Leaving Certificate Examination, 2018

Sample paper prepared by Leamy Maths Community

Mathematics

Paper 2

Higher Level

29 April 2018

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LEAMY MATHS COMMUNITY

Name ____

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Total

300 marks

Sample Instructions

There are two sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer questions as follows:

In Section A, answer all six questions.
In Section B, answer all three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination.

You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Section A	Concepts and Skills	150 Marks

Answer **all six** questions from this section.

Question 1 (25 Marks)

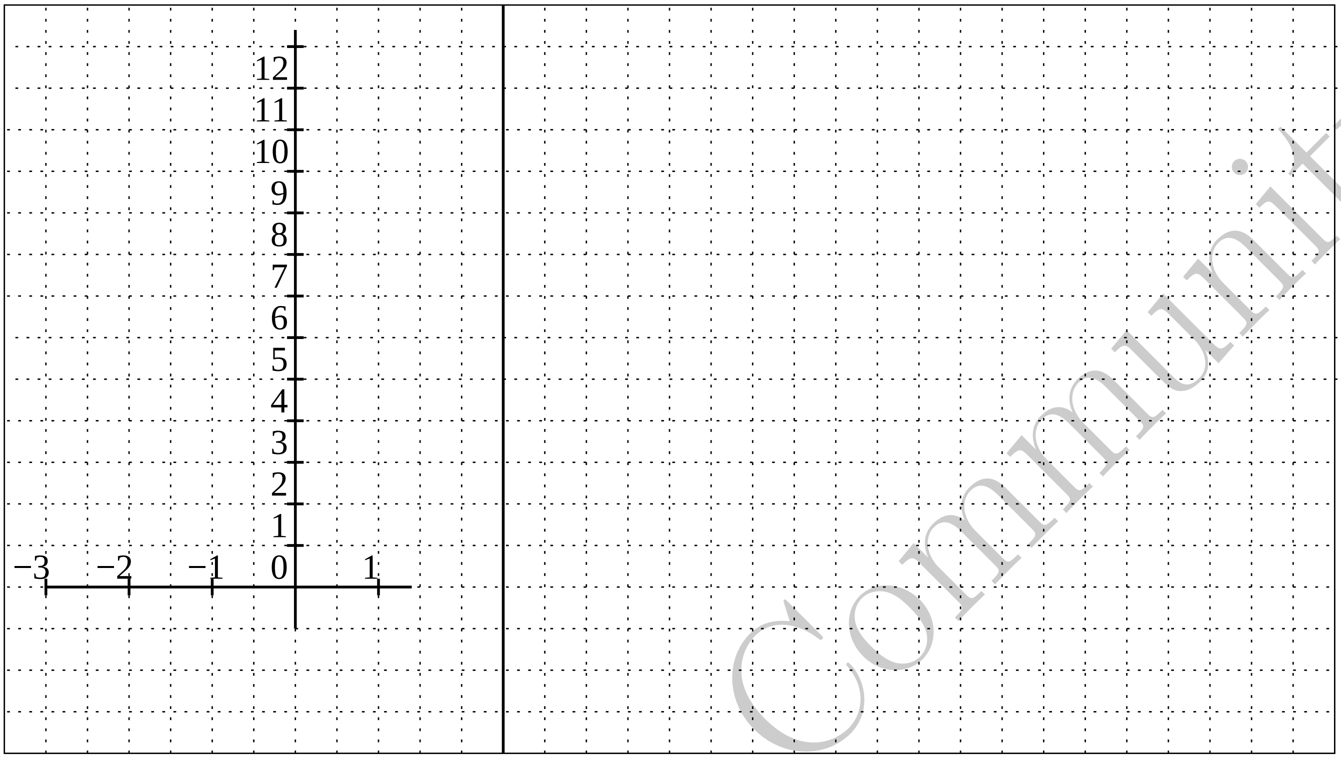
Consider the following line equations:

Line 1	y=2x+4	Line 3	x=16-2y
Line 2	y=-3x+3	Line 4	3y=-9x+2

(a) Which two lines are parallel and which two lines are perpendicular? Justify your answers.

(b) Calculate the equation of the line parallel to line 1 and passing though point A (-1, -1).

(c) Plot lines 2 and 3 on the graph below and calculate their common point.



(d) Calculate the distance between line 4 and point A. Express the value in the form $a\sqrt{b}/c$ where a, b and c are integers.

Question 2 (25 Marks)

(a) Find the equation of the circle $\mathcal{C}_1$ with centre (2,3) and which passes through point (-1,-1).

(b) Calculate the equation of line (T) which is tangent to the circle at point (-1,-1).

(c) Calculate the equation of the circles $\mathcal{C}_2$ and $\mathcal{C}_3$ of radius 10 tangent to line T at point (-1,-1).

(d) Which of the circles $\mathcal{C}_1$, $\mathcal{C}_2$ and $\mathcal{C}_3$ touch internally and which circles turn externally? Justify your answer.

Question 3 (25 Marks)

When Munster plays home, they have an 80% probability of winning. In the 2017/2018 season, Munster played 10 games at home in Thomond Park. Assume that the probability of winning a game is independent from the results of the previous games.

(a) What is the probability that Munster wins 7 out of their 10 games at home? Give your result in percentage form accurate to 2 decimal places.

(b) Calculate the probability that Munster wins 8 games when they play at home, including their last match of the season at home? Give your result in percentage form accurate to 2 decimal places.

(c) What is the probability that Munster wins 8 games or more when they play home this season? Give your result in percentage form accurate to 2 decimal places.

(d) What is the probability that the team wins 7 games or less when they play at hone this season? Give your result in percentage form accurate to 2 decimal places.

Question 4 (25 Marks)

In a fair, you organise the following game. You throw two dice:

- If you get the same number on the two dice, you gain €6.
- You gain €3 if the two numbers are different but their total is even.
- You don't win anything if the total is odd.

(a) Calculate the probability of winning €6 and the probability of winning €3

(b) What is the expected gain for a player?

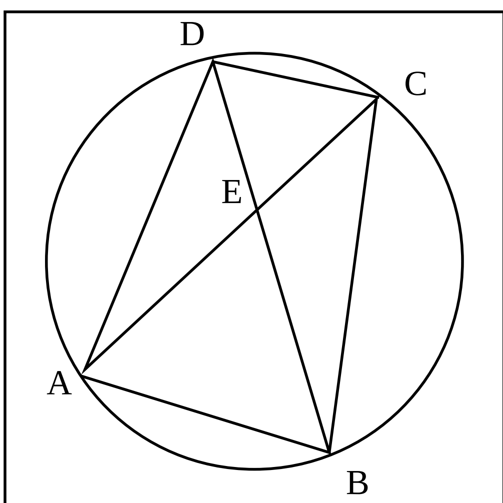
(c) What should be the minimum price to play a game? Justify your answer.

(d) In the first 25 played tried the game. On average, they won €3.5 with a standard deviation of 1.5. Construct a 95% confidence interval for the average gain.

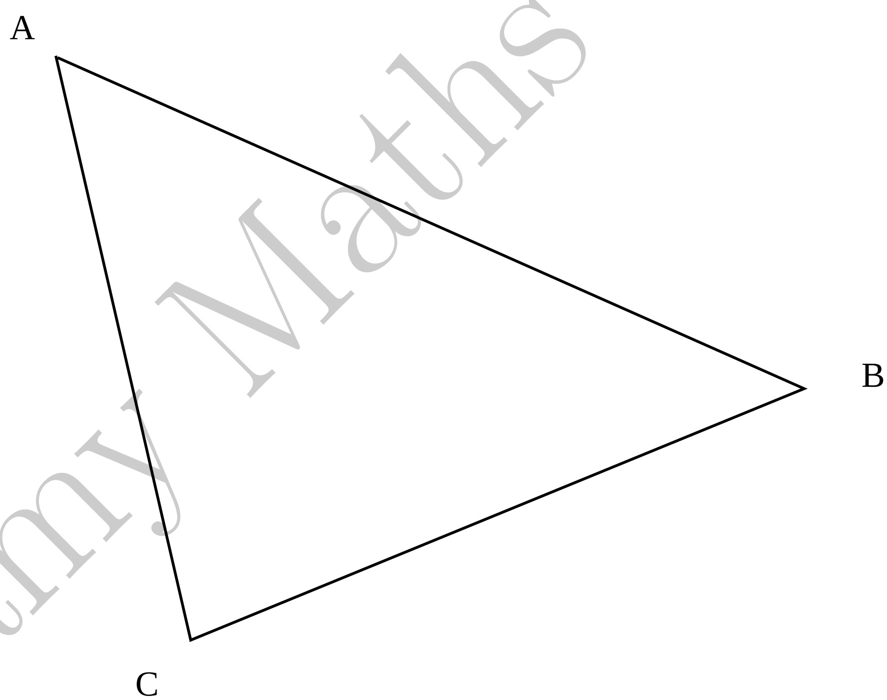
(e) Using the results of the previous part, do you think the dice are biased? Justify your answer.

Question 5 (25 Marks)

(a) In the diagram below, show that if E is the centre of the circle, ABCD is a rectangle.



(b) Construct the orthocentre of the triangle below. Please leave all your constructions apparent



Question 6 (25 Marks)

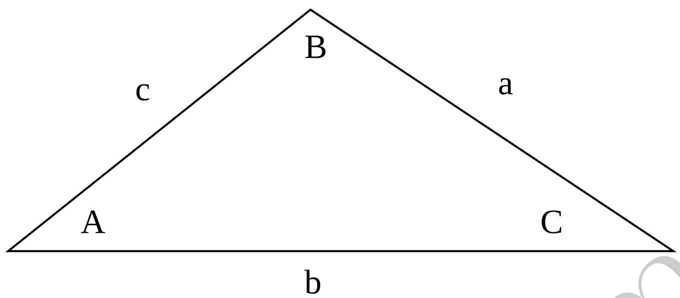
(a) Show that

$$\sin(3x) = 3\sin x - 4\sin^3 x$$

(b) In the triangle ABC, $|AB| = 5$, $|AC| = 9$, $|BC| = 12$. Draw an approximate picture and calculate $\cos < BAC$ in degrees accurate to 2 decimal places.

(c) Show that

$$\frac{a}{\sin < A} = \frac{b}{\sin < B} = \frac{c}{\sin < C}$$



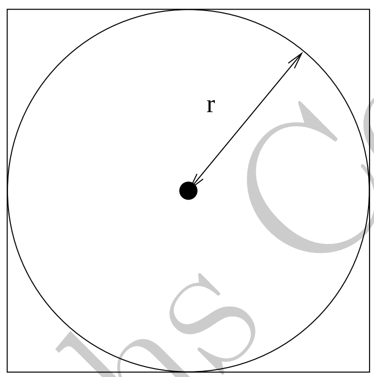
Section B	Contexts and Applications	150 Marks

Answer **all three** questions from this section.

Question 7 (50 Marks)

Aoife sells fruits on the market. She wants to store the fruits in a transparent plastic box which would be easy to store and would contain a single fruit. She can either use a cubic box or a pyramidal shaped box. In the following, the fruits are represented by spheres of radius r.

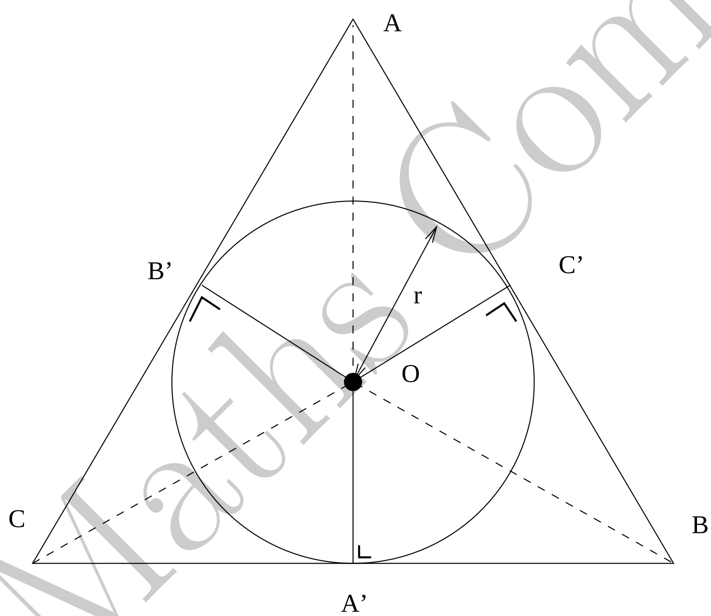
(a) She first considers the cubic display.



(i) Using the figure above, show that the volume of the cube can be expressed as $8r^3$ and surface of plastic necessary to manufacture all faces of the cube is $24r^2$ (neglect the thickness of the plastic).

(ii) Show that the proportion of the volume occupied by the fruit inside the box is $\frac{\pi}{6}$.

(b) She now considers the pyramidal storage with a square base of length $|BC|$. A cross section of the pyramid can be seen in the sketch below.

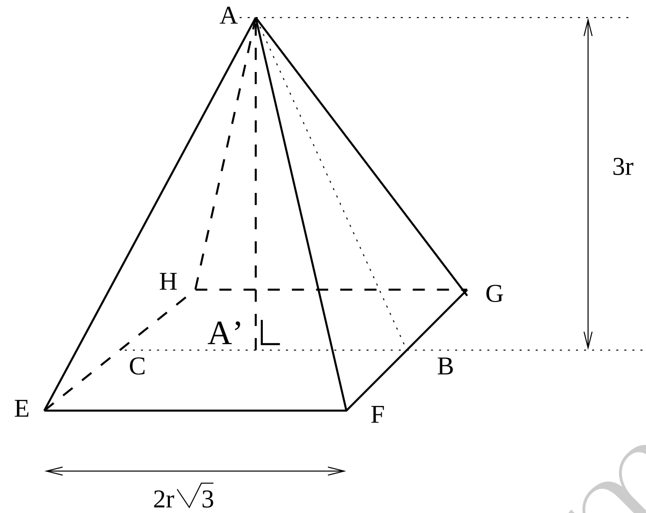


(i) Since $|A'B| = |A'C|$, $|AB'|$=$|B'C|$ and $|AB'| = |AC'|$, show that the 2 triangles $AB'O$ and $A'CO$ are similar and that the two triangles $A'CO$ and $BC'O$ are similar (Use space on the next page).

(ii) It can be shown that the 6 triangles $AB'O$, $A'BO$, $AC'O$, $A'CO$, $B'CO$ and $BC'O$ are congruent. Hence or otherwise, show that the 6 angles $| < AOB'|$, $| < A'OB|$, $| < AOC'|$, $| < A'OC|$, $| < BOC'|$ and $| < B'OC|$ are equal to 60°

(iii) Show that the vertical height of the pyramid $|A'A| = 3r$ and that the length $|BC| = 2r\sqrt{3}$.

(c) Aoife decides to pack her fruits in a pyramid of height $3r$ and a square base of side length $2r\sqrt{3}$.



(i) Calculate the volume of the pyramid in terms of length r and then deduce the proportion of space occupied by the fruit.

(ii) Calculate the distance $|AB|$ where B is the midpoint of [FG]

(iii) Hence or otherwise, calculate the surface of plastic necessary to produce the pyramid in terms of length r.

Question 8 (50 Marks)

A school in Limerick claims that the time students have to travel to come to school can be modelled using a normal distribution with mean $\mu = 25$ minutes and standard deviation $\sigma = 10$ minutes. The head of school wants to verify this claim. First she choses a student randomly.

(a) Using the information above,

(i) What is the probability that the student travelled less than 30 minutes

(ii) What is the probability that the student has travelled between 15 and 45 minutes

(b) The principal decides to interview 20 students and finds the following data

Student	1	2	3	4	5	6	7	8	9	10
Travel time (min)	28	17	17	43	46	45	26	27	28	38
Distance travelled (km)	2.2	1.3	1.3	3.1	3.5	3.3	2.1	2.3	2.1	2.6

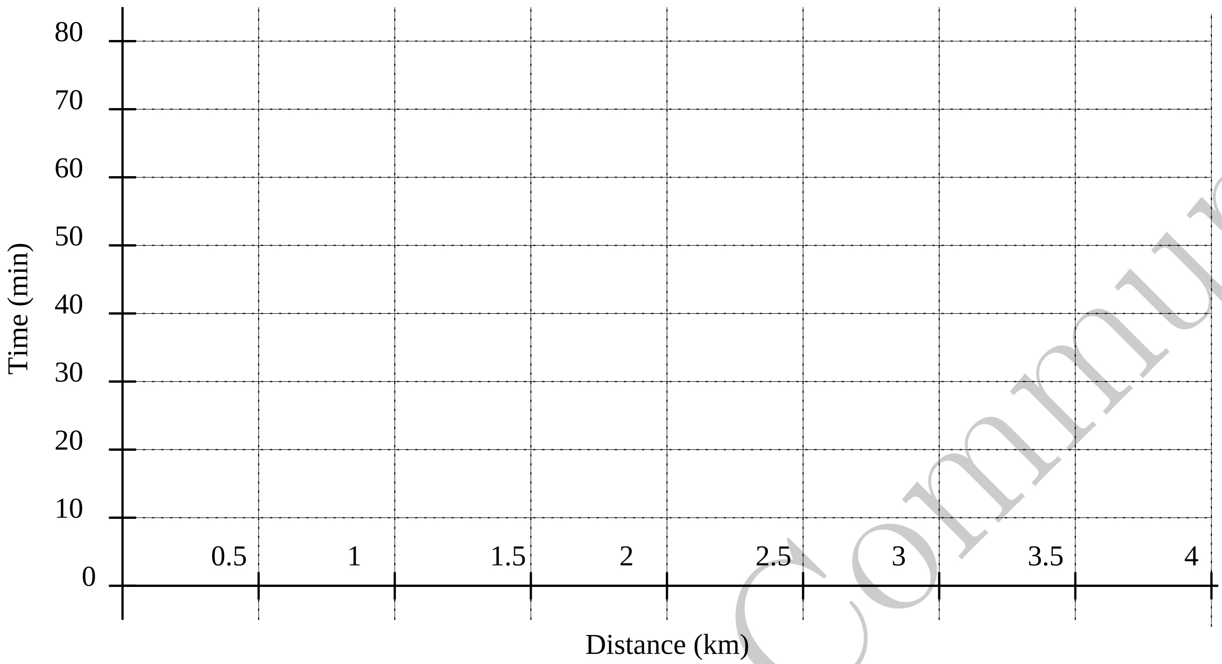
Student	11	12	13	14	15	16	17	18	19	20
Travel time (min)	23	49	18	30	45	33	27	28	22	13
Distance travelled (km)	1.9	3.6	1.7	2.1	3.1	2.2	2	2.2	1.9	1.3

(i) Calculate the mean travelling time corresponding to these data.

(ii) Using a standard deviation of 10 for the population, test the hypothesis, at the 5% level of significance, that it takes students 20 minutes to travel to school. Clearly highlight every step of the process.

(c) The principal now wants to relate the distance and time the students need to travel.

(i) Plot the results on a scatter graph and add an approximate line of best fit to your plot.



(ii) Calculate the correlation coefficient. What does this value tell you about the relationship between distance and travel time?

(iii) Find the equation of the line of best fit.

(iv) How far would you expect a student travel if she needed 30 minutes to go to school?

(v) Can you trust this result? Justify your answer.

Question 9 **(50 Marks)**

When applying appropriate scaling, the intensity of the current in a resistance can be described by the function

$$f(t) = \cos\left(\frac{\pi}{10}t\right)$$

where t is in millisecond and $f(t)$ is in mA.

(a) The instantaneous electrical power P (in mW) delivered to a component with appropriate scalings may be described as

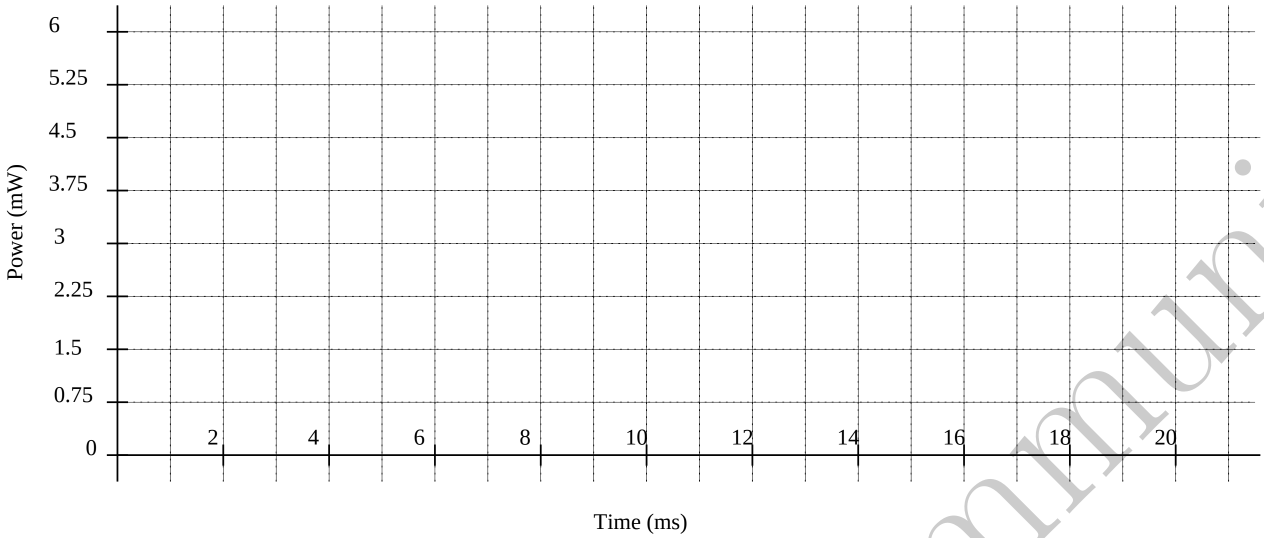
$$P(t) = 6f^2(t)$$

Show that the function P can be written as

$$P(t) = 3 + 3\cos\left(\frac{\pi}{5}t\right)$$

(b) Find the period and the range for both functions $f(t)$ and $P(t)$.

(c) Plot the function P(t) on the graph below.



(d) The component works best when the instantaneous power $P(t)$ is above 1.5 (mW). Identify on the graph for what values of the time t this happens.

(e) Retrieve these results by solving the equation.

$$P(t) = 1.5$$

(f) For what proportion of a period is the power above the 1.5 (mW) threshold?

(g) A new component is added to the electrical system and the power function becomes

$$P(t) = 6\cos\left(\frac{\pi}{10}t + \frac{\pi}{2}\right)\cos\left(\frac{\pi}{10}t\right)$$

Show that the the power function can be expressed as

$$P(t) = -3\sin\left(\frac{\pi}{5}t\right)$$

Rough Work

Rough Work

Rough Work

Rough Work