

Junior Certificate Examination, 2019

Sample paper prepared by Leamy Maths Community

## Mathematics

Paper 2

Higher Level

28 April 2019

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Solutions

300 marks

## Sample Instructions

There are 11 questions on this examination paper. Answer all questions:

Questions do not necessarily carry equal marks. To help you manage your time during this examination, a maximum time for each question is suggested. If you remain within these times, you should have about 15 minutes left to review your work.

Write your answers in the spaces provided in this booklet. You may lose marks if you do not do so. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of Formulae and Tables. You must return it at the end of the examination.

You will lose marks if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

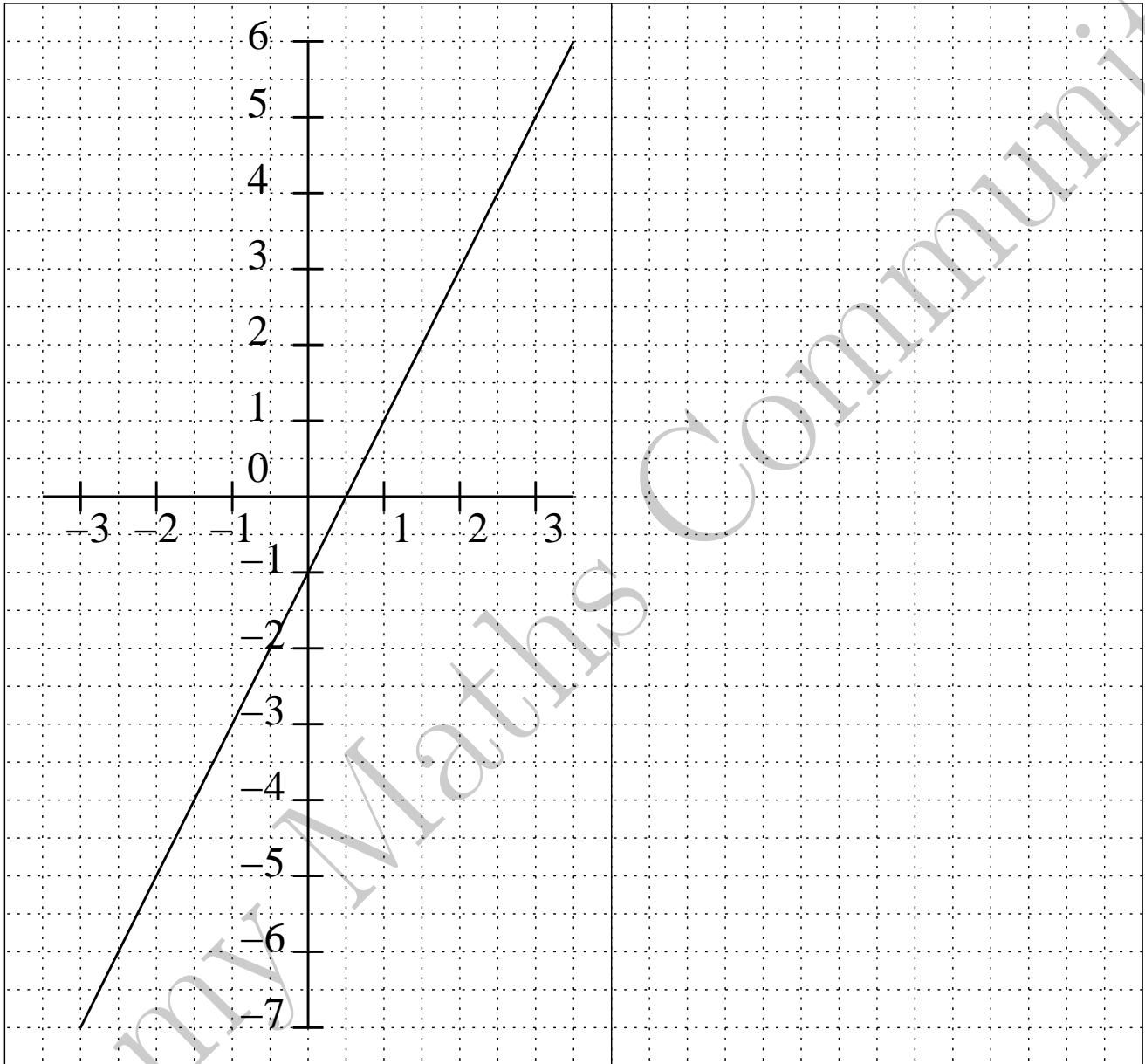
Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

**Question 1** (Suggested maximum time: 10 minutes)

Consider the line  $\mathcal{L}: y = 2x - 1$ .

(a) Plot the line  $\mathcal{L}$  on the graph.



x	-3	-2	-1	0	1	2	3
y	-7	-5	-3	-1	1	3	5

Marks: 0, 3, 5, 8, 10

- (b) Calculate the coordinates of the points where the line crosses the  $x$  and  $y$  axis.  
 $x = 0 \implies y = -1$  and  $y = 0 \implies 0 = 2x - 1 \implies x = \frac{1}{2}$ . The two points are  $(0, -1)$  and  $(\frac{1}{2}, 0)$ .

Marks: 0, 3, 5

- (c) Explain how you can verify your results on the graph  
 You can see on the graph where the line crosses the axis and read the coordinates.

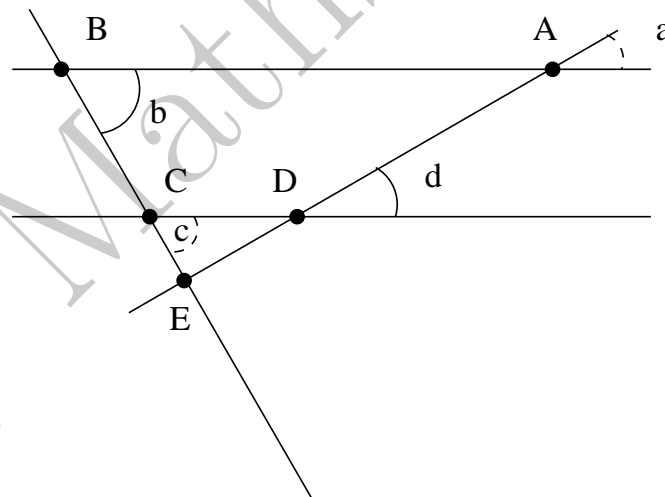
Marks: 0, 3, 5

- (d) Line  $D_1$  is parallel to  $\mathcal{L}$  and line  $D_2$  is perpendicular to  $\mathcal{L}$ . Fill in the table below and justify your answer.

Line	$\mathcal{L}$	$D_1$	$D_2$
Slope	2	2	-2

Parallel lines have the same slopes and the product of the slopes of two perpendicular lines is -1. Marks: 0, 3, 5

## Question 2 (Suggested maximum: 20 time minutes)



The two lines  $(AB)$  and  $(CD)$  are parallel

- (a) Why do the angles  $\angle a$ ,  $\angle b$ ,  $\angle c$ ,  $\angle d$  verify  $\angle a = \angle d$  and  $\angle b = \angle c$ ?  
 These are corresponding angles. Marks: 0, 3, 5

- (b)  $\angle a = 30^\circ$ . Calculate  $\angle BAD$ .  
 $\angle BAD = a = 30^\circ$

Marks: 0, 3, 5

(c)  $\angle b = 60^\circ$ . Show that ABE is a right angle triangle.

$$\angle BAE + \angle ABE + \angle AEB = 180 \implies 30 + 60 + \angle AEB = 180 \implies \angle AEB = 90$$

Marks: 0, 3, 5

(d) Show that the two triangles ABE and CDE are similar

$$\angle ABE = \angle DCE \quad \angle BAE = \angle CDE \quad \angle AEB = \angle DEC$$

The two triangles have three equal angles, they are similar.

Marks: 0, 3, 5, 8, 10

(e)  $|AB| = 9\text{cm}$ ,  $|BE| = 6\text{cm}$ ,  $|BC| = 4\text{cm}$

(i) Calculate the distance  $|AE|$ . Give your result in the form  $a\sqrt{b}$ ,  $a, b \in \mathbb{N}$ .

$$|AB|^2 = |AE|^2 + |BE|^2 \implies 81 = |AE|^2 + 36 \implies |AE|^2 = 45 \implies |AE| = \sqrt{45} = 3\sqrt{5}$$

Marks: 0, 3, 5, 7

(ii) Calculate the distance  $|CD|$ .

$$\cos 60 = \frac{|CE|}{|CD|} \implies \frac{1}{2} = \frac{2}{|CD|} \implies |CD| = 2 \times 2 = 4$$

Marks: 0, 3, 5, 8

There is an inconsistency in the question. An alternative solution is also valid:

$$\frac{|CE|}{|BE|} = \frac{|CD|}{|AB|} \implies \frac{2}{6} = \frac{x}{9} \implies |CD| = 3$$

### Question 3 (Suggested maximum time: 15 minutes)

The co-ordinate diagram below shows the triangle ABC.

(a) Write down the co-ordinates of the points A, B and C.

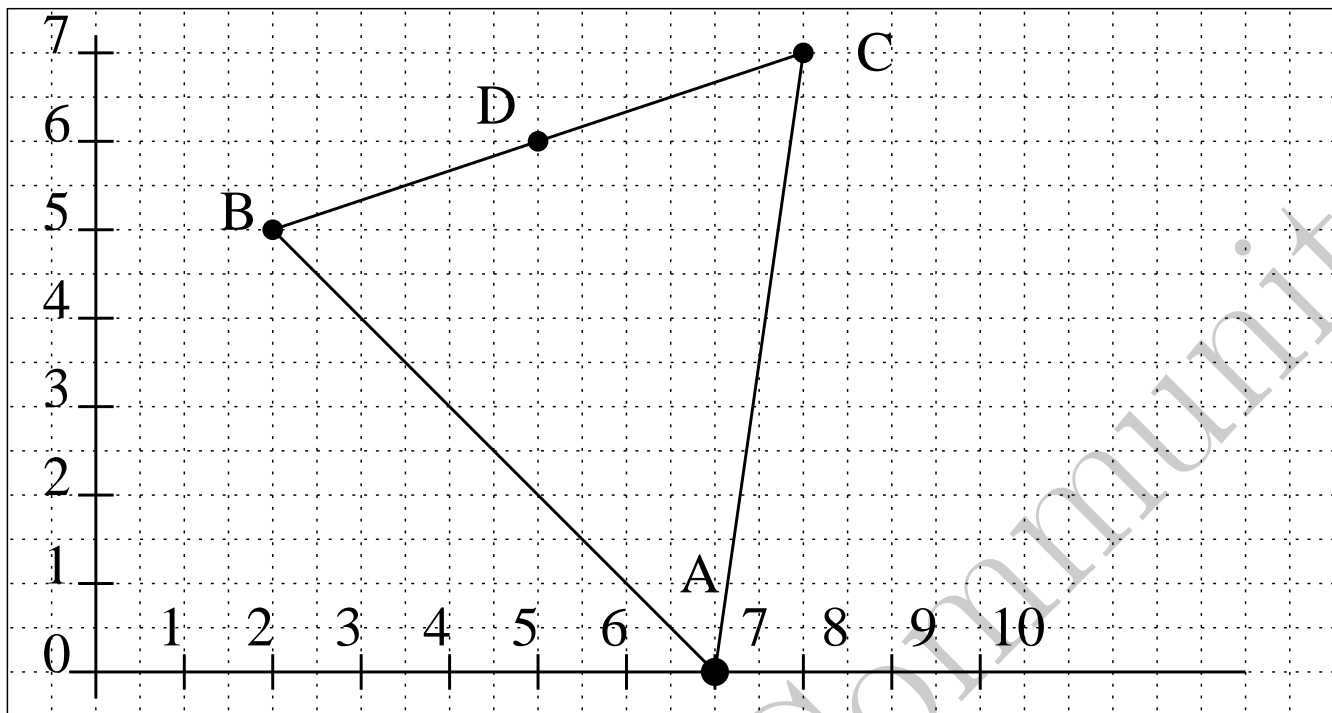
$$A=(7, 0), B=(2, 5), C=(8, 7)$$

Marks: 0, 3, 5

(b) Calculate the co-ordinates of point D, the midpoint of  $[BC]$ . Put the point on the graph.

$$D = \left( \frac{2+8}{2}, \frac{5+7}{2} \right) = (5, 6)$$

Marks: 0, 3, 5



(c) Calculate the distances  $|BC|$  and  $|AD|$ .

$$|BC| = \sqrt{(x_C - x_B)^2 + (y_C - y_B)^2} = \sqrt{6^2 + 2^2} = \sqrt{40} = 2\sqrt{10}$$

$$|AD| = \sqrt{(x_D - x_A)^2 + (y_D - y_A)^2} = \sqrt{2^2 + (-6)^2} = \sqrt{40} = 2\sqrt{10}$$

Marks: 0, 3, 5, 8, 10

(d) Calculate the slopes of lines  $(BC)$  and  $(AD)$  and show that the two lines are perpendicular.

$$\text{Slope}_{BC} = \frac{y_C - y_B}{x_C - x_B} = \frac{7 - 5}{8 - 2} = \frac{1}{3}$$

$$\text{Slope}_{AD} = \frac{y_D - y_A}{x_D - x_A} = \frac{6 - 0}{5 - 7} = -3$$

The product of the two slopes is  $-1$  so the two lines  $(BC)$  and  $(AD)$  are perpendicular. Marks: 0, 3, 5, 8, 10

(e) Calculate the area of triangle  $ABC$ .

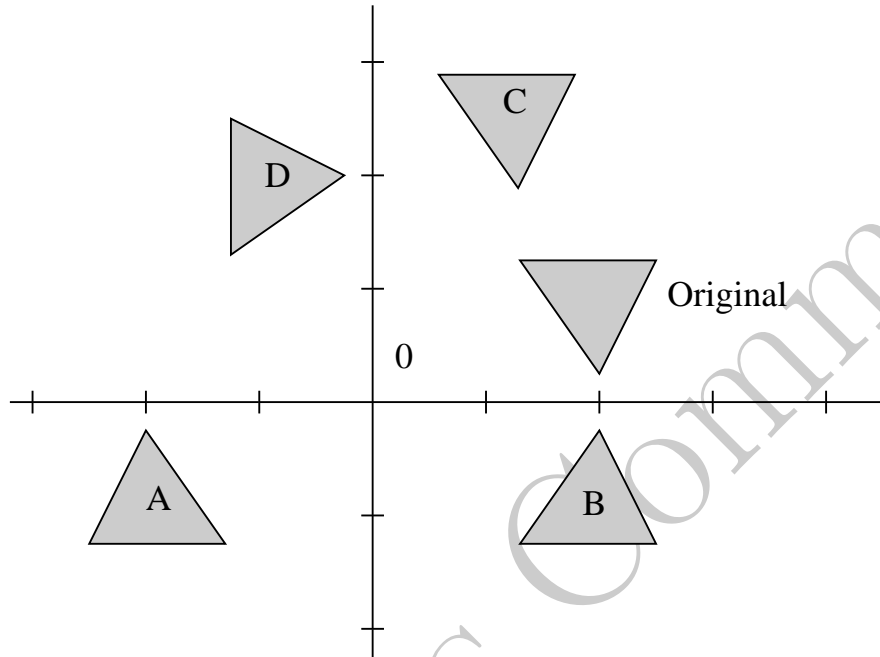
$$\text{Area} = \frac{1}{2}|BC| \times |AD| = \frac{1}{2}\sqrt{40}\sqrt{40} = 20$$

Marks: 0, 3, 5, 8, 10

### Question 4

(Suggested maximum time: 5 minutes)

The co-ordinate diagram above shows an original shape and its image under five transformations. Complete the table below, each letter must be used once only. Justify your answer using the box on the next page.

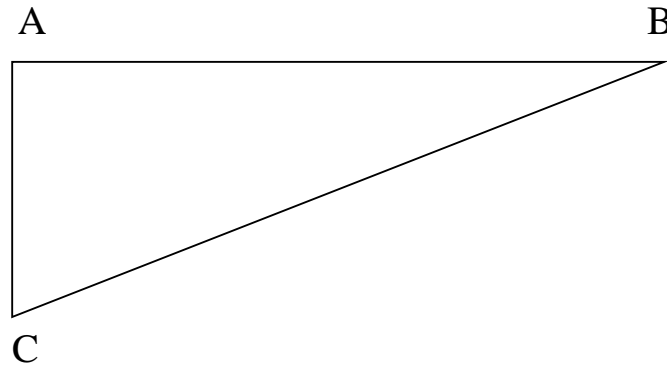


Central symmetry in $(0, 0)$	A
Rotation about $(0, 0)$	D
Translation	C
Axial symmetry	B

Marks: 0, 3, 5, 8, 10

## Question 5 (Suggested maximum time: 10 minutes)

Triangle ABC is a right angle triangle with  $|AB| = 8\text{cm}$ ,  $|AC| = 6\text{cm}$  and  $|BC| = 10\text{cm}$ .



- (a) Calculate  $\cos \angle ABC$  and  $\sin \angle ABC$ . Express your results in the form  $a/b$  where  $a, b \in \mathbb{N}$

$$\cos \angle ABC = \frac{8}{10} = \frac{4}{5} \quad \sin \angle ABC = \frac{6}{10} = \frac{3}{5}$$

Marks: 0, 3, 5

- (b) Without using your calculator, calculate  $\cos^2 \angle ABC + \sin^2 \angle ABC$

$$\cos^2 \angle ABC + \sin^2 \angle ABC = \left(\frac{4}{5}\right)^2 + \left(\frac{3}{5}\right)^2 = \frac{16+9}{25} = 1$$

Marks: 0, 3, 5

- (c) Calculate the value in degrees of angle  $\angle ABC$  correct to 4 decimal places.

$$\angle ABC = \cos^{-1}\left(\frac{4}{5}\right) = 36.8699^\circ$$

Marks: 0, 3, 5

- (d) Write this result in DMS format (i.e. degrees, minutes, and seconds).

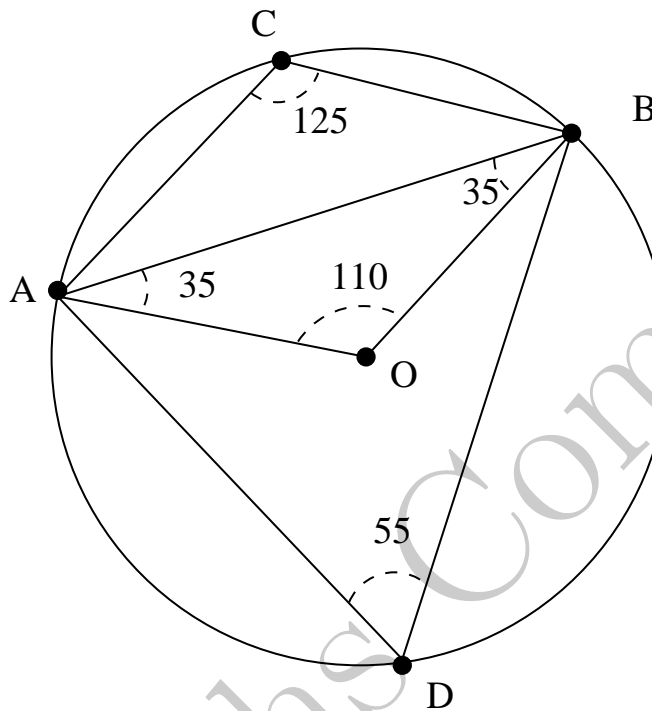
$$\angle ABC = 36^\circ 52' 12''$$

Marks: 0, 3, 5



## Question 6 (Suggested maximum time: 10 minutes)

The diagram below shows the two triangles ABO and ABC. O is the centre of the circle and the angle  $\angle OAB = 35^\circ$ . Calculate the value of the angles  $\angle AOB$ ,  $\angle ADB$  and then  $\angle ACB$ . Show all your working out.



$$\begin{aligned} \angle OAB = \angle OBA = 35 &\implies \angle AOB = 180 - \angle OAB - \angle OBA = 110 \\ \implies \angle ADB = 55 &\implies \angle ACB = 180 - \angle ADB = 180 - 55 = 125 \end{aligned}$$

Marks: 0, 5, 10, 15, 20, 25

## Question 7 (Suggested maximum time: 10 minutes)

(a) Calculate the volume of the pyramid with a square base shown above.

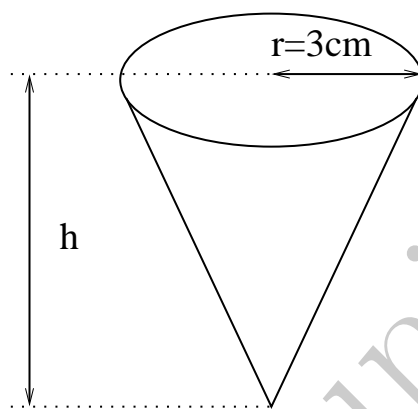
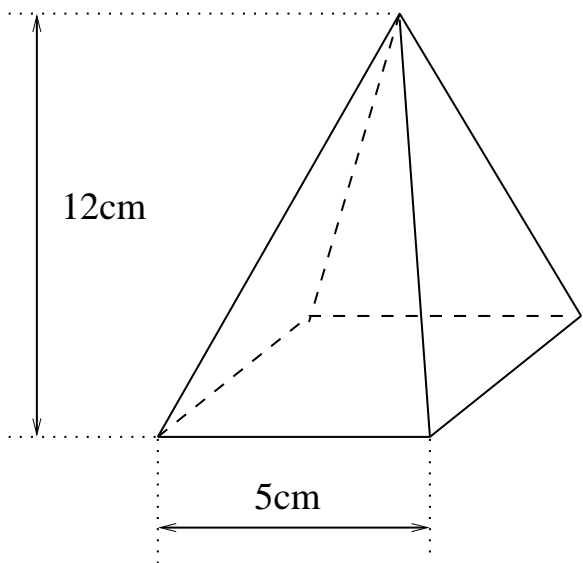
$$V = \frac{1}{3}(12)(5)^2 = 100\text{cm}^3$$

Marks: 0, 3, 5

(b) Calculate the height  $h$  of the cone shown above if the volume of the cone is  $V = 15\pi \text{ cm}^3$ .

$$15\pi = \frac{1}{3}h\pi(3)^2 \iff 15\pi = 2h\pi \iff h = 5\text{cm}$$

Marks: 0, 3, 5, 8, 10



### Question 8 (Suggested maximum time: 15 minutes)

In a bookshop, the black, blue, green and red pens are mixed. You pick a pen randomly. The probability of picking a black pen is  $\frac{2}{9}$ , the probability of picking a blue pen is  $\frac{4}{9}$  and the probability of picking a red pen is  $\frac{2}{9}$

- (a) Calculate the probability of picking a green pen.

$$\frac{2}{9} + \frac{4}{9} + \frac{2}{9} + G = 1 \iff G = \frac{1}{9}$$

Marks: 0, 3, 5

- (b) Calculate the probability of picking a black or a red pen.

$$p = \frac{2}{9} + \frac{2}{9} = \frac{4}{9}$$

Marks: 0, 3, 5

- (c) There are 90 pens in the box. Calculate the number of pens of each color.

$$\text{Green} = 10 \quad \text{Blue} = 40 \quad \text{Black} = 20 \quad \text{Red} = 20$$

Marks: 0, 3, 5

- (d) 10 blue pen and 20 black pen are added in the box.

- (i) Calculate the new number of pens in the box for each colour.

$$\text{Green} = 10 \quad \text{Blue} = 40 + 10 = 50 \quad \text{Black} = 20 + 20 = 40 \quad \text{Red} = 20$$

Marks: 0, 3, 5

- (ii) What is the new probability of picking up a blue pen. Give you results in the format  $\frac{a}{b}$  where a, b  $\in \mathbb{N}$

$$p = \frac{50}{120} = \frac{5}{12}$$

Marks: 0, 3, 5

## Question 9 (Suggested maximum time: 15 minutes)

Kate owns a shop. She wants to know if people spend the same amount of money in the morning and in the afternoon. She collects the following data

- Morning: 141, 192, 178, 153, 182, 145, 175, 184, 147, 168, 155, 157, 184, 147, 197.
- Afternoon: 145, 185, 141, 151, 186, 154, 189, 151, 186, 172, 179, 165, 157, 190, 154.

- (a) Represent the data using the back to back stem leaf diagram below.

Morning		Afternoon
7,7,5,1	14	1,5
7,5,3	15	1,1,4,4,7
8	16	5
8,5	17	2,9
4,4,2	18	5,6,6,9
7,2	19	0

Marks: 0, 3, 5, 8, 10, 12, 15

- (b) Find the median for both morning and afternoon.

Morning=168, Afternoon=165

Marks: 0, 3, 5, 8, 10

(c) Calculate the mean for morning and afternoon.

Morning=167, Afternoon=167

Marks: 0, 3, 5, 8, 10

(d) Calculate the inter quartile range for morning and afternoon.

Morning=184-147=37, Afternoon=186-151=35

Marks: 0, 3, 5, 8, 10

(e) Do customers in the morning and afternoon spend the same amount?

Yes, there is the same median and average and the interquartile range is the same.

Marks: 0, 3, 5

## Question 10 (Suggested maximum time: 10 minutes)

The amount of time students spend on a computer daily is described in the table below

0-60 min	60-120 min	120-180 min	180-300 min
7	21	18	5

(a) Use mid-interval values to estimate the mean time the students spend on their computer. Give your answer correct to one decimal place.

$$\text{Mean} = \frac{7 \times 30 + 21 \times 90 + 18 \times 150 + 5 \times 210}{7 + 21 + 18 + 5} = 117.6 \text{ min}$$

Marks: 0, 3, 5, 8, 10

(b) On average, students actually spend 125 minutes on their computer. Why is this value different from the value you calculated in the previous questions?

Mid values only lead to an approximative value.

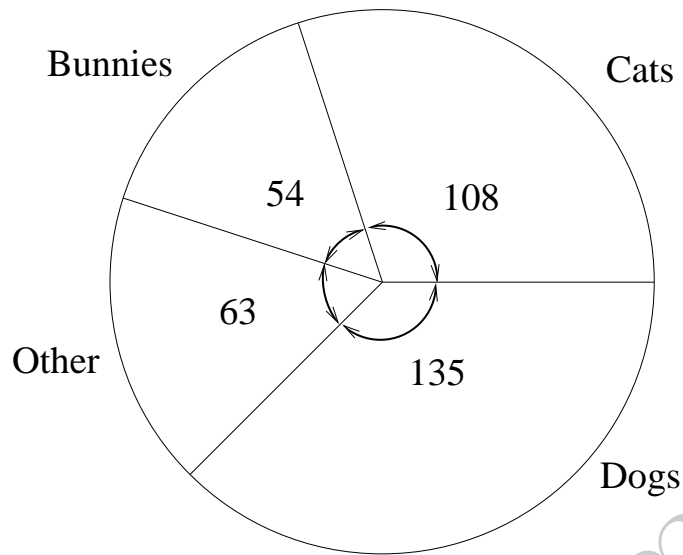
Marks: 0, 3, 5

## Question 11 (Suggested maximum time: 15 minutes)

A survey is carried out among students in a school about their favourite pet.

- 30 students preferred dogs,
- 24 students preferred cats,
- 12 students preferred bunnies,
- 14 students preferred another type of pet.

(a) Represent the data in a pie chart. Justify your results in the box on the next page.



Marks: 0, 3, 5

	Dogs	Cats	Bunnies	Other	Total
Number of students	30	24	12	14	80
Angle	135	108	54	63	360

Marks: 0, 3, 5, 8, 10

(b) What percentage of students preferred a pet dog?

$$p = \frac{30}{80} = 37.5\%$$

Marks: 0, 3, 5

(c) What is the difference in percentage between the number of people preferring a cat and the number of people preferring bunnies?

$$p = \frac{24 - 12}{80} = 15\%$$

Marks: 0, 3, 5

- (d) If approximately 14.3% of people who prefer another type of pet actually prefer donkeys, how many donkey lovers are there in the school?

$$14 \times .143 \approx 2$$

Marks: 0, 3, 5

- (e) If a person is chosen randomly, what is the probability that the student's favourite pet is a cat or a dog?

$$p = \frac{54}{80} = 0.675$$

Marks: 0, 3, 5

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