Junior Certificate Examination, 2018

Sample paper prepared by Leamy Maths Community

Mathematics

Paper 2

Higher Level

Sunday 29 April

Paper written by J.P.F. Charpin and S. King

Solutions

300 marks

Sample Instructions

There are 13 questions on this examination paper. Answer all questions:

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times, you should have about 10 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may loose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of Formulae and Tables. You must return it at the end of the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

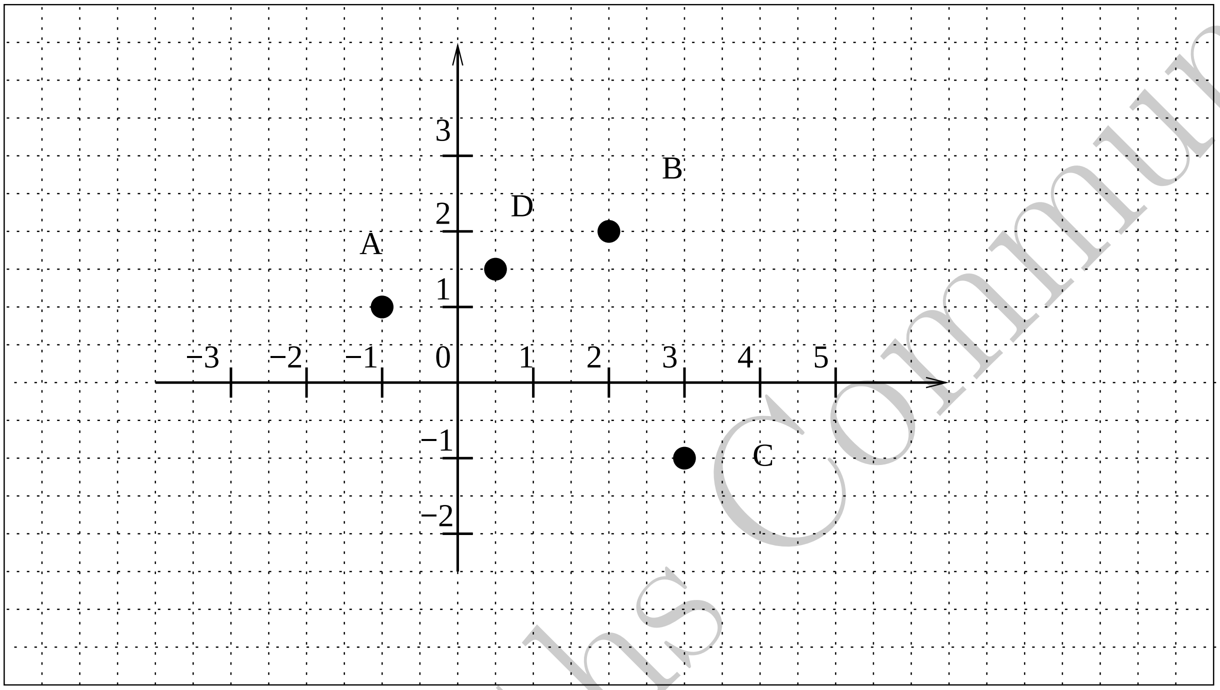
Write the make and model of your calculator(s) here:

Question 1 (Suggested maximum time 15 minutes)

(a) Write down the coordinates of the three points A, B, C on the graph below.

$$A = (-1, 1) \qquad B = (2, 2) \qquad C = (3, -1)$$

Marks: 0, 3, 5



(b) Calculate the coordinates of point D the midpoint of [AB]

$$D = \left(\frac{2-1}{2}, \frac{2+1}{2}\right) = \left(\frac{1}{2}, \frac{3}{2}\right)$$

Marks: 0, 3, 5

(c) Calculate the three distances $|AB|$, $|BC|$ and $|AC|$. Give your results in surd form.

$$AB = \sqrt{10} \qquad BC = \sqrt{10} \qquad AC = \sqrt{20}$$

Marks: 0, 3, 5

(d) Without measuring, show that the triangle ABC is an isosceles right angle triangle.
AB=AC so the triangle is isosceles.

$$AB^2 + BC^2 = AC^2$$

The this is an isosceles right angle triangle. Marks: 0, 3, 5, 8, 10, 15

(e) Hence or otherwise, calculate the area of the triangle.

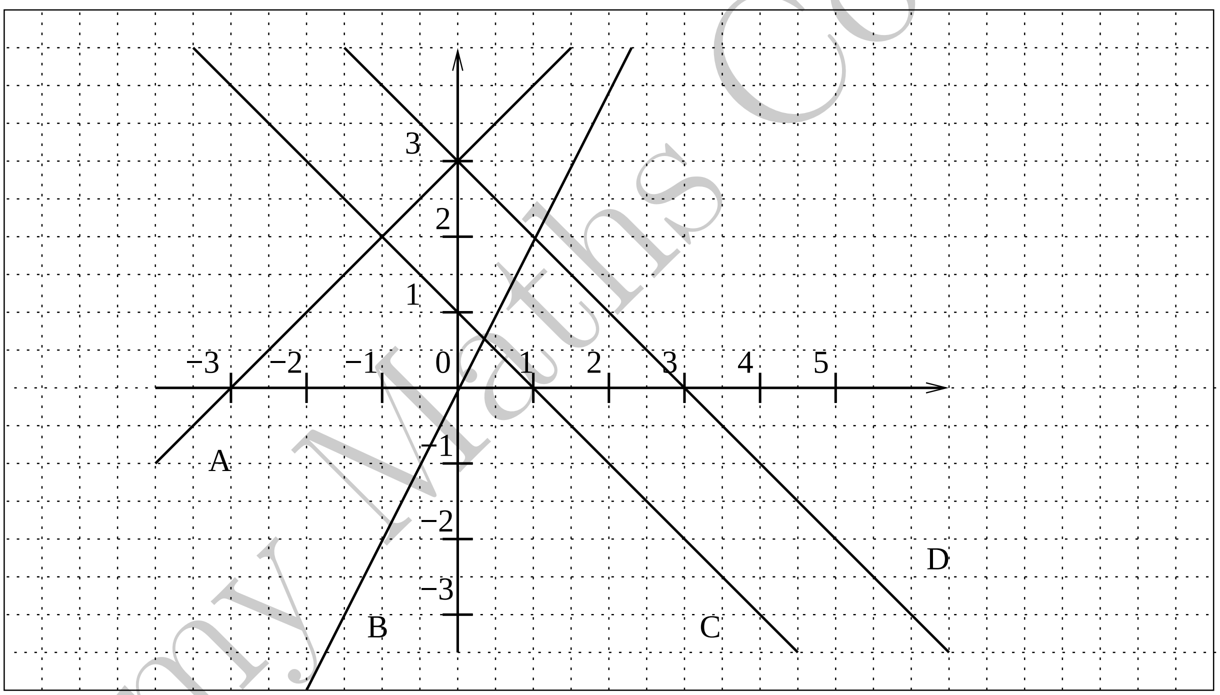
$$Area = \frac{1}{2} AB \times AC = 5$$

Marks: 0, 3, 5

Question 2 (Suggested maximum time 15 minutes)

(a) Using the digram below, match the four lines with the equations in the table below

Line 1	y=-x+1	C	Line 2	y=-x+3	D
Line 3	y=x+3	A	Line 4	y=2x	B



.

Marks: 0, 3, 5

(b) Show that point (4,-3) belongs to Line 1.

$$y = -4 + 1 = -3$$

so the point is on the line

Marks: 0, 3, 5

(c) Calculate the intersection of Line 2 with the x-axis and y-axis.

$$-x+3=0 \Longrightarrow x=3 \Longrightarrow (3,0)$$
$$y=-0+3 \Longrightarrow y=3 \Longrightarrow (0,3)$$

Marks: 0, 3, 5

(d) Calculate the intersection point between the Line 3 and Line 4.

$$2x=x+3 \Longrightarrow x=3 \Longrightarrow y=6$$

The point is (3,6)
Marks: 0, 3, 5

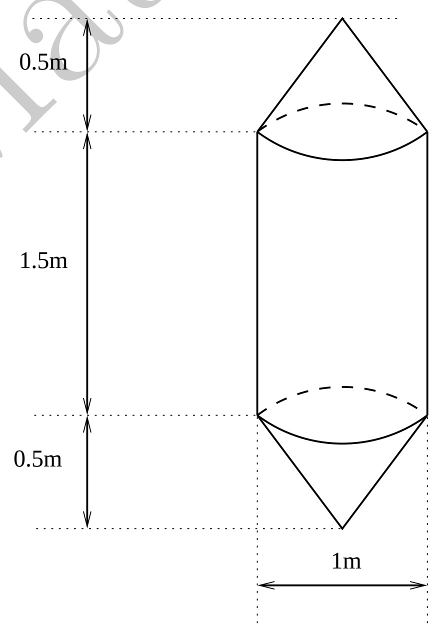
(e) Which two lines can be mapped using

(i) a translation? Justify your answer.
The two lines 1 and 2 (C and D) are parallel so they can be mapped with a translation.
Marks: 0, 3, 5

(ii) an axial symmetry through the y-axis? Justify your answer.
Lines 2 and 3 (D and A) have opposite slopes and they cross on the y axis. They can be mapped using an axial symmetry through the y-axis.
Marks: 0, 3, 5

Question 3 (Suggested maximum time 10 minutes)

(a) A water tank has the shape described in the figure below.



(i) Calculate the volume of the cylindrical part. Give your result correct to 2 decimal places.

$$\begin{aligned} V &= \pi R^2 h \\ &= \pi(0.5)^2(1.5) \\ &= 1.18m^3 \end{aligned}$$

Marks: 0, 3, 5, 7

(ii) Calculate the volume of the conical parts. Give your result correct to 2 decimal places.

$$V = 2 \times \frac{1}{3}\pi R^2 h = \frac{2}{3}\pi(0.5)^2(1.5) = 0.26m^3$$

Marks: 0, 3, 5, 8

(iii) Calculate total volume of the water tank. Give your result correct to 2 decimal places.

$$V = 1.18 + 0.26 = 1.44m^3$$

Marks: 0, 3, 5

(b) If the water tank was only cylindrical with height 1m, what diameter should be used so the volume of the tank is the same as the volume calculated in section **a(iii). Give your result correct to 2 decimal places.**

$$V = \pi R^2 h \Longrightarrow 1.44 = \pi(1)R^2 \Longrightarrow D = 2R = 2\sqrt{\frac{1.44}{\pi}} = 1.35m$$

Marks: 0, 3, 5, 8, 10

Question 4 (Suggested maximum time 10 minutes)

Stephen walks up and down the stairs of Leamy House a few times a day. One of his students asks what height he must climb up every time. To approximate this, they leave the light on and get down to the street. Stephen stands away from the building and he just avoids the light from the window. Together with the student, they measure the length indicated in the figure below.

$$\frac{DE}{BC} = \frac{AD}{AB} \Longrightarrow DE = BC \times \frac{AD}{AB} = 1.96 \times \frac{5}{1.4} = 7$$

(a) Using the information provided, calculate the distance $|DE|$.

$$\frac{DE}{BC} = \frac{AD}{AB} \Longrightarrow DE = BC \times \frac{AD}{AB} = 1.96 \times \frac{5}{1.4} = 7$$

Marks: 0, 3, 5, 8

(b) Hence or otherwise, calculate the height of point F. How many meters does Stephen climb up when he uses the stairs up to the Leamy Maths community?

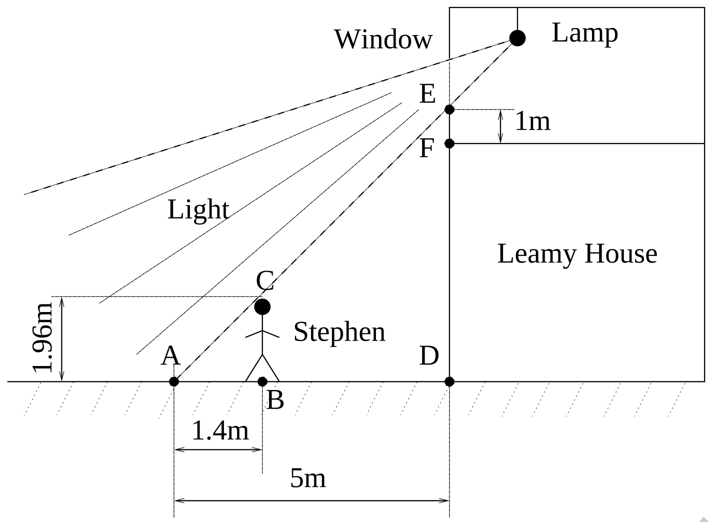
$$DF = 7 - 1 = 6m$$

Marks: 0, 3, 5

(c) Calculate the value of angle $| < CAB|$ in degrees rounded to the nearest degree.

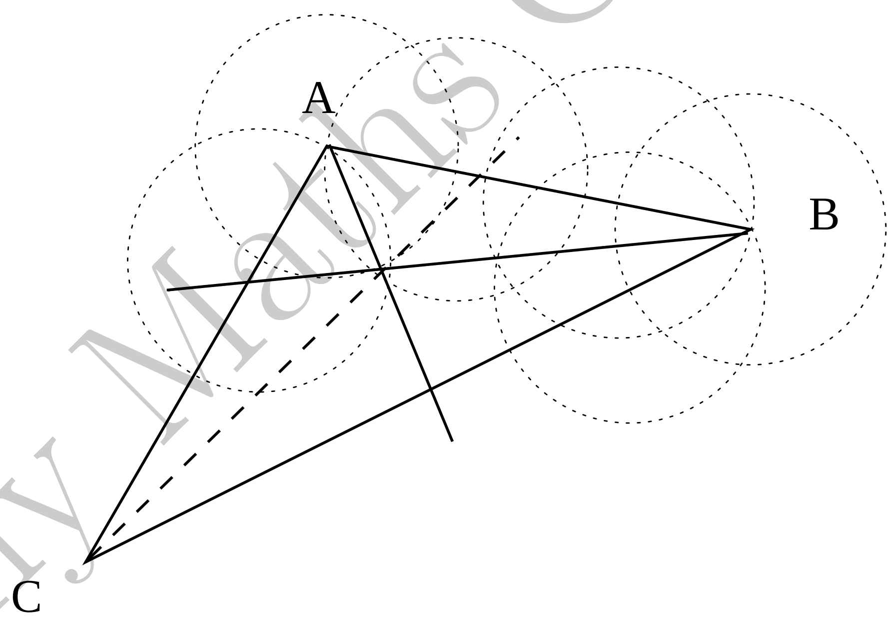
$$\tan | < CAB| = \frac{1.96}{1.4} \Longrightarrow | < CAB| = 54^o$$

Marks: 0, 3, 5, 7



Question 5 (Suggested maximum time 10 minutes)

(a) Construct the bisectors of angles $|\angle CAB|$ and $|\angle ABC|$ of the triangle below using only a compass and a straight edge. Show all construction work.

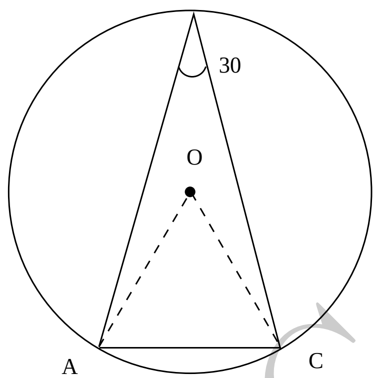


Marks: 0, 5, 10, 15

(b) Using only your straight edge, construct the bisector of of angle $|<ACB|$. Justify your answer. You only need to join point C with the common point of the other 2 bisectors.
Marks: 0, 3, 5

Question 6 (Suggested maximum time 10 minutes)

(a) Show that the angle $|<AOC| = 60°$.



$$|<AOC| = 2|<ABC| = 2 \times 30 = 60$$

Marks: 0, 3, 5

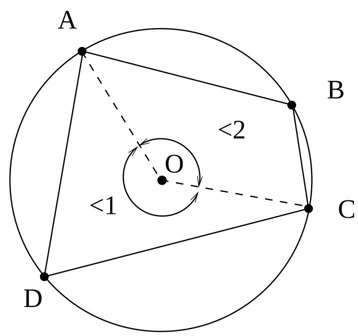
(b) Hence or otherwise show that triangle AOC is equilateral.
AO=OC, so the triangle is isosceles and $|<OAC| = |<OCA|$. Since the three angles of a triangle add up to 180, therefore

$$|<OAC| + |<OCA| + |<AOC| = 180 \implies 2|<OAC| + 60 = 180$$
$$\implies |<OAC| = |<OCA| = \frac{180-60}{2} = 60^o$$

All three angles in the triangle are equal to 60^o so the triangle is equilateral.
Marks: 0, 3, 5, 8, 10, 13, 15

Question 7 (Suggested maximum time 15 minutes)

Show that opposite angles in ABCD add up to 180°. Marks: 0, 3, 5



To prove

$$|<ABC| + |<ADC| = 180^o \qquad |<BAD| + |<BCD| = 180^o$$

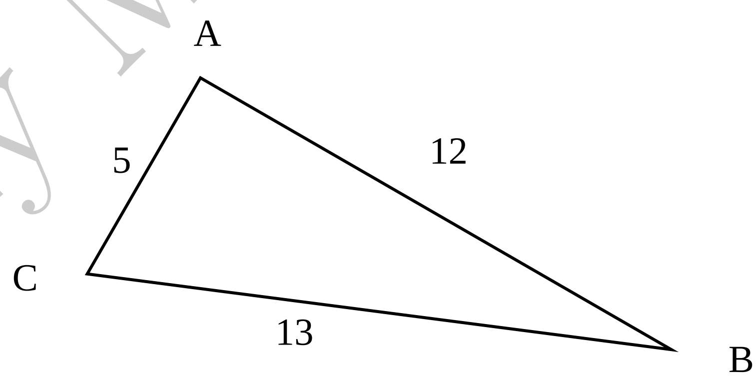
Marks: 0, 3, 5

Proof

$$|<1| = 2\times|<ADC| \qquad |<2| = 2\times|<ABC|$$
$$|<1| + |<2| = 360^o \Longrightarrow 2\left(|<ABC| + |<ADC|\right) = 360^o$$
$$\Longrightarrow |<ABC| + |<ADC| = \frac{360}{2} = 180^o$$

Marks: 0, 5, 10, 15, 20

Question 8 (Suggested maximum time 5 minutes)



(a) Show that ABC is a right angle triangle.

$$BC^2 = 13^2 = 169 \qquad AB^2 + AC^2 = 12^2 + 5^2 = 144 + 25 = 169$$

so ABC is a right angle triangle.

Marks: 0, 3, 5

(b) Calculate $\cos < ABC$ and $\cos < ACB$. Give your answer as a fraction

$$\cos < ABC = \frac{12}{13} \qquad \sin < ACB = \frac{5}{13}$$

Marks: 0, 3, 5

(c) Using fractions only, show that

$$\cos^2 < ABC + \cos^2 < ACB = 1$$
$$\cos^2 < ABC + \cos^2 < ACB = \left(\frac{12}{13}\right)^2 + \left(\frac{5}{13}\right)^2 = \frac{144}{169} + \frac{25}{169} = 1$$

Marks: 0, 3, 5, 8,10

Question 9

Note all parts of this questions are independent and can be answered separately.

Question 9a (Suggested maximum time 20 minutes)

(a) Aoife has to conduct a series of surveys for a project. She asks people in her school the following questions:

(1) How tall are you?

(2) What is you favourite topic? (possible answers: English, Irish, Mathematics, French, History)

(3) How long does it take you to come to school (possible answers: short time (0 to 10 minutes), medium time (10 to 30 minutes), long time (30 to 40 minutes), very long time (40 to 60 minutes))

(4) How many school days did you miss in the last three months?

Use the following table to classify the questions.

	Question number		Question number
Numerical Continuous	1	Numerical Ordinal	4
Categorical Ordinal	3	Categorical Nominal	2

Marks: 0, 3, 5

(b) Aoife collected the following heights in cm:

- Females: 170, 149, 151, 176, 171, 157, 168, 164, 173, 162, 175, 147, 182, 175, 171, 164, 176, 174, 166, 180, 172, 155, 173, 153, 170.
- Males: 161, 187, 162, 171, 165, 180, 175, 168, 170, 177, 174, 170, 172, 160, 172, 190 176, 181, 171, 159, 182, 184, 192, 194, 164.

(i) Represent the data using the back to back stem leaf diagram below. Marks: 0, 5, 10, 15

Females		Males
9, 7	14	
7, 5, 3, 1	15	9
8, 6, 4, 4, 2,	16	0, 1, 2, 4, 5, 8
6, 6, 5, 5, 4, 3, 3, 2, 1, 1, 0, 0	17	0, 0, 1, 1, 2, 2, 4, 5, 6, 7
2, 0	18	0, 1, 2, 4, 7
	19	0, 2, 4

(ii) Find the median for both males and females.

$$Females = 170cm \qquad Males = 172cm$$

Marks: 0, 3, 5

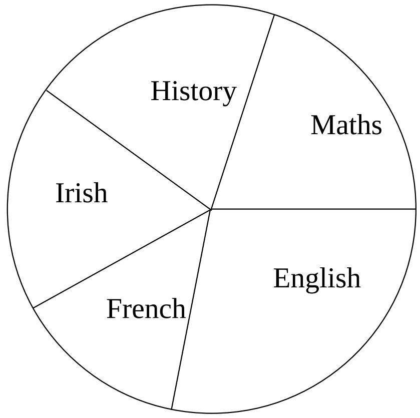
(iii) Find the mode for both males and females.
Females: 164, 170, 171, 173, 175, 176
Males 170, 171, 172
Marks: 0, 3, 5

Question 9b (Suggested maximum time 10 minutes)

(c) The responses for the favourite topic are as follows

- English: 14,
- Irish: 9,
- Mathematics: 10,
- French: 7,
- History: 10

(i) Represent the data in a pie chart. Marks: 0, 3, 5, 8, 10, 13, 15



(ii) If a person is chosen randomly, what is the probability that the student's favourite topic is Mathematics?

$$p = \frac{10}{20} = 0.2 = 20\%$$

Marks: 0, 3, 5

(iii) If a person is chosen randomly, what is the probability that the student's favourite topic is English or history?

$$p = \frac{24}{50} = \frac{12}{25} = 0.48 = 48\%$$

Marks: 0, 3, 5

Question 9c (Suggested maximum time 10 minutes)

(d) The travelling time results are shown the table below

Short (0-10 min)	Medium (10-30 min)	Long (30-40 min)	Very Long (40-60 min)
15	25	8	2

(i) Use mid-interval values to estimate the mean time it takes students to come to school. Give your answer correct to one decimal place.

$$\begin{aligned} Time &= \frac{1}{50}[5\times 15+20\times 25+8\times 35+50\times 2] \\ &= \frac{1}{50}[75+500+280+100] \\ &= \frac{955}{50} = 19.1min \end{aligned}$$

Marks: 0, 3, 5, 8, 10, 13, 15

(ii) On average, it actually takes 26 minutes for students to go to school. Why is this value different form the value you calculated in the previous part?
Used mid-term values so result is less precise.
Marks: 0, 3, 5

Question 10 (Suggested maximum time 10 minutes)

In a fair, the following game is organised: you throw two dice. If the total is a prime number, you win €2 otherwise, you lose.

(i) Calculate the total of the two dice in the following table: the top raw indicates the result of the first dice and the first column is the result for the second dice.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Marks: 0, 3, 5

(ii) What is the probability of winning?
Prime numbers in the set are 2,3,5,7,11. The probability of winning is therefore

$$p = \frac{1+2+4+6+2}{36} = \frac{15}{36} = 41.6\%$$

Marks: 0, 3, 5, 8, 10, 15