



9 Financial Maths

9.1 Compound Interest

1. Find the future value of an investment of €5000, earning interest at an AER of 2.5 % for 8 years.
2. €25000 is invested at an AER of 3.2 %. Find the future value after 15 years.
3. What is the monthly interest rate for an investment with an AER of 3.5 %?
4. What is the monthly interest rate for an investment with an AER of 4.2 %?
5. What is the AER of an investment earning 0.2% interest per month?
6. What is the quarterly interest rate for an investment with an AER of 5.3 %?
7. What is the AER of an investment earning interest at 1.1 % per quarter?
8. What is the future value of €10000 invested for 27 months with an AER of 3.5 %?
9. €3500 was invested at an AER of 4.3 %. What will the investment be worth after 5 years and 5 months?
10. €500 was invested at an AER of 3.8%. After how long will the investment be valued at €800?
11. How long will it take an investment to double if it is earning interest at 5.1 % per annum?
12. An investment of €3500 amounted to €4937.10 after 10 years. What was the AER?
13. An investment of €5000 amounted to €6895.66 after 8 years. What was the AER?
14. Find the present value of €2,000 to be paid in 3 years time, at an AER of 3.7%.
15. What amount of money needs to be invested at 2.8 %, in order to have €10000 in five years?
16. As part of a business agreement, Company A will pay Company B €250000 in 10 years. What is the present value of this payment, if the AER is 4.2 %?
17. What is the present value of €3600 to be paid in 34 months time at rate of 5%?
18. As part of a corporate bond, Company X must pay its creditors €3000 in 3 years and 3 months, and a further €5000 in 5 years and 5 months. What is the total present value of these payments, assuming an AER of 4.4%?





19. Kevin will turn 21 in 2 years and 7 months. Upon his 21st birthday he is entitled to an inheritance of €21000. What is the present value of this inheritance, assuming an AER of 3.8%?
20. Mary borrows €20000 to buy a car. If she makes no repayments, how much will she owe the bank in 5 years time if the AER is 4.6%?
21. Jack borrows €15000 from the bank for 5 years. The AER for the first 3 years is 4.5%. After 3 years he pays back €10000, so the AER drops to 3.5%. How much will Jack need to fully repay the loan after the fifth year? How much interest did he pay?

9.2 Depreciation

1. A car purchased for €30000 depreciates at 15% per annum. What will the car be worth five years after it is bought?
2. A lorry which cost €75000 new, depreciates at 20% per annum for the first two years, then at 10% per annum after two years. What will the lorry be worth after 5 years?
3. A machine bought for €500000 new was worth €220000 after 7 years. What was the annual depreciation rate?
4. A lorry bought for €125000 depreciates at a rate of 17% per annum. If the lorry is to be sold before the price drops below €75000, how long before it is sold?
5. A company buys a factory machine for €250000. The value of this machine will depreciate at 22% per annum. At the same time the company also invests €50000 into a fund with a return of 5% per annum.
 - i. What will the factory machine be worth after 6 years?
 - ii. What will be the value of the investment after 6 years?
 - iii. If inflation is 1.8% per annum. How much will a new version of the same factory machine cost in 6 years?
 - iv. After 6 years, the company will sell the original machine, and upgrade by buying a new one. They will cash in their investment and use this money plus the money they receive from selling the old machine, to buy the new machine. How much extra funding will they need to buy the new machine?

9.3 Financial Series

9.3.1 Saving Schemes

1. Anna will deposit €1000 at the beginning of every year for 10 years. Calculate the value of her savings at the end of 10 years, if the AER is 3%.
2. Calculate the future value of 48 monthly installments of €100, at an AER of 5.1%. What is the total interest earned on this investment?
3. €50 is deposited weekly into an account paying 4.3% interest per annum. How much will the savings be worth after a year?





4. When Michael turned 18 he decided to open a bank account and deposit €50 at the beginning of every month. If the AER is 4.2%, how much will Michael have in his account on his 21st birthday?
5. When Jean was born, her parents set up a college fund for her. They plan to put a set amount into the bank at the beginning of every month. They want to have \$250000 in Jeans college fund on her 18th birthday. How much should they deposit every month if the fixed rate on the fund is 6%?
6. A couple are saving for a deposit for a house. They want to deposit the same amount at the beginning of every month. They would like to have €20000 saved after 3 years. If the bank are offering a savers rate of 3.8%, how much do the couple need to deposit every month to reach their target?
7. A company sets up a sinking fund for their employees. How much has to be paid into the fund every week, if the fund is to be worth €50000 in 5 years time? Assume an AER of 5%.

9.3.2 Bonds and Assets

8. A ten year bond offers a 30% total return on the investment. What is the AER of this bond?
9. A twenty year bond offers a 100% total return on the investment. What is the AER of this bond?
10. Company Y has issued ten year corporate bonds to raise money. They will have to pay their creditors €1000 at the end of every year for the next ten years. What is the fair price for this bond, if the expected market interest rate is 6% per annum?
11. Company Z has issued ten year corporate bonds to raise money. They will have to pay their creditors €500 at the end of every year for ten years, plus pay an additional lump sum of €10000 at the end of the ten years. What is the fair price for this bond, if expected market interest rates are at 7%?
12. A technology company issues a five year corporate bond to raise money. Because technology companies are seen as high risk investments, they will have to pay a market interest rate of 12.5% per annum. As part of the structure of the bond, they will pay their creditors €5000 per quarter for 5 years, plus a lump sum of €50000 at the end of the five years. What is the fair market price for this bond?

9.3.3 Lottery Payments

13. In the USA lottery winners have a choice of taking a lump sum now or receiving regular payments for a certain length of time. Matt has just won the lottery and has the choice of receiving \$1m now, or \$6000 at the beginning of each month for the next 20 years. Assuming an AER of 5%, which option should Matt choose?





14. Emma just won the New York powerball lottery. She can choose between a lump sum of \$15m now, or receive payments of \$37000 at the beginning of every week for the next 10 years. Which option should she choose if the interest rate is 4% per annum?
15. After winning the lottery in the USA, Amy is offered a choice between a lump sum of \$10m now or receive a set payment at the beginning of every month for 20 years. If the expected interest rate is 5%, what should the monthly payments be, so the sum of the present values of these payments is the same value as the lump sum?

9.3.4 Loans

16. Calculate the monthly loan repayments on a 35 year mortgage of €300000, at 3.5% interest.
17. Calculate the monthly loan repayments on a 20 year business loan of €155000, at an interest rate of 7.5%.
18. Jessica wants to take out a 35 year mortgage. She can afford to make repayments of €800 per month. If the fixed interest rate is 4%, what is the largest mortgage she can take out?
19. A small business wants to take out a 10 year commercial loan to buy a new premises. They can afford to make repayments of €2500 a month. If the interest rate on small business loans is 8%, what is the largest loan that the business can afford?

9.3.5 Pension Schemes

20. Mark is thinking of setting up a retirement fund for his pension. The pension scheme is designed in such a way that he will receive annual payments of €30000, for twenty five years, with the first payment on the day he retires. The fixed interest rate for the entire pension scheme is 5%.
 - (a) How much money needs to be in the retirement account on the day that Mark retires, in order to fund the twenty five annual payments of €30000?
 - (b) Mark starts saving for his pension 20 years before he will retire. How much must he deposit annually, at the beginning of each year, to ensure he has adequate funds in his retirement account on the day that he retires?
21. Marie is setting up her retirement fund. She will retire in twenty five years. Her pension will provide monthly payments of €2500, at the beginning of every month, beginning on the day she retires. Her pension payments will last for twenty years. The expected rate of interest is 6%.
 - (a) How much money needs to be in her retirement account on the day that Marie retires, in order to fund her pension payments?
 - (b) Marie will make equal deposits at the beginning of every week in order to save for her retirement. How much should she deposit every week, to ensure there is adequate funds in her retirement account on day that she retires?





22. How much must John save at the beginning of every month, for twenty years up until retirement, in order to fund a pension that will pay him €3000 at the beginning of every month, for fifteen years, from the day he retires? Let the AER be a fixed 5.5%.

9.4 Exam Questions

1. **2018 Paper 2!** Acme Confectionery has an employee pension plan. For an employee who qualifies for the full pension, Acme Confectionery will pay a sum of €20000 on the day of retirement. It will then pay a sum on the same date each subsequent year for the next 25 years. Each year the employee is paid a sum that is 1% more than the amount paid in the previous year. What sum of money must the company have set aside on the day of retirement in order to fund this pension? Assume an annual interest rate (AER) of 2.4%

2. **2017 Paper 1**

- (a) When a loan of eP is repaid in equal repayments of amount € A , at the end of each of t equal periods of time, where i is the periodic compound interest rate (expressed as a decimal), the formula below can be used to find the amount of each repayment.

$$A = P \frac{i(1+i)^t}{((1+i)^t - 1)}$$

Show how this formula is derived. You may use the formula for the sum of a finite geometric series.

- (b) Alex has a credit card debt of €5000. One method of clearing this debt is to make a fixed repayment at the end of each month. The amount of this repayment is 2.5% of the original debt.
- What is the fixed monthly repayment, € A , required to pay the debt of €5000?
 - The annual percentage rate (APR) charged on debt by the credit card company is 21.75%, fixed for the term of the debt. Find as a percentage, correct to 3 significant figures, the monthly interest rate that is equivalent to an APR of 21.75%.
 - Assume Alex pays the fixed monthly repayment, € A each month and does not have any further transactions on that card. Complete the table below to show how the balance of the debt of €5000 is reducing each month for the first three months, assuming an APR of 21.75%, charged and compounded monthly.
 - Using the formula you derived on the previous page, or otherwise, find how long it would take to pay off a credit card debt of €5000, using the repayment method outlined at the beginning of part (b) above.
Give your answer in months, correct to the nearest month
 - Alex decides to borrow €5000 from the local Credit Union to pay off this credit card debt of €5000. The APR charge for the Credit Union loan is 8.5% fixed for the term of the loan. The loan is to be repaid in equal weekly repayments, at the end of each week, for 156 weeks. Find the amount of each weekly repayment.





Payment number	Fixed monthly payment, €A	€A		New balance of debt (€)
		Interest	Previous balance reduced by (€)	
0				5000
1			42.50	4957.50
2				
3				

- vi. How much will Alex save by paying off the credit card debt using the loan from the Credit Union instead of paying the fixed repayment from part (b)(i) each month to the credit card company?

3. 2015 Paper 1

- (a) Donagh is arranging a loan and is examining two different repayment options.
- Bank A will charge him a monthly interest rate of 0.35%. Find, correct to three significant figures, the annual percentage rate (APR) that is equivalent to a monthly interest rate of 0.35%.
 - Bank B will charge him a rate that is equivalent to an APR of 4.5%. Find, correct to three significant figures, the monthly interest rate that is equivalent to an APR of 4.5%.
- (b) Donagh borrowed 80000 at a monthly interest rate of 0.35%, fixed for the term of the loan, from Bank A. The loan is to be repaid in equal monthly repayments over ten years. The first repayment is due one month after is issued. Calculate, correct to the nearest euro, the amount of each monthly repayment.

4. 2013 paper 1

- (a) Niamh has saved to buy a car. She saved an equal amount at the beginning of each month in an account that earned an annual equivalent rate (AER) of 4%.
- Show that the rate of interest, compounded monthly, which is equivalent to an AER of 4% is 0.327%, correct to 3 decimal places.
 - Niamh has €15000 in the account at the end of 36 months. How much has she saved each month, correct to the nearest euro?
- (b) Conall borrowed to buy a car. He borrowed €15000 at a monthly interest rate of 0.866%. He made 36 equal monthly payments to repay the entire loan. How much, to the nearest euro, was each of his monthly payment?

5. **2014 Sample Paper 1** Pdraig is 25 years old and is planning for his pension. He intends to retire in forty years' time, when he is 65. First, he calculates how much he wants to have in his pension fund when he retires. Then, he calculates how much he needs to invest in order to achieve this. He assumes that, in the long run, money can be invested at an inflation-adjusted annual rate of 3%. Your answers throughout this question should therefore be based on a 3% annual growth rate.





- (a) Write down the present value of a future payment of €20000 in one years' time.
- (b) Write down, in terms of t , the present value of a future payment of €20000 in t years' time.
- (c) Padraig wants to have a fund that could, from the date of his retirement, give him a payment of €20000 at the start of each year for 25 years. Show how to use the sum of a geometric series to calculate the value, on the date of retirement, of the fund required.
- (d) Padraig plans to invest a fixed amount of every month in order to generate the fund calculate in part (c). His retirement is $40 \times 12 = 480$ months away.
- Find, correct to four significant figures, the rate of interest per month that would, if paid and compounded monthly, be equivalent to an effective annual rate of 3%.
 - Write down, in terms of n and P , the value on the retirement date of a payment of € P made n months before the retirement date.
 - If Padraig makes 480 equal monthly payments of € P from now until his retirement, what value of P will give the fund he requires?
- (e) If Padraig waits for ten years before starting his pension investments, how much will he then have to pay each month in order to generate the same pension fund?
6. **2011 Paper 1** Most lottery games in the USA allow winners of the jackpot prize to chose between two forms of the prize: an *annual-payments* option or a *cash-value* option. In the case of the *New York Lotto*, there are 26 annual payments in the *annual payments* option, with the first payment immediately, and the last payment in 25 years' time. The payments increase by 4% each year. The amount advertised as the jackpot prize is the total amount of these 26 payments. The *cash value* option pays a smaller amount than this.
- If the amount of the first annual payment is A , write down, in terms of A , the amount of the second, third, fourth and 26th payments.
 - The 26 payments form a geometric series. Use this fact to express the advertised jackpot prize in terms of A .
 - Find, correct to the nearest dollar, the value of A that corresponds to an advertised jackpot prize of \$21.5 million.
 - A winner who chooses the *cash-value* option receives, immediately, the total of the present values of the 26 payments. The interest rate used for the present-value calculations is 4.78%. We want to find the cash value of the prize referred to in part (c).
 - Complete the table below to show the actual amount and the present value of each of the first three annual payments.

Payment Number	time to payment (years)	actual amount	present value
1	0		
2	1		
3	2		
 - Write down, in terms of n , an expression for the present value of the n th annual payment.





- iii. Find the amount of prize money payable under the *cash-value* option. That is, find the total of the present values of the 26 annual payments. Give your answer in millions, correct to one decimal place.
- (e) The jackpot described in parts (c) and (d) above was won by an Irish woman earlier this year. She chose the *cash-value* option. After tax, she received \$7.9 million. What percentage of tax was charged on her winnings?

Leamy Maths Community

