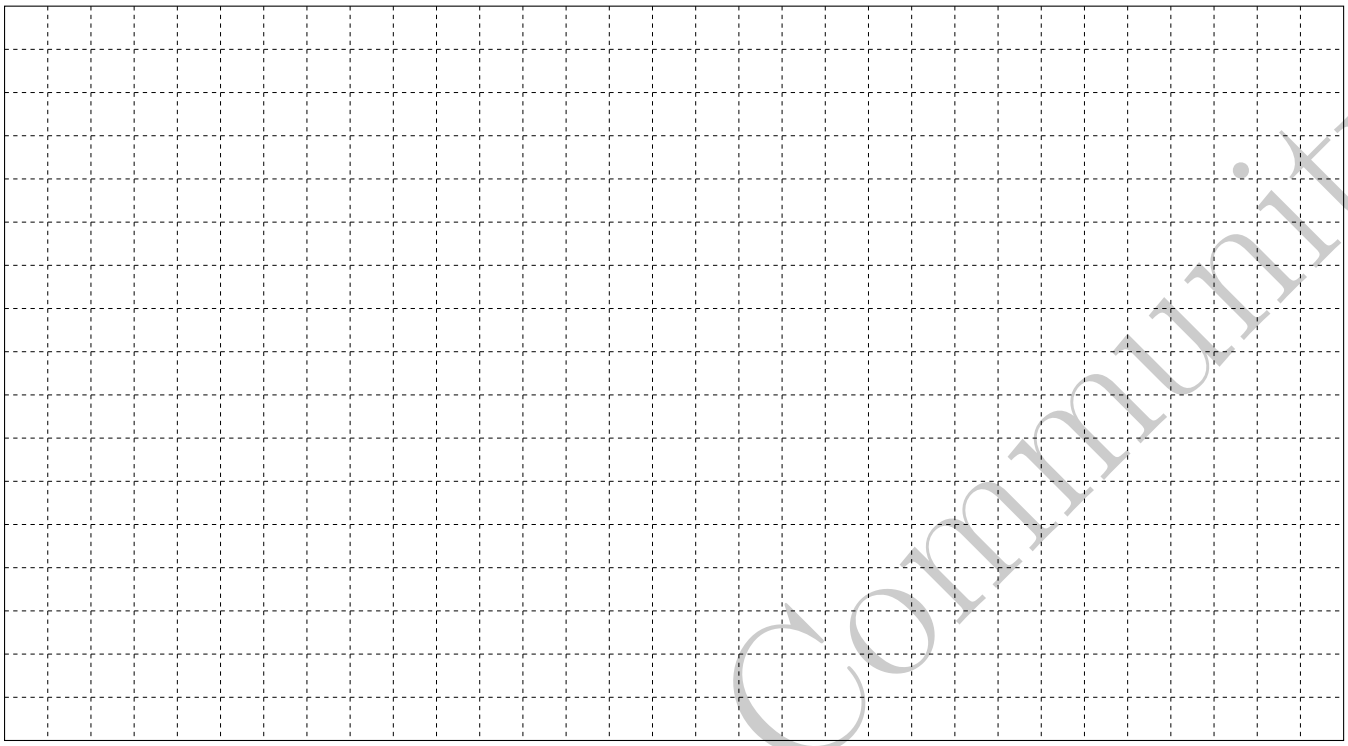
(i) Can whether or not a person finishes the race be considered a Bernoulli trial? You should specify the conditions for Bernoulli trials (also called binomial trials) and then conclude if it is possible to use this method here.



(ii) What is the probability that exactly 10 people finish the race?

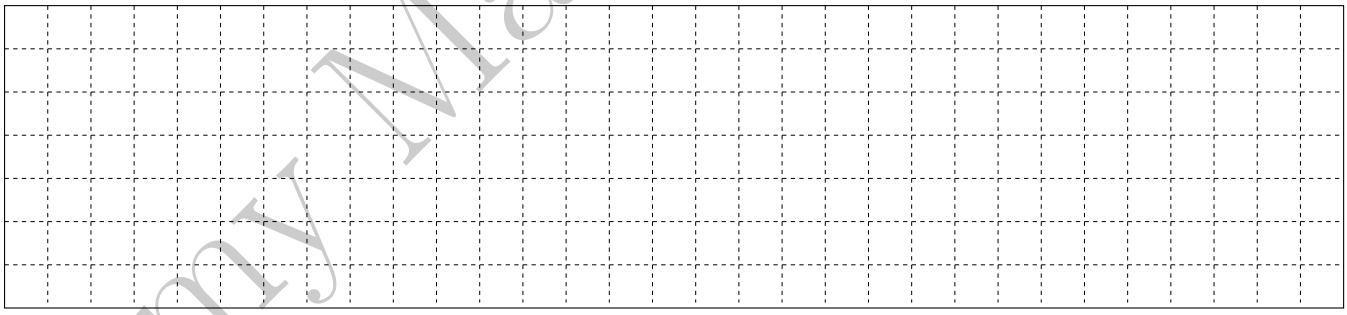


(iii) What is the probability that 10 or more people finish the race?

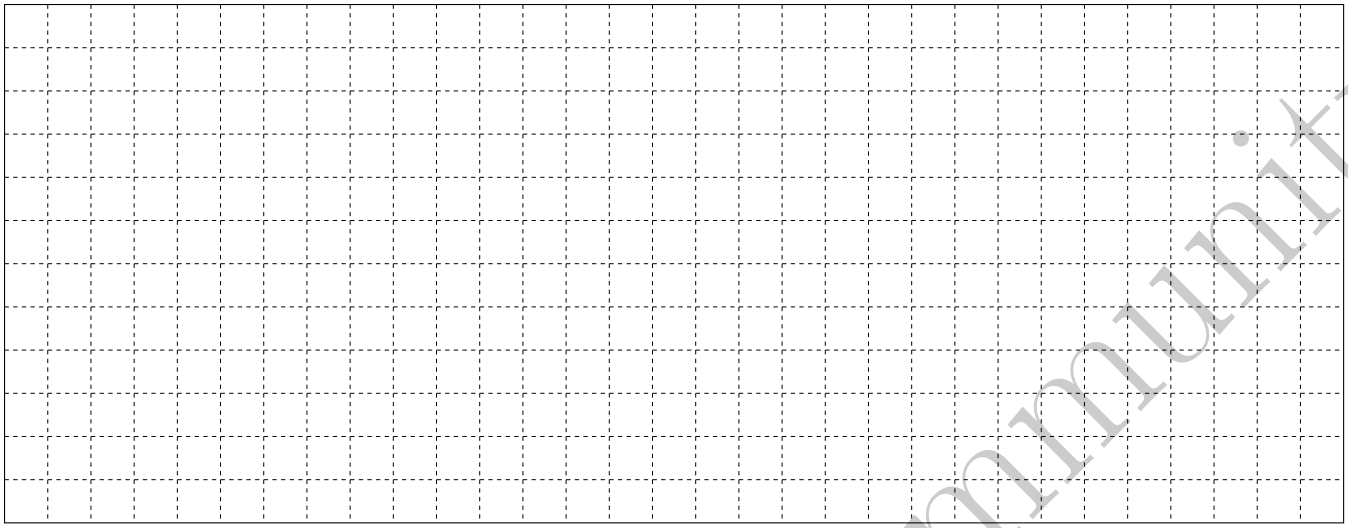


(d) The organisers want to study the average running time in the sample. The following probabilities should be calculated using the table provided (these runners have completed the race).

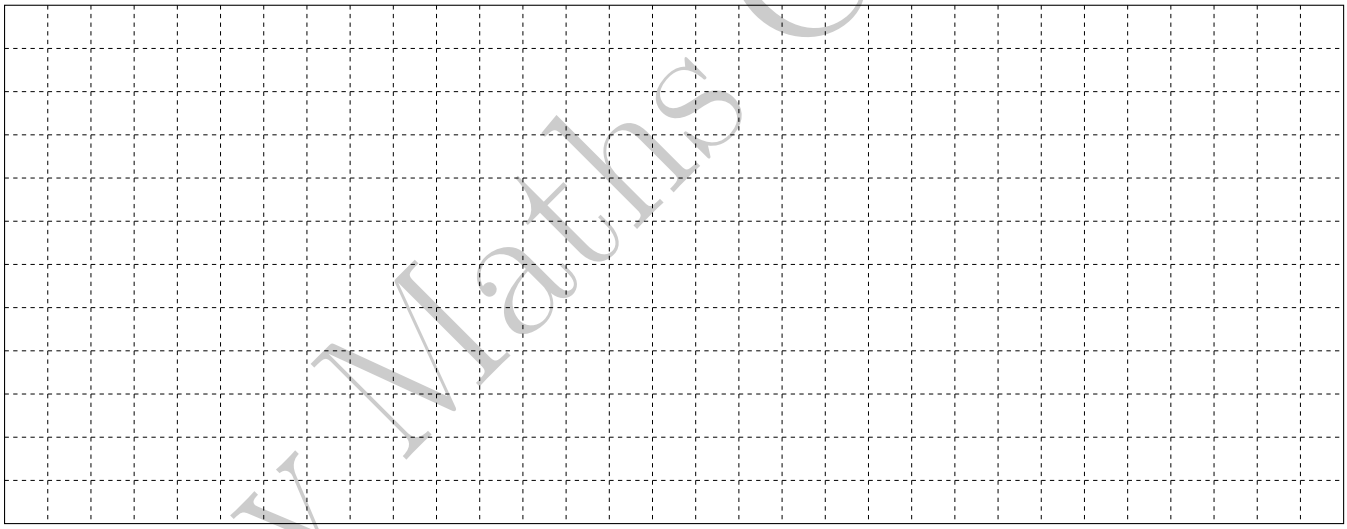
(i) What is the probability that a randomly selected runner finishes the race between 3 and 4 hours?



(ii) What is the probability that 3 randomly selected runners all finish the race between 3 and 4 hours?

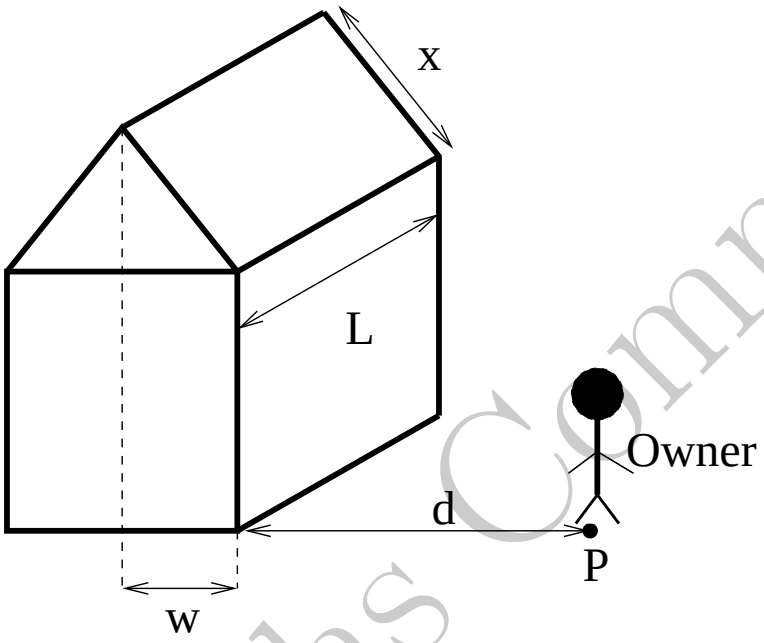


(iii) If two runners are randomly chosen, what is the probability that 1 runner finishes between 3 and 4 hours and 1 runner finishes between 4 and 5 hours?



Question 8 **(75 Marks)**

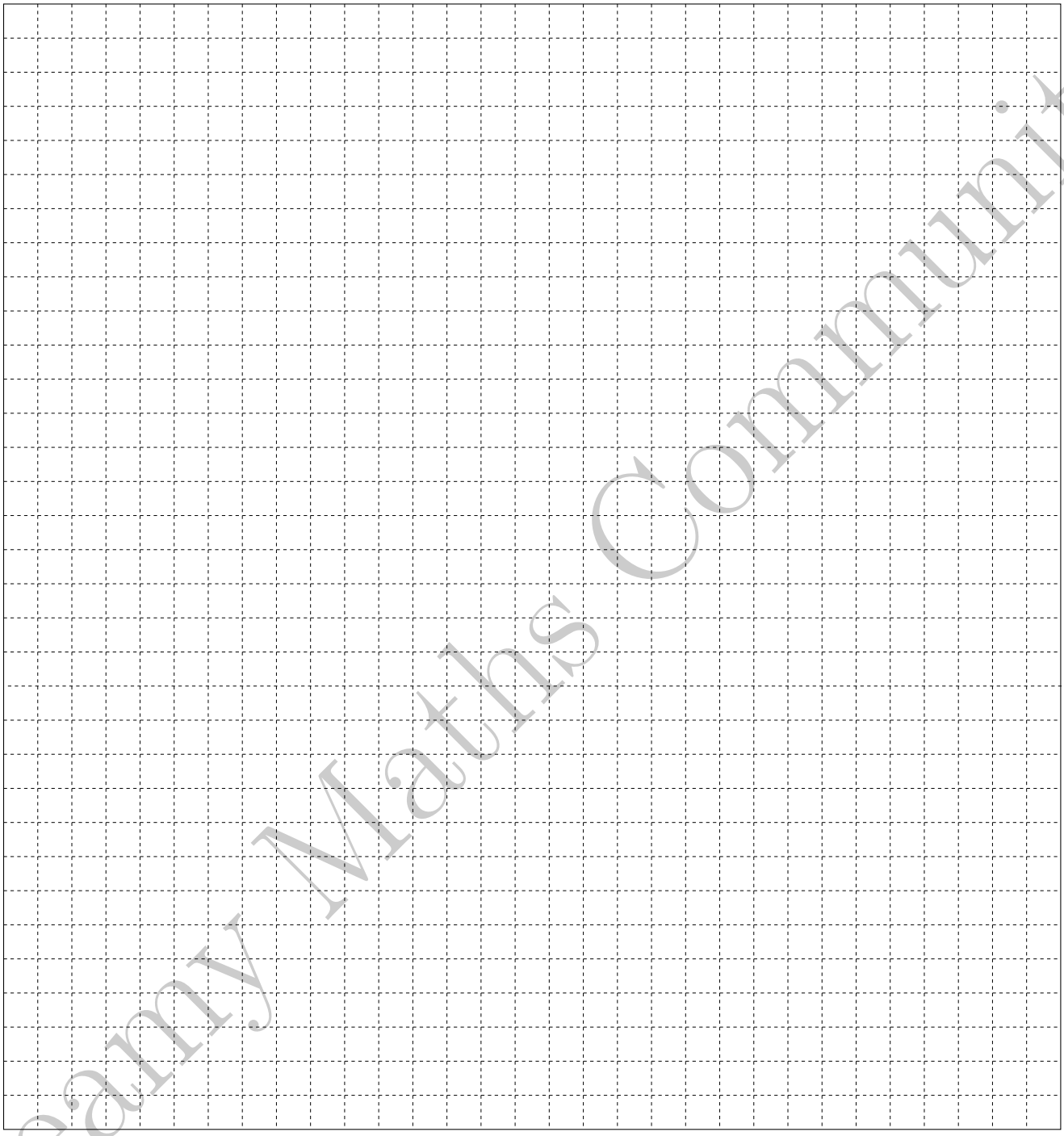
(a) A house is to become eco-friendly. The roof is going to be covered with solar panels and rain collectors will be installed. The owner wants to estimate the area of the roof without climbing on top of the building but is happy to do all the necessary measurements on the ground. He uses a clinometer to calculate the angles of elevation from point P on the ground to the bottom and top of the roof and finds $45°$ to the bottom of the roof and $52.12°$ to the top of the roof.

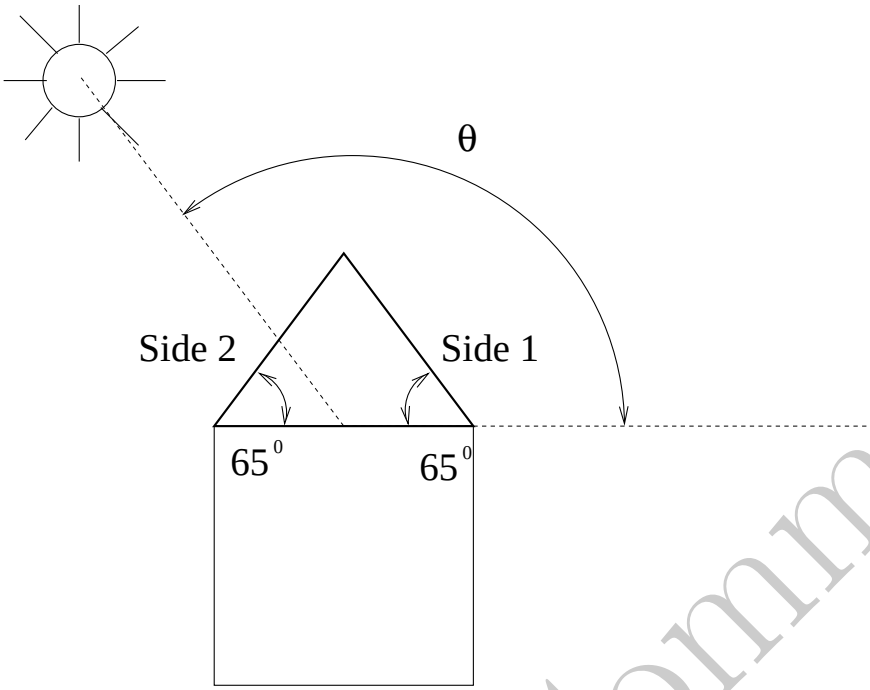


The owner measured the following: $w = 4$m, $L = 10$m and $d = 10$m. The dimension x should be calculated correct to one decimal place.

(i) Explain how to use a clinometer.

(ii) Calculate the area of the roof correct to one decimal place and the angle of the roof to the horizontal in degrees, rounded up to the nearest degree.





(b) The left side of the house is facing West, the right side is facing East. It is assumed that there are 12 hours of light every day from 6 in the morning till 6 in the evening. At sun rise, $\theta = 0$, at sun set, $\theta = 180$.

(i) For what values of θ will side 1 face the sun ?

(ii) For what values of θ will side 2 face the sun?

(iii) Assuming the sun moves 15° per hour, estimate from what time to what time each side of the roof will be lit.

(iv) The efficiency of the solar panels is maximum when the sun shines perpendicularly to the roof. Identify which expression represent the efficiency on side 1 and side 2 and for both case, specify for which values of θ it may be used (the expressions are in degrees).

$A:\qquad E = \cos(\theta - 25)$ $\qquad\qquad$ $B:\qquad E = \cos(\theta - 155)$

(v) Calculate for which values of the angle θ the efficiency is equal to 50% (E=0.5). Perform the calculation for both sides.

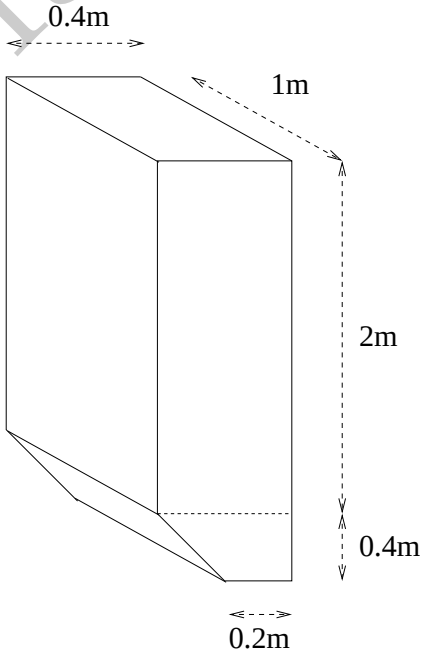
(vi) Using the results of the previous question, determine for what time of the day the efficiency of the solar panel will exceed 50% for side 1 and for side 2.

(c) Rain is falling vertically at a rate of 5mm per hour per square metre. A rain collector gathers of all the water falling on the roof. You want to calculate the volume of water falling on the roof. You therefore need to use the catchment area (area in the direction perpendicular to the rain direction).

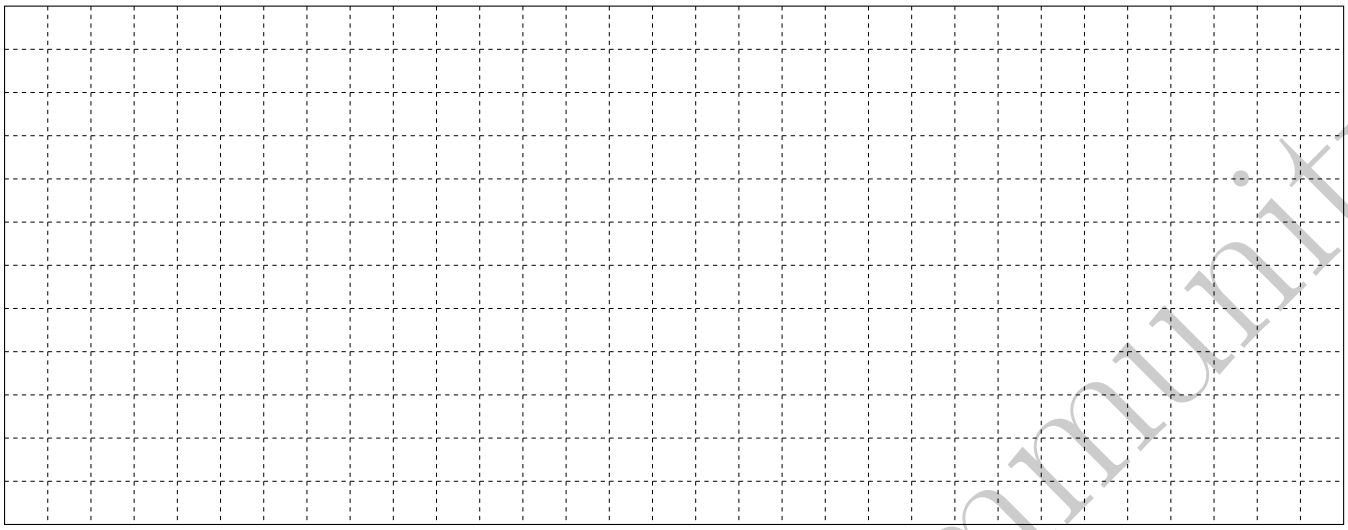
(i) Explain why the catchment area of the house must be 80m^2 and not the answer you found in part a (ii).

(ii) It rains for 2 hours. What volume of water fell on the roof during this time (in m^3)?

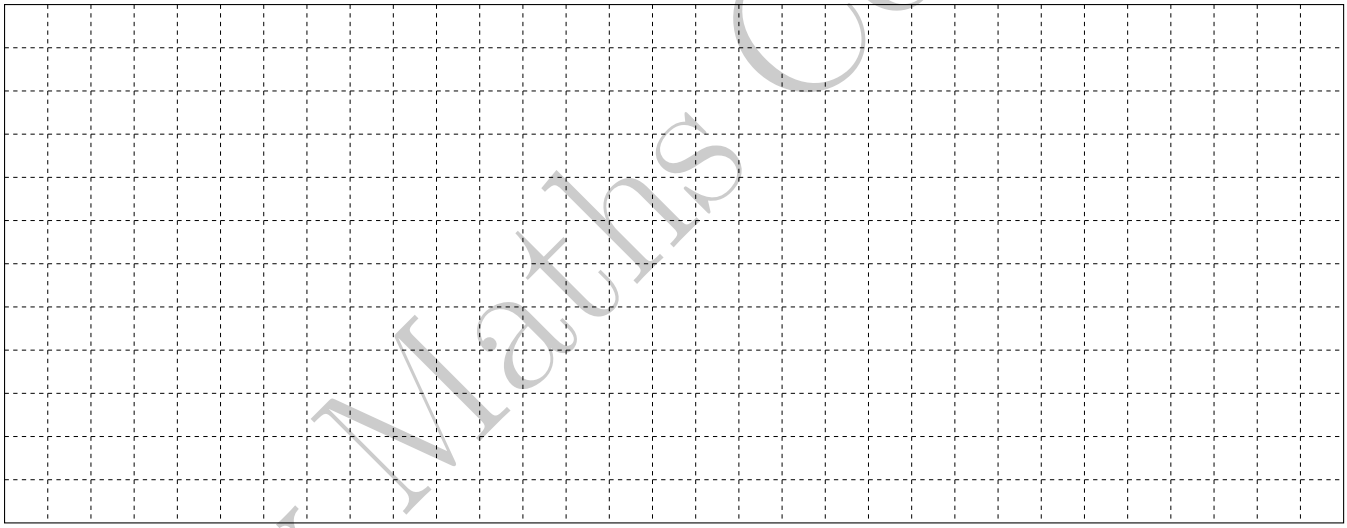
(iii) The rain collector has the following characteristics



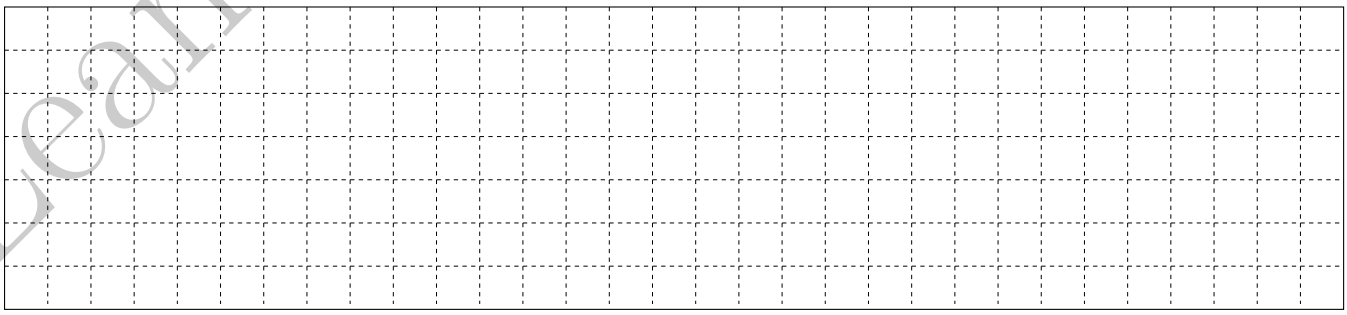
What is the maximum volume of water contained in this rain collector?



(d) What is the level of water in the tank if 0.8m^3 of rain fell on the roof?



(e) How much longer should it rain at this rate to fill the tank?



Rough Work