Sample paper prepared by Leamy Maths Community

Mathematics

Paper 1

Higher Level

Sunday 22 April

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LEAMY
MATHS COMMUNITY

Solutions

Q1	Q2	Q3	Q4	Q5	Q6	Q7	Q8

Q9	Q10	Q11	Q12	Q13	Q14	Q15	Total

300 marks

Sample Instructions

There are 15 questions on this examination paper. Answer all questions:

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times, you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may loose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of Formulae and Tables. You must return it at the end of the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Question 1 (Suggested maximum time 5 minutes)

Jane wants to buy a flight ticket from Shannon to London. On a website, she sees that today, the ticket costs €75.

(a) Within the following week, the price of the ticket increases by 15%.

   (i) Calculate the price of the ticket after one week.

$$P = 75.15 = 86.25$$

   Marks: 0, 3, 5

   (ii) At the end of the week, the website reduces the price of the ticket by €10. Calculate the percentage of decrease in the ticket price compared to the price you calculated in (i).

$$New\ Price = 86.25 - 10 = 76.25$$
$$r = \frac{76.25}{86.25} = 0.884$$
$$d = 100(1 - .884) = 11.6$$

   The price decreaes by 11.6%.
   Marks: 0, 3, 5

(b) In the end, Jane finds a flight ticket for €65. The website takes €5 then airport taxes correspond to 34% of the rest of the price. How much will the airline receive for this ticket?

$$Airport = (65 - 5) \times \frac{34}{100} = 20.4$$
$$Airline = 65 - 5 - 20.4 = 39.6$$

   Marks: 0, 3, 5

Question 2 (Suggested maximum time 10 minutes)

There are 30 client a restaurants which offers a choice of starters, mains and desserts.
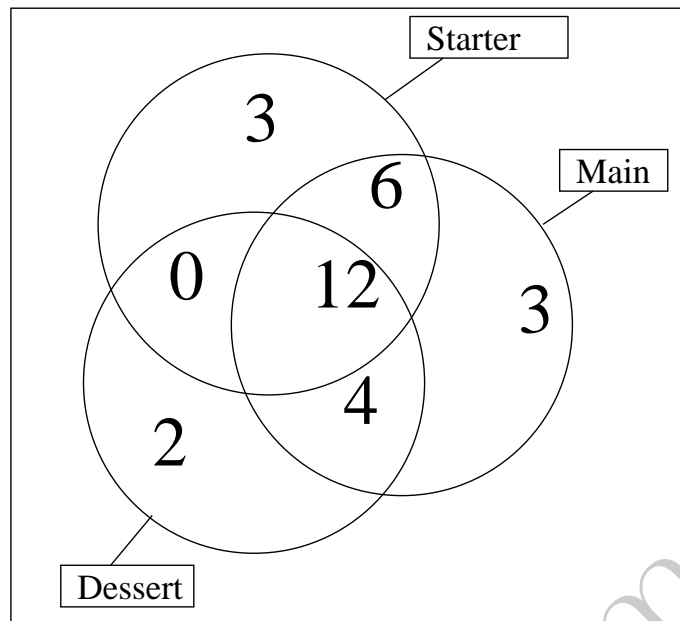
- 21 had a starter,
- 25 had a main,
- 18 had a dessert,
- 3 had only a starter,

- 3 had only a main,
- 6 had a main and a starter (no dessert),
- 4 had a main and a dessert (no starter).

(a) Put this information in the Venn diagram below. Marks: 0, 3, 5

(b) Find the missing values. Marks: 0, 3, 5, 7

(c) The restaurant charges

   - €7 for a starter,
   - €12 for a main,

- €5 for a dessert,
- A two course meal, starter and main or main and dessert, costs €16,
- A three course meal costs €20.

How much will the clients pay in total?

$$T = 3 \times 7 + 12 \times 3 + 5 \times 2 + 16 \times 10 + \times 20 = 467$$

Marks: 0, 3, 5, 8

Question 3 (Suggested maximum time 5 minutes)

(a) In a particular linear sequence, the third term is 20 and the fifth term is 44. Fill in the boxes below to show the rest of the first six terms of this sequence.

-4	8	20	32	44	56

Difference between 2 consecutive terms $= \dfrac{44 - 20}{2} = 12$

Marks: 0, 3, 5

(b) Dónal is asked to study a quadratic sequence: 12, 15, 20, 28, 36, 47 60. He knows one of the terms in the sequence is incorrect. Which term is it? Justify your answer.

$$12 + 3 = 15 \quad 15 + 5 = 20 \quad 20 + 7 = 27$$
$$27 + 9 = 36 \quad 36 + 11 = 47 \quad 47 + 13 = 60$$

Marks: 0, 3, 5

(c) Dónal needs to find the next two terms in the sequence. What are they? Justify your answer.

$$60 + 15 = 75 \quad 15 + 17 = 92$$

Question 4 (Suggested maximum time 5 minutes)

(a) Find the solutions of the equation

$$2x^2 - 5x + 3 = 0$$

$$2x^2 - 5x + 3 = (2x - 3)(x - 1) \implies x = 1 \quad x = \frac{3}{2}$$

(b) Specify if each of the solutions belongs to the set of integers $\mathbb{Z}$, the set of rationals $\mathbb{Q}$ or the set of irrational s numbers $\mathbb{R}\backslash\mathbb{Q}$
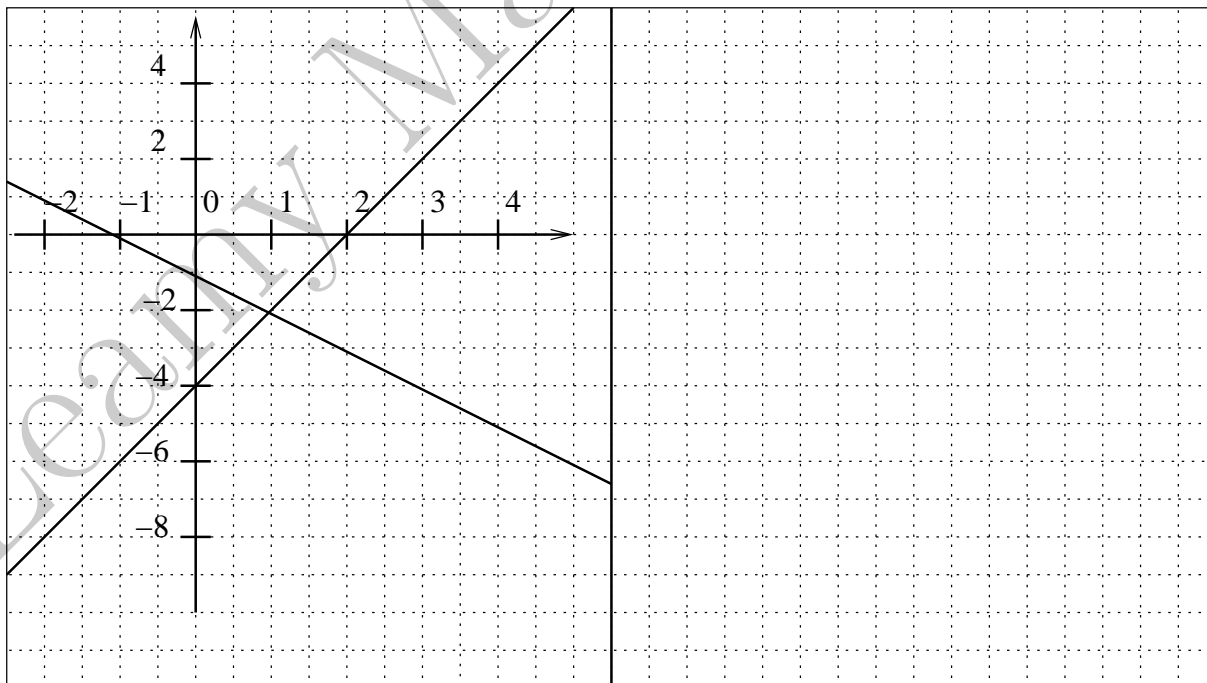
$$1 \in \mathbb{Z} \ and \ \mathbb{Q} \quad \frac{3}{2} \in \mathbb{Q}$$

Question 5 (Suggested maximum time 10 minutes)

(a) Plot the two lines $y = 2x - 4$ and $y = -1 - x$ on the diagram below. Justify your calculations.

(b) Calculate where the two lines cross using simultaneous equations.

$$x = 1 \qquad y = -2$$

Marks: 0, 3, 5, 8

(c) Explain how you can see this solution on your graph.
This is where the two lines cross
Marks: 0, 3, 5

Question 6 (Suggested maximum time 10 minutes)

In an electric circuit with a capacitance C and an inductance L, the frequency f at which resonance occurs is defined by

$$f = \frac{1}{2\pi\sqrt{LC}}$$

(a) Calculate the resonance frequency f when $L = 0.020$ (H) and $C = 2 \times 10^{-6}$ (F). Round up your result to the nearest integer.

$$f = 796Hz$$

Marks: 0, 3, 5

(b) Express L in terms of f and C.

$$L = \frac{1}{4\pi^2 f^2 C}$$

Marks: 0, 3, 5, 7

(c) The angular frequency ω is defined as $\omega = 2\pi f$. Show that

$$LC\omega^2 = 1$$

$$LC\omega^2 = 4LC\pi^2 f^2 = 4LC\pi^2 \left(\frac{1}{2\pi\sqrt{LC}}\right)^2 \frac{4LC\pi^2}{4LC\pi^2} = 1$$

Marks: 0, 3, 5, 8

Question 7 (Suggested maximum time 10 minutes)

(a) Factorise $9a^2 - 81b^2$

$$9a^2 - 81b^2 = 9(a - 3b)(a + 3b)$$

Marks: 0, 3, 5

(b) Use factors to simplify

$$\frac{x^2 - x - 12}{x^2 - 9}$$

$$\frac{x^2 - x - 12}{x^2 - 9} = \frac{(x-4)(x+3)}{(x-3)(x+3)} = \frac{x-4}{x-3}$$

Marks: 0, 3, 5, 8, 10

(c) Factorise $3a^2 - 3ab + 4a - 4b$

$$3a^2 - 3ab + 4a - 4b = 3a(a-b) + 4(a-b) = (3a+4)(a-b)$$

Marks: 0, 3, 5

Question 8   (Suggested maximum time 10 minutes)

(a) Solve the equation

$$\frac{1-x}{4} + \frac{3x-1}{2} = 6$$

$$\frac{1-x}{4} + \frac{3x-1}{2} = 6 \implies 1 - x + 2(3x - 1) = 24 \implies 5x = 25 \implies x = 5$$

Marks: 0, 3, 5, 8, 10

(b) Solve the equation below. Give your answers in the simplest possible form $a/b =$ where $a, b \in \mathbb{Z}$.

$$\frac{2}{3-x} = 4$$

$$4(3 - x) = 2 \implies 12 - 4x = 2 \implies 4x = 10 \implies x = \frac{5}{2}$$

Marks: 0, 3, 5, 8, 10

Question 9   (Suggested maximum time 5 minutes)

(a) Write the following expression in the form 5^p where $p \in \mathbb{Q}$.

$$5^3 \times 5^4 \times 5 = 5^8$$

Marks: 0, 2

(b) Write the following expression in the form 2^q where $q \in \mathbb{Q}$.

$$\frac{4}{8^3} = 2^{-7}$$

Marks: 0, 3

(c) Write the following expression in the form a^r where $r \in \mathbb{Q}$.

$$a^2 \times \sqrt{a} = a^{\frac{5}{2}}$$

Marks: 0, 3, 5

Question 10 (Suggested maximum time 15 minutes)

(a) Show that 171 is not a prime number.

$$171 = 9 \times 19$$

Marks: 0, 3, 5, 7

(b) A formula used to calculate prime numbers is

$$n = \frac{2^p + 1}{3}$$

Calculate the values for p=1, 3, 5, 7, 9 and fill in the table below.

p	3	5	7	9
n	3	11	43	171

Marks: 0, 3, 5, 8, 10

(c) According to number theory, the formula provides a prime number only when p is an odd prime number. Can you verify this with your results above?
It works for 3,5,7 which are prime numbers but not for 9 which is not a prime number

Marks: 0, 3, 5, 7

(d) Calculate a prime number with $p \geq 10$. Justify your choice.

$$\frac{2^1 1 + 1}{3} = 683$$

Marks: 0, 3, 5

Question 11 (Suggested maximum time 15 minutes)

Eimar and her five friends decide to create a group to collect funds for the local children hospital. They challenge themselves to recruit new people every week: each of the 6 original members should recruit 2 people every week.

(a) Fill in the table below describing the number of members of the group.

Week	1	2	3	4	5
No. of members	6	18	30	42	54

(b) Find a formula giving the number of members N in terms of the number of week W.

$$W_n = 12(n-1) + 6 = 12n - 6$$

(c) After k weeks, there are 90 members. Using the formula above or any other method, find the value of k.

$$96 = 12k - 6 \implies k = 8$$

(d) Now there are 90 members, the team starts to collect money. The amount of money in euros they collect is given by the formula

$$Val = -200 + 700W$$

where W is the number of weeks they have been collecting (starts at W=1).

(i) How much have they collected in week 3 (W=3)?

$$W_3 = -200 + 2100 = 1900$$

(ii) In what week will they reach €6,800?

$$6800 = -200 + 700W \implies W = 10$$

(iii) They need collect over €10,000 for the hospital. What is the minimum number of weeks they need to collect to reach this amount?

$$10000 > -200 + 700W \implies W > \frac{10200}{700} = 14.57$$
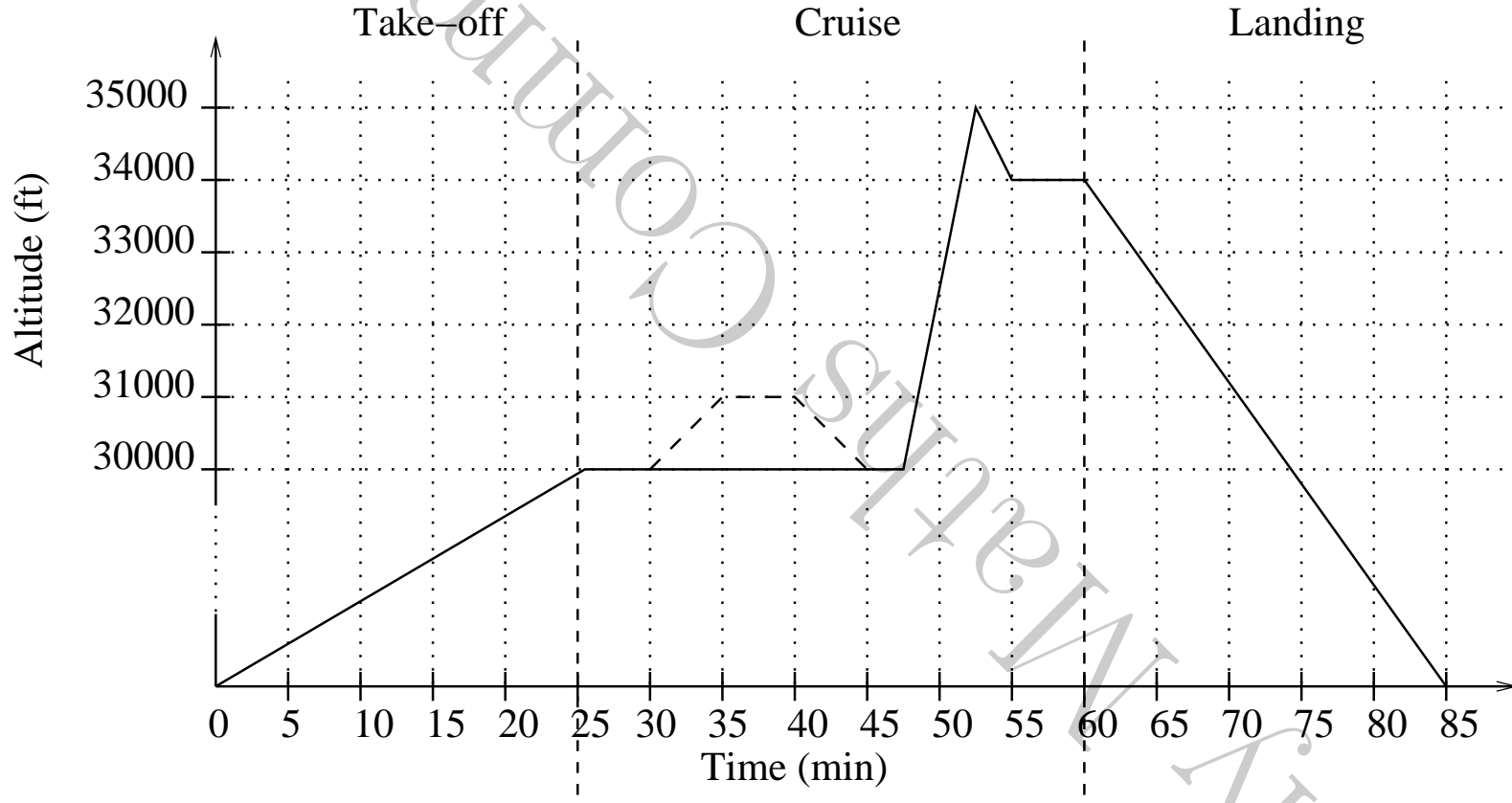
They need 15 weeks.

Question 12 (Suggested maximum time 15 minutes)

The graph on the next page shows the altitude flight plan for a plane travelling from Shannon to London Heathrow.

(a) Complete the following table using the information on the graph on the next page.

Flight time (min)	85
Time for the take-off (min)	25
Amount of time spent at 30,000 ft (min)	22.5
Maximum altitude reached during cruise	35000
Altitude of the plane at the start of the landing phase	34000

(b) During the take-off and landing phases, on average, the plane flies at approximately 300km/hour while the cruising speed is approximately 720km/hour. Calculate the distance covered by the plane with the new flight plan. Give your answer correct to two decimal places.

$$D = \frac{25}{60} \times 300 + \frac{35}{60} \times 720 + \frac{25}{60} \times 300 = 125 + 420 + 125 = 670 km$$

Marks: 0, 3, 5, 8, 10

(c) Another plane is flying on the same flight path. To avoid collision, the pilot needs to change the altitude of the plane. The plane must climb another 1000ft between time t=30 min and t=35 min, stay at this new altitude for 5 minutes and then descend back to the originally planned altitude in the following 5 minutes.

   (i) Plot the new course on the graph. Marks: 0, 3, 5

  (ii) With this new flight path, calculate the altitude the plane had to climb up during the entire flight.

$$30000 + 1000 + 5000 = 36000$$

Marks: 0, 3, 5

(iii) Engines can be run at reduced speed while the plane loses altitude. Calculate the percentage of flight time when engines run at reduced speed.

$$\frac{32.5}{85} = 38.2\% \qquad \frac{27.5}{85} = 32.4\%$$
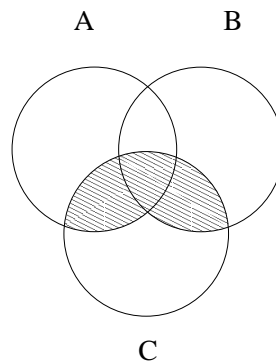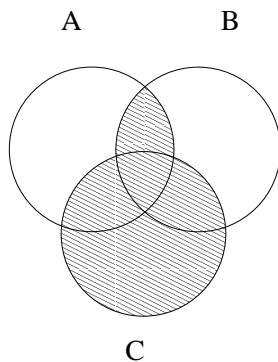
Marks: 0, 3, 5

Question 13   (Suggested maximum time 5 minutes)

Shade the region indicated for both diagrams

$$(A \cap B) \cup C \qquad\qquad\qquad (A \cup B) \cap C$$
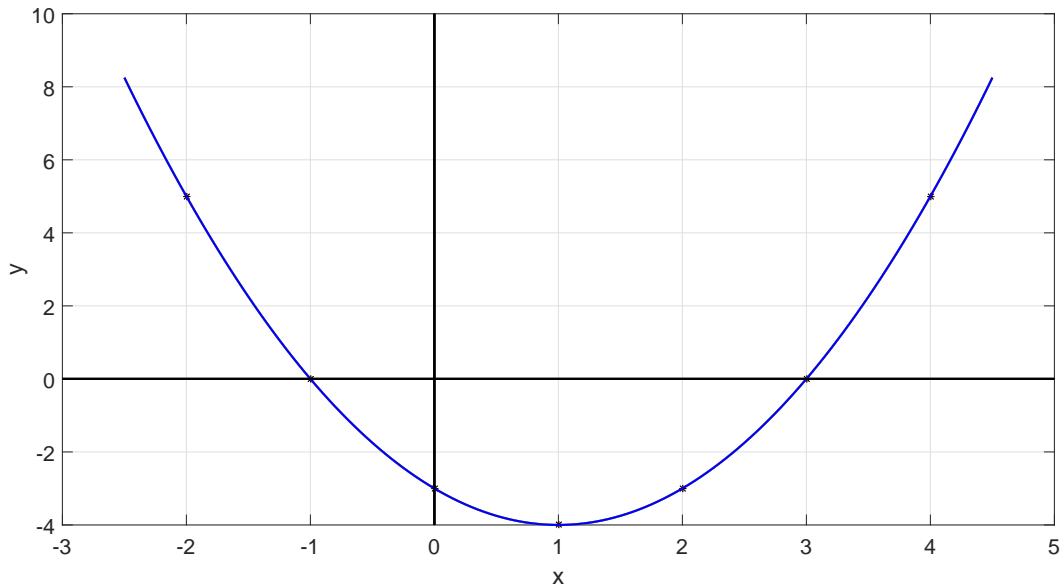
Marks: 0, 3, 5 Marks: 0, 3, 5

A   B   A   B

C   C

Question 14 (Suggested maximum time 5 minutes)

Plot the function

$$f(x) = x^2 - 2x - 3$$

x	-2	-1	0	1	2	3	4
y	5	0	-3	-4	-3	0	5

Marks: 0, 3, 5

Marks: 0, 3, 5

Question 15 (Suggested maximum time 15 minutes)

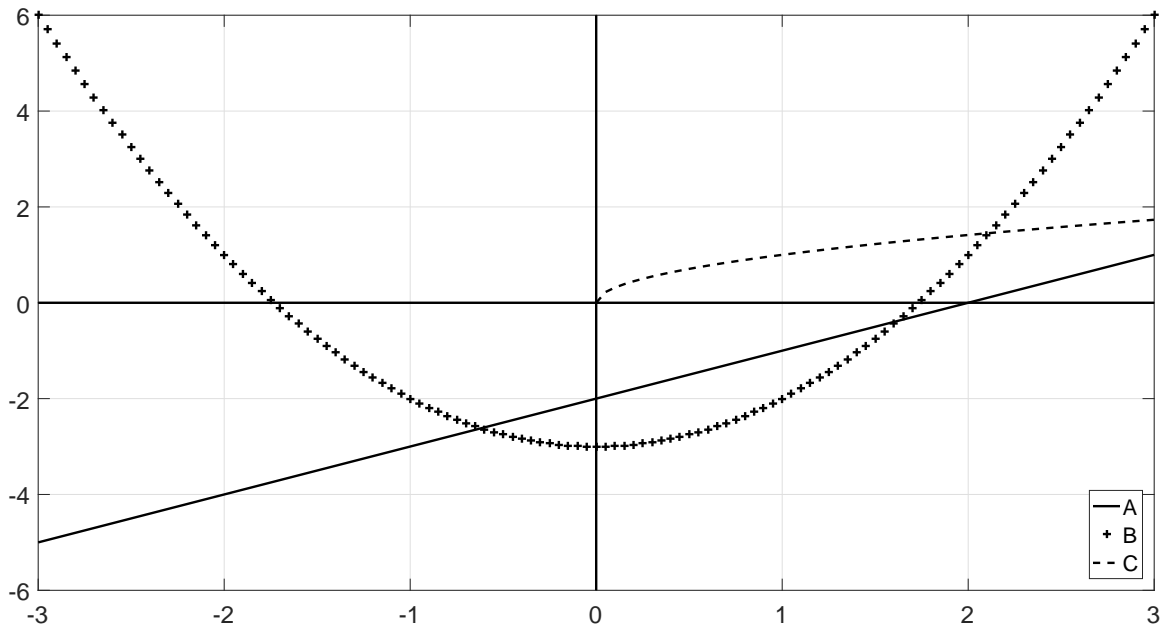**(a)** Associate the 3 graphs with the following functions

   (i) Function 1

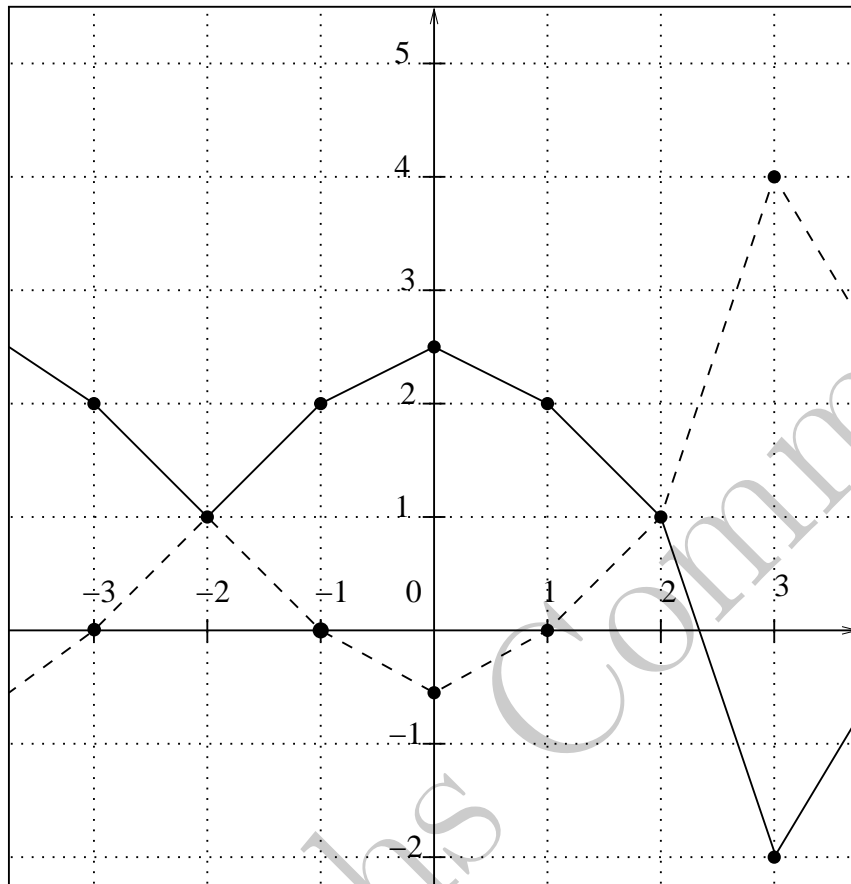$$y = x^2 - 3$$

   (ii) Function 2

$$y = x - 2$$

   (iii) Function 3

$$y = \sqrt{x}$$

Function	1	2	3
Graph	B	A	C

Marks: 0, 3, 5, 8

(b) The graph below defines the function f(x).

(i) Using the graph fill in the table below

x	-3	-2	-1	0	1	2	3
f(x)	2	1	2	2.5	2	1	-2
2-f(x)	0	1	0	-0.5	0	1	4

Marks: 0, 3, 5, 7 Marks: 0, 3, 5, 8, 10

(ii) Hence or otherwise, plot the function $2 - f(x)$ on the graph below.
Marks: 0, 3, 5