Leaving Certificate Examination, 2019

Sample paper prepared by Leamy Maths Community

Mathematics

Paper 1

Higher Level

18 April 2019

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Name ____

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8	Q9	Total

300 marks

Sample Instructions

There are two sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer questions as follows:

In Section A, answer all six questions.
In Section B, answer all three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination.

You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

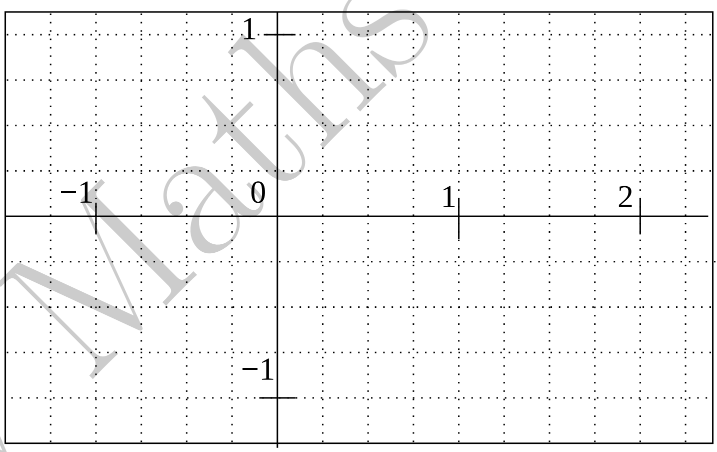
Section A	Concepts and Skills	150 Marks

Answer **all six** questions from this section.

Question 1 (25 Marks)

(a) The equation $z^3 - z^2 - z - 2 = 0$ has one integer solution z_1 and two complex solutions $z_2 = a + ib$ and $z_3 = a - ib$ where a, b $\in \mathbb{R}$ and $b \geq 0$. Calculate the three solutions z_1, z_2 and z_3 of the equation.

(b) Express the three solutions in polar form and plot them on the Argand diagram below.



(c) What unique geometrical transformation maps both z_2 to z_3 and z_3 to z_2?

Question 2 (25 Marks)

(a) Solve the system

$$\begin{cases} a^2 + b^2 = 13 \\ ab = 6 \end{cases}$$

(b) Solve the inequality

$$\frac{2x-3}{x+1} > 1$$

(c) Solve the inequality

$$\left(\frac{1}{2}\right)^x > 2$$

Question 3 (25 Marks)

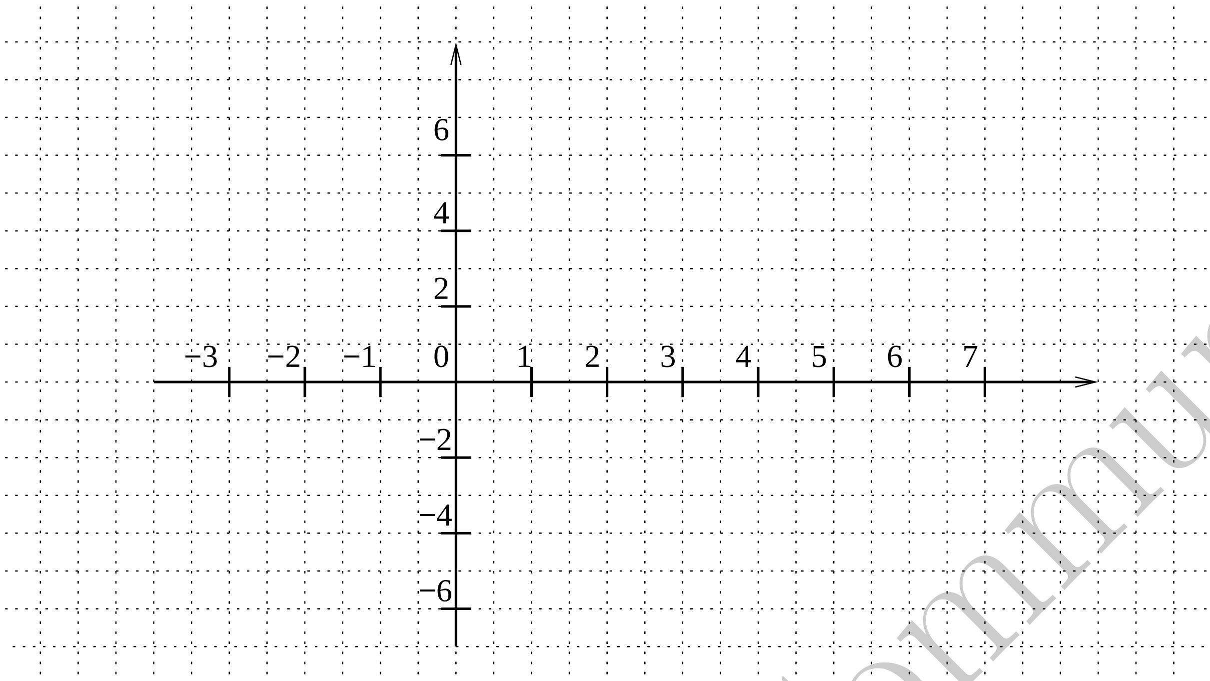
(a) Differentiate the function $f(x) = x^2 - 3x + 8$ from first principles.

(b) Calculate the equation of the tangent for function $g(x)$ at point $x = -1$. Use space on the next page if necessary.

$$g(x) = xe^{x^2+x}$$

(c) Identify the asymptotes of function and specify if they are vertical or horizontal asymptotes. Plot the asymptotes and roughly sketch function $h(x)$ using the grid on the next page.

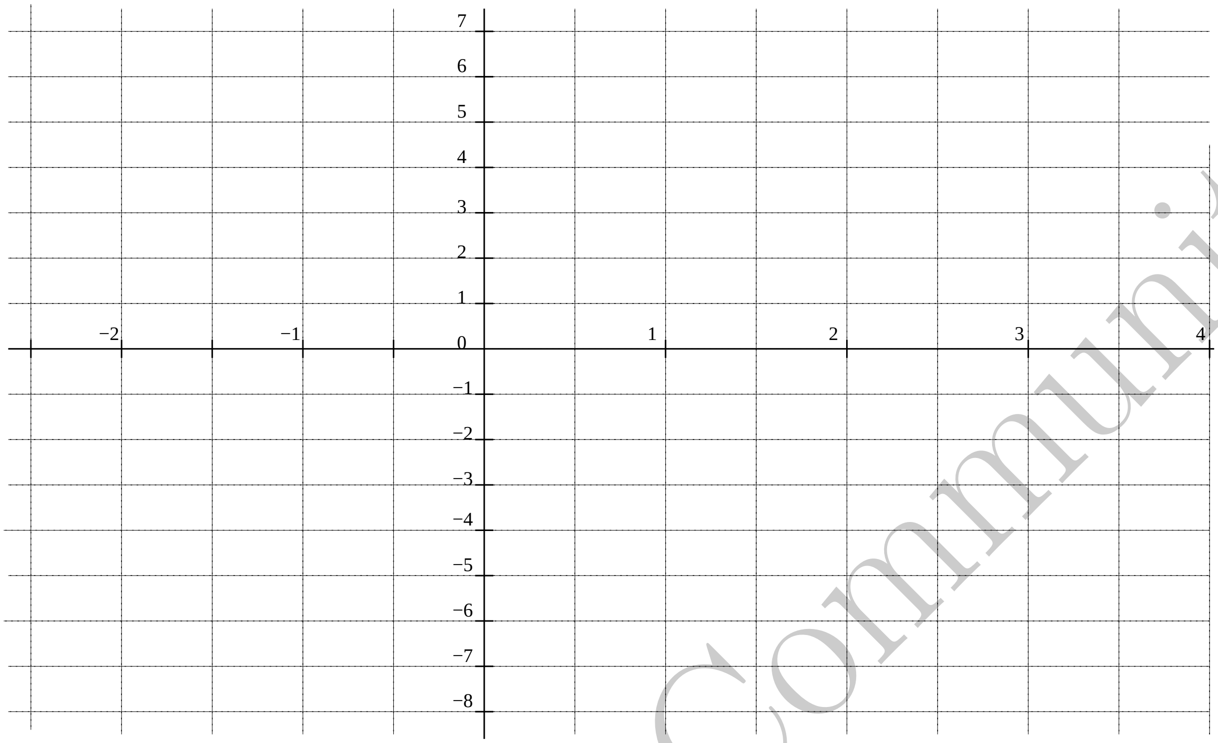
$$h(x) = \frac{2x+1}{x-2}$$



Question 4 **(25 Marks)**

(a) Plot the two functions $f(x)$ and $g(x)$. Use the grid on the next page and the space below for your rough work.

$$f(x) = 2x^2 - 2x - 6 \qquad g(x) = x^2 - x$$



(b) Identify where the two function cross on the graph and verify this analytically.

(c) Calculate the area between the two functions. Give your result in the form $\frac{a}{b}$ where $a, b \in \mathbb{N}$.

Question 5 (25 Marks)

The first three terms of a series are: 1, $x + 2$, $x^2 + 3$

(a) Arithmetic series

(i) Find the values of x if the series is arithmetic.

(ii) Hence, for both values of x from part **(i)**, write the general term of the series in the form $T_n = p + nq$ where $p,\ q \in \mathbb{Z}$.

(b) Geometric series

(i) Find the values of x if the series is geometric.

(ii) Calculate the value of the 4^{th} term of this series. Give your result in the form $\frac{a}{b}$ where a, b $\in \mathbb{N}$

Question 6 **(25 Marks)**

In an experiment, meteorologists are observing the formation of rain drops. The drops are spherical and their radius increases at the rate of 2 μm/s where μm is a small fraction of a metre.

(a) Find the rate of increase of the droplet volume when the radius is 5 μm. Give your result in the form $a\pi$ where $a \in \mathbb{N}$.

(b) Calculate the rate of increase for the surface of the sphere when the rate of increase for the volume of the sphere is $288\pi\ (\mu m)^2$.

(c) The droplet stops being spherical when the rate of increase of the surface area grows at a pace larger than the critical value $128\pi\ (\mu m)^2/\text{s}$. Calculate the radius of the droplet when the surface increases at this critical rate.

(d) Is it possible to obtain spherical droplets with a radius r=$10\mu m$? Justify your answer.

Section B	Contexts and Applications	150 Marks

Answer **all three** questions from this section.

Question 7 (50 Marks)

An engineering company has to build a new concrete dam. Concrete is a mixture of water, cement and sand or gravel. When these ingredients are mixed, a chemical reaction occurs which creates a lot of heat. To prevent this heat from damaging the structure of the dam, a network of pipes is installed inside the dam to cool it down.

The average temperature in the dam is calculated using Newton's law of cooling:

$$T = T_a + (T_0 - T_a)e^{-kt}$$

where t is the **time in months**, T_a is the ambient or surrounding temperature, T_0 is the temperature at time $t = 0$ and k is a positive constant.

(a) Engineers want to estimate how long it will take the dam to cool down without water flowing through the pipes. They think that a couple of months after completion, the average temperature in the dam will be $T_0 = 60°C$ (this corresponds to t=0). They estimate that two months later, the average temperature in the dam will be $T = 59.75°C$.

(i) Using these two temperatures estimated by the engineers and using an average ambient temperature of $10°C$, identify T_a and calculate k correct to 4 decimal places.

(ii) What will be the average temperature in the dam after 50 years? Give your results correct to 1 decimal place.

(iii) How long will it take the dam to reach $12°C$ (express your answer in years)? Does this make sense?

(b) To reduce the cooling down time, engineers decide that the dam will not be built as a single concrete block and that cold water will be flown through the pipe system. In this situation, the average temperature in the dam is

$$T = 10 + 50e^{-0.0025t(2t-a)}$$

(i) Calculate the value of a for which, at t=2: $10 + 50e^{-0.0025t(2t-a)} = 10 + 50e^{-0.0025t}$

(ii) For what value of the time t is the following inequality true?
$10 + 50e^{-0.0025t(2t-3)} > 10 + 50e^{-0.0025t}$

(iii) Calculate the average temperature in the dam after 2 years, correct to one decimal place when the new construction method is used.

(iv) Interpret the results of the two previous questions for the dam.

Question 8 (50 Marks)

Munster wants to build a second stadium in Limerick. To finance this project, they decide to issue a 20-year bond.

(a) If the bond offers a 50% return on investment, what is the AER of this bond? Give your result correct to 3 decimal places.

(b) The bond will actually pay €6 at the end of every year plus an additional €100 lump sum at the end of the 20 years.

(i) Calculate the present value of the €100 lump sum if the expected market interest rate is 4%. Give your result correct to 4 decimal places.

(ii) Prove by induction that

$$a + ar + ar^2 + ...ar^{n-1} = a\frac{1 - r^n}{1 - r}$$

(iii) Hence show that if the expected market interest rate is 4%, the present value of all coupon payments is equal to

$$V = 6\frac{1-\dfrac{1}{1.04^{20}}}{0.04}$$

(iv) Calculate the total present value of bond if the expected market interest rate is 4%. Give your result correct to 4 decimal places.

(v) The bond is sold for €120. Should you buy it? Justify your answer.

(c) The income generated by ticket sales corresponds to €N per bond at the end of each year, so management can set aside €N-6 at the end of every year.

(i) Show that after 20 years, if the expected rate is 4%, management has set aside

$$M = (N-6)\frac{1.04^{20}-1}{0.04}$$

(ii) If every year ticket sales generate €12 per bond. How much money can management put aside in 20 years? Give your result correct to 2 decimal places.

(iii) How much should ticket sales generate every year so management can exactly repay the final €100 of the bond after 20 years? Give your answer correct to 3 decimal places.

Question 9 (50 Marks)

Consider the function

$$f(x) = \ln\left(1 + x^2\right)$$

(a) Function properties

(i) What are the domain and range of the function?

(ii) Show that $f(x) \geq 0$ for all $x \in \mathbb{R}$. Is the function surjective? Justify your answer.

(iii) Show that $f(2) = f(-2)$. Is the function injective? Justify your answer.

(iv) Calculate the inverse of the function for the limited domain $x \geq 0$.

(b) Some calculus

(i) Calculate the first order derivative of the function.

(ii) Calculate the second order derivative of the function.

(iii) Identify the turning point(s) and specify if each turning point is a minimum or a maximum.

(iv) Calculate the coordinates of the inflection point(s).

(v) Using one of the previous results, calculate

$$\int_0^1 \frac{2x}{1+x^2}dx$$

Rough Work

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