

Junior Certificate Examination, 2019

Sample paper prepared by Leamy Maths Community

Mathematics

Paper 1

Higher Level

18 April 2019

Paper written by J.P.F. Charpin and S. King



Solutions

300 marks

Sample Instructions

There are 12 questions on this examination paper. Answer all questions:

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times, you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of Formulae and Tables. You must return it at the end of the examination.

You will lose marks if all necessary work is not clearly shown.

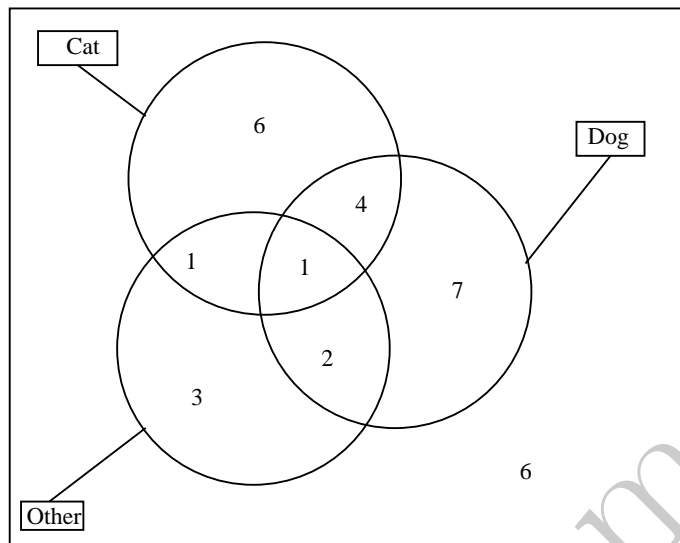
Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1

(Suggested maximum time 10 minutes)



There are 30 students in the class.

- 14 have a dog,
- 12 have a cat,
- 6 do not have a pet.

(a) Find the missing values in the Venn diagram. Justify your answers.

Marks: 0, 3, 5, 7

(b) How many students have a cat or a dog?

$N=21$

Marks: 0, 3

(c) What proportion of students does not have a pet? Give your answer as a percentage correct to one decimal place.

$$\frac{6}{30} = 20\%$$

Marks: 0, 3, 5

(d) What proportion of students have a cat or an other type of pet? Give your answer as a percentage correct to one decimal place.

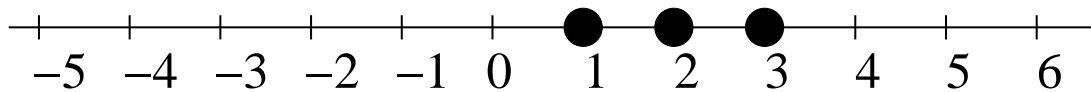
$$\frac{17}{30} = 56.7\%$$

Marks: 0, 3, 5

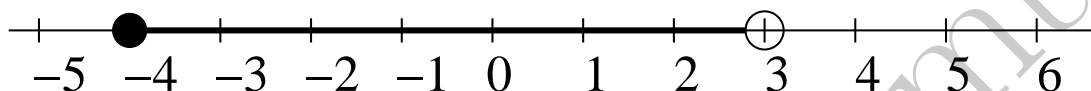
Question 2 (Suggested maximum time 5 minutes)

Solve the inequalities and draw your solution on the graph.

- (a) $x < 4$, $x \in \mathbb{N}$ Marks: 0, 3, 5



- (b) $-4 \leq x < 3$, $x \in \mathbb{R}$. Marks: 0, 3, 5



Question 3 (Suggested maximum time 5 minutes)

- (a) Write the following expression in the form 7^p where $p \in \mathbb{Q}$.

$$7^8 \times 7^4 \times 7$$

$$7^{13}$$

Marks: 0, 3, 5

- (b) Write the following expression in the form 2^q where $q \in \mathbb{Q}$.

$$\frac{8}{4^3}$$

$$2^{-3}$$

Marks: 0, 3, 5

- (c) Write the following expression in the form a^r where $r \in \mathbb{Q}$.

$$(\sqrt{a})^3 \times a$$

$$a^{5/2}$$

Marks: 0, 3, 5

Question 4 (Suggested maximum time 5 minutes)

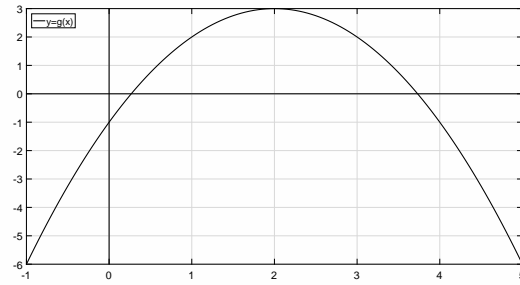
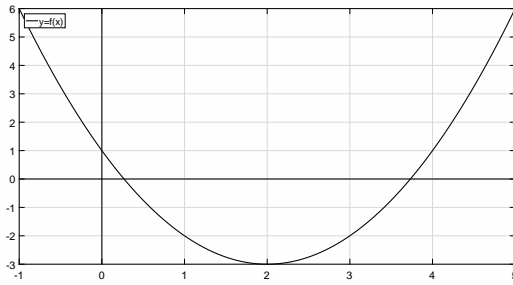
- (a) Solve the equation

$$x^2 - 4x + 1 = 0$$

Give your answers in surd form and then correct to two decimal places.

$$x_1 = 2 + \sqrt{3} = 3.73 \quad x_2 = 2 - \sqrt{3} = 0.27$$

Marks: 0, 3, 5, 7



- (b) Identify the function $h(x) = x^2 - 4x + 1$ in the graph below. Justify your answer
 Solution f(x) because U shaped function

$$y=f(x)$$

$$y=g(x)$$

Marks: 0, 3

Question 5 (Suggested maximum time 10 minutes)

Factorise the following expressions

- (a) $9x^2 - 25$
 $9x^2 - 25 = (3x - 5)(3x + 5)$

Marks: 0, 3, 5

- (b) $x^2 - 4x + 4$

$$x^2 - 4x + 4 = (x - 2)^2$$

Marks: 0, 3, 5

- (c) $4ac - 3ad + 8bc - 6bd$
 $4ac - 3ad + 8bc - 6bd = (a + 2b)(4c - 3d)$

Marks: 0, 3, 5, 8, 10

Question 6 (Suggested maximum time 15 minutes)

- (a) Use factors to simplify

$$\frac{3x^2 - 2x - 8}{x^2 - 4}$$

$$\frac{3x^2 - 2x - 8}{x^2 - 4} = \frac{(3x + 4)(x - 2)}{(x - 2)(x + 2)} = \frac{3x + 4}{x + 2}$$

Marks: 0, 3, 5, 8, 10

(b) Fill the following table. Use the space below to justify your answers.

x	-4	-3	3	6
$\frac{6x + 8}{x + 2}$	8	10	5.2	5.5
$\frac{3x^2 - 2x - 8}{x^2 - 4}$	4	5	2.6	2.75

Marks: 0, 3, 5

(c) What do you observe when you compare the results of the second and third line of the table above?

There is a factor 2 between the two lines

Marks: 0, 3, 5

(d) Explain it using the result of part a.

$$\frac{6x + 8}{x + 2} = 2 \frac{3x + 4}{x + 2} = 2 \frac{3x^2 - 2x - 8}{x^2 - 4} = \frac{(3x + 4)(x - 2)}{(x - 2)(x + 2)}$$

Marks: 0, 3, 5, 8, 10

Question 7 (Suggested maximum time 10 minutes)

Solve the equations

(a) $\frac{2x - 4}{3} + \frac{2x - 1}{5} = 7$

$$10x - 20 + 6x - 3 = 105 \iff 16x = 128 \iff x = 8$$

Marks: 0, 3, 5, 8, 10

(b) $\frac{12}{2x + 1} = 4$

$$2x + 1 = 3 \iff x = 1$$

Marks: 0, 3, 5, 8, 10

Question 8

(Suggested maximum time 15 minutes)

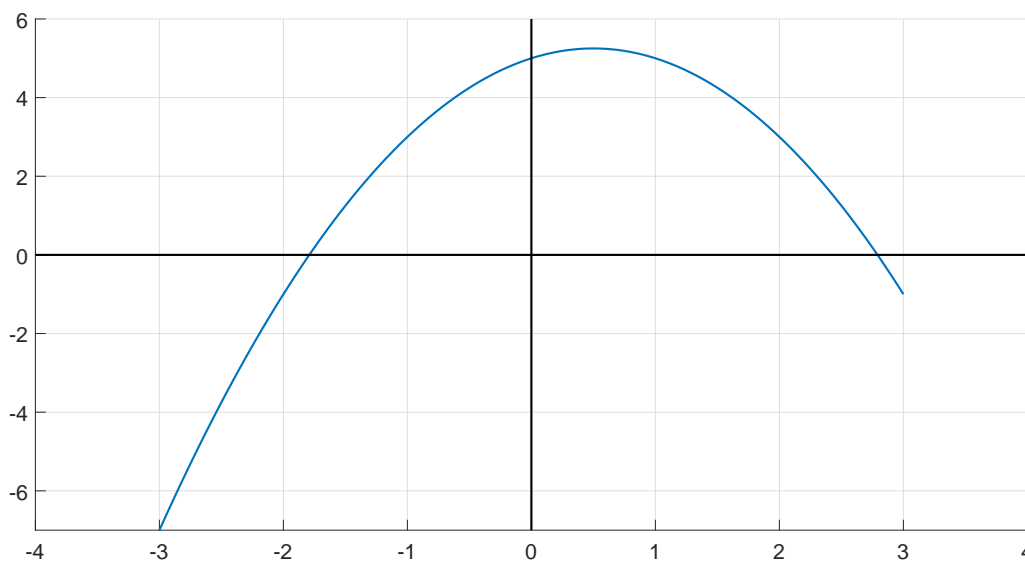
Consider the function

$$f(x) = -x^2 + x + 5$$

- (a) Fill the table below and then plot the function. Show details of your calculations in the space provided.

x	-3	-2	-1	0	1	2	3
f(x)	-7	-1	3	5	5	3	-1

Marks: 0, 3, 5, 8, 10



- (b) Using the graph, identify where the function is equal to 2.5.
 $x \approx 2.1$ and $x \approx -1.2$ Marks: 0, 3, 5
- (c) Retrieve these results using the function definition. Give your answer correct to 3 decimal places.

$$-x^2 + x + 5 = 2.5 \iff x = \frac{1 \pm \sqrt{10}}{2}$$

$$x = 2.16 \text{ and } x = -1.16$$

Marks: 0, 3, 5, 8, 10

- (d) Use the graph and the results above to identify for what values of x the function verifies $f(x) \leq 2.5$.
 $x \leq -1.16$ and $x \geq 2.16$
Marks: 0, 3, 5, 8, 10

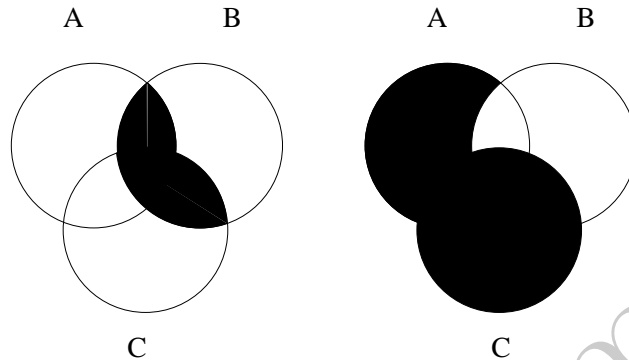
Question 9 (Suggested maximum time 5 minutes)

Shade the region indicated for both diagrams

$$(A \cup C) \cap B$$

$$(A \cap B') \cup C$$

Marks: 0, 3, 5, 8, 10



Question 10 (Suggested maximum time 20 minutes)

Ciara has decided to do the Great Limerick Run on the 5th May 2019. The race is 42 km long. She wants to finish the race in 5 hours. She plans to

- Run the first 16km in two hours at a constant speed,
- Run the next 11 km at a constant speed during the third hour,
- Run 7km at a constant speed in the next 30 minutes
- Run the remaining distance at a constant speed during the last 90 minutes

(a) Complete the graph on the next page to show the distance Ciara has run as a function of time. Justify your answer.

Marks: 0, 3, 5, 8, 10

(b) Read on the graph the distance Ciara will cover in 4 hours. Justify your answer.

$$N \approx 37\text{km}$$

Marks: 0, 3, 5

(c) Read on the graph the time it will take Ciara to complete half the race (21km). Justify your answer.

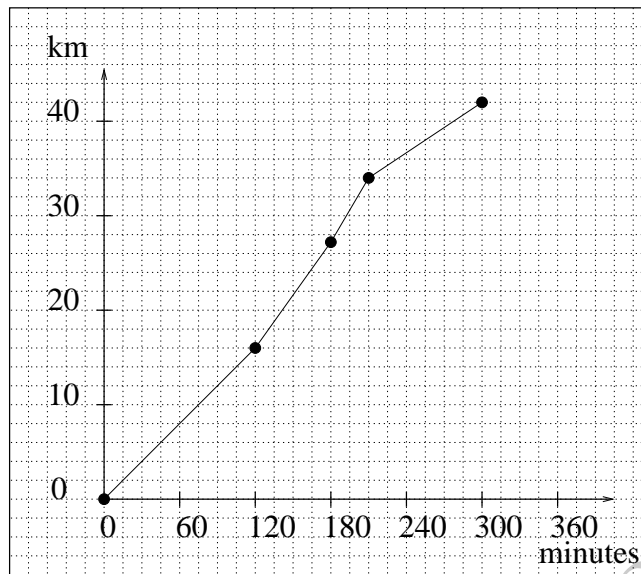
$$\approx 150\text{minutes} = 2 \text{ hours } 30 \text{ minutes}$$

Marks: 0, 3, 5

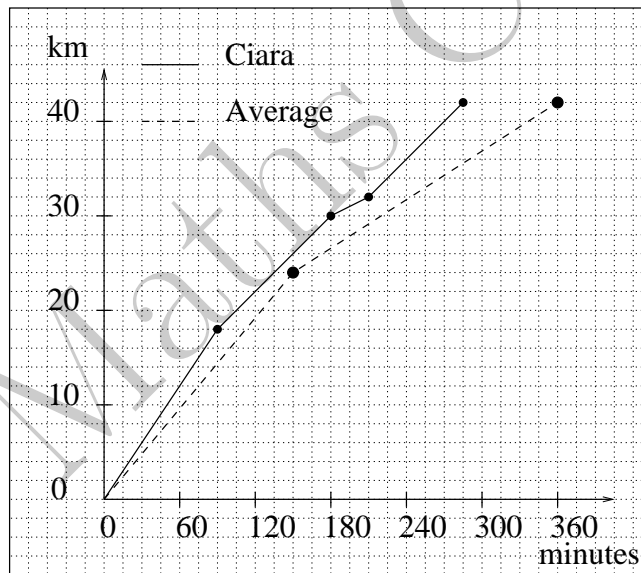
(d) Calculate Ciara's average speed in km/h during the first three hours of the race. Justify your answer.

$$v = \frac{27}{3} = 9 \text{ km/h}$$

Marks: 0, 3, 5



(e) The figure below shows Ciara's actual progression on the race day as well as the average progression of runners.



Answer the following questions and justify your answers either on the graph or on the space provided.

How long did Ciara need to complete the race?	285 minutes
What was Ciara's average speed between 0 and 90 minutes	$\frac{18}{1.5} = 12 \text{ km/h}$
When did Ciara run the slowest during the race?	$180 \leq t \leq 210$
How long did it take the average runner to complete the race?	6 hours
When Ciara finished the race, how long did the average runner still have to run?	7 km

Marks: 0, 3, 5, 8, 10, 12, 15

Question 11 (Suggested maximum time 15 minutes)

A mobile phone company charges its clients according to the number of minutes they spend on calls every month:

Hour/month	1	2	3	4	5	6
Price (€)	15	18	21	24	27	30

(a) Complete the table given the price forms a linear pattern.

Marks: 0, 3, 5, 8, 10

(b) Find a formula for the price P paid by the clients as a function of the number n of minutes they use every month (the prices form a linear pattern). $P = 12 + 3n$

Marks: 0, 3, 5, 8, 10

(c) For another plan, the company charges the clients

$$P = 15 + 2n$$

(i) Stephen uses 3 hours every month. How much will he be charged?

€21

Marks: 0, 3, 5

(ii) Kate pays €29 every month. How many hours does she use every month?

$$\frac{29 - 12}{2} = 7 \text{ hours}$$

Marks: 0, 3, 5

(d) VAT must be added to the price charged by the mobile phone company. Including VAT, Kate paid €35.67 (she paid €29 for her phone usage).

(i) How much VAT did she pay?

$$35.67 - 29 = 6.67$$

Marks: 0, 3, 5

(ii) Find the VAT rate applied. Express your result as a percentage.

$$\frac{6.67}{29} = 0.23 = 23\%$$

Marks: 0, 3, 5

Question 12 (Suggested maximum time 25 minutes)

A Pythagorean triple is a set of three numbers (a, b, c) which verify

- a, b and $c \in \mathbb{N}$
- $a < b < c$
- $a^2 + b^2 = c^2$

(a) Check if the following sets are Pythagorean triples

Triple	a^2	b^2	$a^2 + b^2$	c^2	Pythagorean triple (Yes/No)
(3, 4, 5)	9	16	25	25	Yes
(5, 12, 13)	25	144	169	169	Yes
(7, 8, 9)	49	64	113	81	No
(7, 24, 25)	49	576	625	625	Yes

Marks: 0, 3, 5, 8, 10

- (b) If $(10, 24, x)$ is a Pythagorean triple, what is the value of x ?
 $x^2 = 10^2 + 24^2 = 676 = 26^2$ so $x = \pm 26$ the only possible value is $x = 26$

Marks: 0, 3, 5

- (c) If $(9, x, 15)$ is a Pythagorean triple, what is the value of x ?
 $9^2 + x^2 = 15^2$ so $x = \pm 12$ the only possible value is $x = 12$

Marks: 0, 3, 5

- (d) Show that if $(x, 2x - 1, 3x - 7)$ is a Pythagorean triple, then x is a solution of the equation:

$$2x^2 - 19x + 24 = 0$$

$$\begin{aligned}x^2 + (2x - 1)^2 &= (3x - 7)^2 &\iff x^2 + 4x^2 - 4x + 1 &= 9x^2 - 42x + 49 \\&&\iff 4x^2 - 38x + 48 &= 0 \\&&\iff 2x^2 - 19x + 24 &= 0\end{aligned}$$

Marks: 0, 3, 5, 8, 10

- (e) Solve the equation above and find the two possible values of x .

$$x = 8 \text{ or } x = 1.5$$

Marks: 0, 3, 5

- (f) Which of the two values forms a Pythagorean triple? Justify your answer and write the triple.
 $x = 8$ is the only integer. $(8, 15, 17)$

Marks: 0, 3, 5

- (g) Darragh says that you can find triples containing two prime numbers. Is he right or wrong? Justify your answer using the triples you found in the previous questions.

True, for example $(3, 4, 5)$

Marks: 0, 3, 5

- (h) Caoimhe says that every triple contains at least one prime number? Is she right or wrong? Justify your answer using the triples you found in the previous questions.

False, for example $(6, 8, 10)$

Marks: 0, 3, 5