

Leaving Certificate Examination, 2018

Sample paper prepared by Leamy Maths Community

Mathematics

Paper 1

Ordinary Level

Sunday 22 April

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Name _____

Solutions

300 marks

Sample Instructions

There are two sections in this examination paper:

Section A	Concepts and Skills	150 marks	6 questions
Section B	Contexts and Applications	150 marks	3 questions

Answer questions as follows:

In Section A, answer all six questions.

In Section B, answer all three questions.

Write your answers in the spaces provided in this booklet. There is space for extra work at the back of the booklet. You may also ask the superintendent for more paper. Label any extra work clearly with the question number and part.

The superintendent will give you a copy of the booklet of *Formulae and Tables*. You must return it at the end of the examination.

You are not allowed to bring your own copy into the examination.

Marks will be lost if all necessary work is not clearly shown.

Answers should include the appropriate units of measurement, where relevant.

Answers should be given in simplest form, where relevant.

Write the make and model of your calculator(s) here:

Answer **all six** questions from this section.

Question 1

(25 Marks)

Cathal sells two types of cakes, cake A and cake B.

- (a) One week, he sells 30 cakes of type A and a number of cakes of type B for €101.25. He charges €1.5 for each cake of type A and €2.25 for cakes of type B. How many cakes of type B did he sell?

$$30(1.5) + x(2.25) = 101.25 \Rightarrow x = 2.25$$

Marks: 0, 3, 5, 7

- (b) Two weeks later, he sells 40 cakes of type A and 20 cakes of type B. If he has increased the price of type A cakes by 20% twice and has kept the same price for cakes of type B, how much will clients pay in total?

$$1.5 \times 1.2^2 = 2.16 \Rightarrow 2.16 \times 40 + 20 \times 2.25 = 131.4$$

Marks: 0, 3, 5, 8, 10

- (c) A large cake is initially sold for €20. The value is decreased by 10% a week for three weeks in a row. What is the new price for this cake?

$$P = 20 \times (0.9)^3 = 14.58$$

Marks: 0, 3, 5, 8

Question 2

(25 Marks)

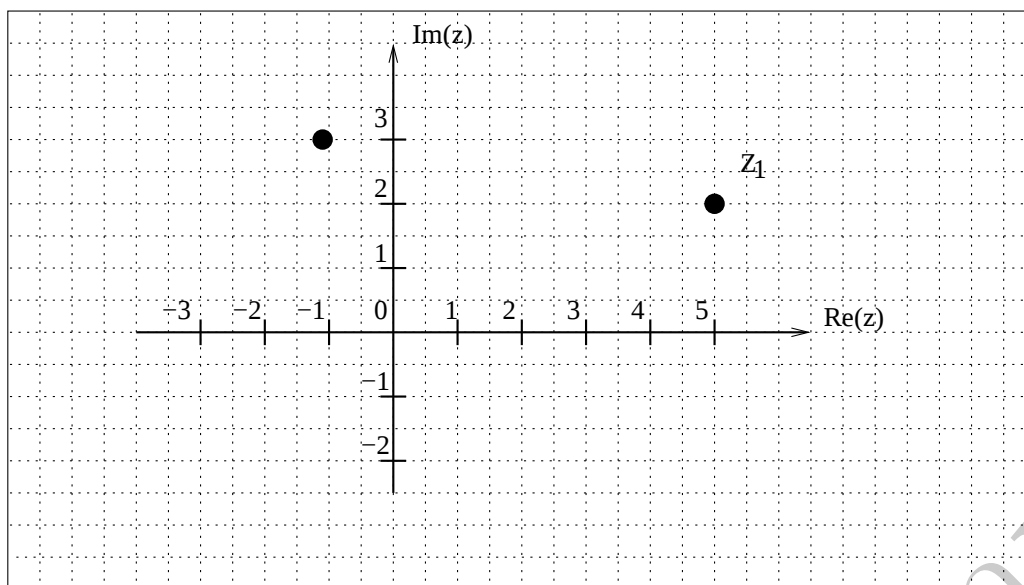
- (a) The complex number $z_1 = a + ib$ where $i^2 = -1$ is shown on the Argand diagram below. Identify the values for a and b .

$$a = 5$$

$$b = 2$$

Marks: 0, 3, 5

- (b) Place the point $z_2 = -1 + 3i$ on the Argand diagram.



(c) $z_3 = z_1/z_2$. Write z_3 in the form $a + ib$ where $a, b \in \mathbb{R}$.

$$\begin{aligned}
 z_3 &= \frac{z_1}{z_2} \\
 &= \frac{5 + 2i}{-1 + 3i} \\
 &= \frac{(5 + 2i)(-1 - 3i)}{(-1 + 3i)(-1 - 3i)} \\
 &= \frac{-5 - 15i - 2i - 6i^2}{1 - 9i^2} \\
 &= \frac{1 - 17i}{10} \\
 &= 0.1 - 1.7i
 \end{aligned}$$

Marks: 0, 3, 5, 8, 10

(d) Compare $|z_1| + |z_2|$ and $|z_1 + z_2|$.

$$|z_1| = \sqrt{25 + 4} = \sqrt{29} \quad |z_2| = \sqrt{1 + 9} = \sqrt{10} \quad |z_1 + z_2| = \sqrt{16 + 25} = \sqrt{41}$$

$$\text{so } |z_1| + |z_2| \neq |z_1 + z_2|$$

Marks: 0, 3, 5, 8, 10

Question 3

(25 Marks)

(a) Solve for x

$$5(x - 3) - (4 - 2x) = 3x - 3$$

$$4x = -3 + 19 \implies x = 4$$

Marks: 0, 3, 5

(b) Solve for x and y in the simultaneous equations below.

$$\begin{aligned}4x + 3y &= 1 \\ -x + 2y &= -3\end{aligned}$$

$$x = 1 \quad y = -1$$

Marks: 0, 3, 5, 8, 10

(c) Find the values of $x \in \mathbb{N}$ which verify

$$4(x + 3) - 2x < 21$$

$$2x < 9$$

The solutions are then $x = 1, 2, 3, 4$. Marks: 0, 3, 5, 8, 10

Question 4

(25 Marks)

Consider the function

$$f(x) = x^3 - 3x^2 - 7x - 2$$

(a) Where does the function graph cross the y axis

$$f(0) = -2 \implies (0, -2)$$

Marks: 0, 3, 5

(b) Show that the point $A(-1, 1)$ is on the graph of f

$$f(-1) = -1 - 3 + 7 - 2 = 1$$

so A is on the graph of f . Marks: 0, 3, 5

(c) Calculate the value of the derivative of f at $x = -1$.

$$f'(x) = 3x^2 - 6x - 7 \implies f'(-1) = 2$$

Marks: 0, 3, 5, 7

(d) Calculate the equation of the tangent to the graph of f at the point $(-1, 1)$

$$y = 2(x + 1) + 1 = 2x + 3$$

Marks: 0, 3, 5, 8

Question 5

(25 Marks)

- (a) Solve the equation:

$$x^2 - 6x + 7 = 0$$

Give your solutions correct to one decimal place.

$$x_1 = \frac{6 + \sqrt{8}}{2} = 4.4 \quad x_2 = \frac{6 - \sqrt{8}}{2} = 1.6$$

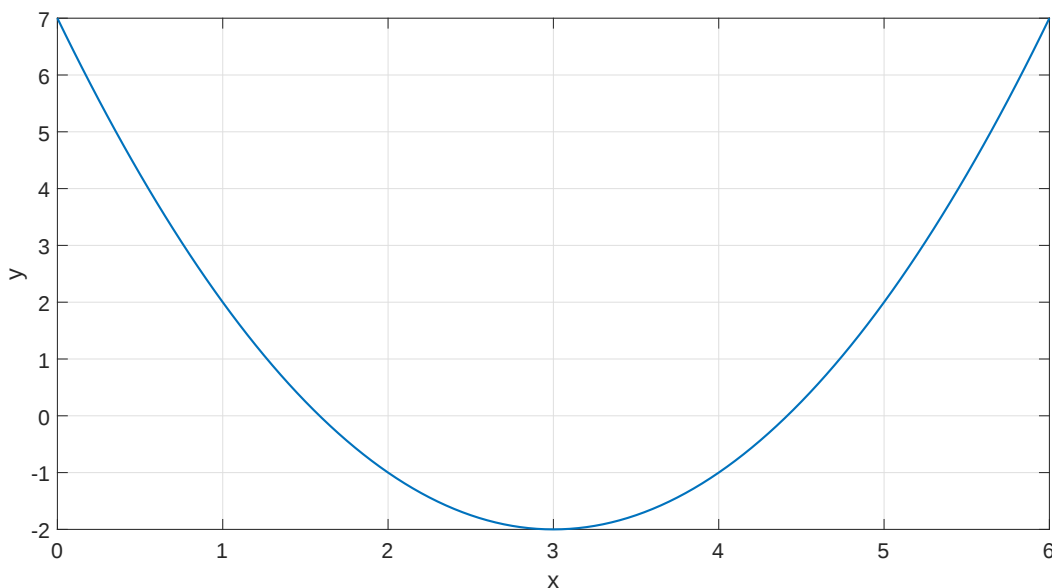
Marks: 0, 3, 5, 7

- (b) Find the turning point of function $f(x) = x^2 - 6x + 7$.

$$f'(x) = 2x - 6 \quad f'(x) = 0 \implies x = 3$$

Marks: 0, 3, 5, 8

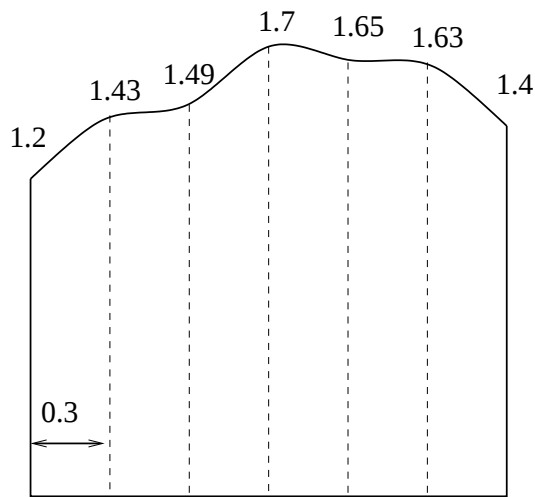
- (c) Using the information you found in the first two parts, sketch the graph for $y = f(x)$ for $0 \leq x \leq 6$ Marks: 0, 3, 5, 8, 10



Question 6

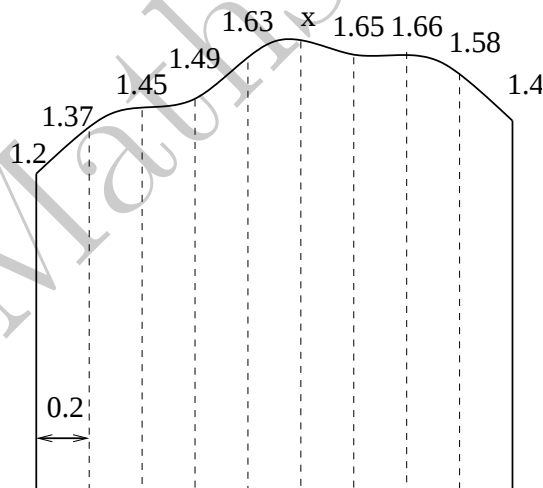
(25 Marks)

- (a) Mary wants to paint her garden gate. She needs to know the surface of the gate. She divides the gate into six sections as shown below. The width of each section is 0.3 metres. The height, in metres, of each section is given in the diagram. Use the Trapezoidal rule to estimate the area of the gate.



$$Area = \frac{0.3}{2} (1.2 + 1.4 + 2 \times (1.43 + 1.49 + 1.7 + 1.65 + 1.63)) = 2.75$$

- (b) Mary found the measurements she made the previous year. The area of the gate was estimated by applying the Trapezoidal rule again. This time, a different section width (0.2m) and a different set of section heights were used, as shown below. The area was found to be 2.75 m². Use this information to find the value of the height marked x on the diagram



$$2.75 = \frac{0.2}{2} (1.2 + 1.43 + 2 \times (1.37 + 1.45 + 1.49 + 1.65 + x + 1.65 + 1.66 + 1.58)) \implies x = 1.62$$

Answer **all three** questions from this section.

Question 7

(50 Marks)

Eimar decides to change her gas provider. She goes for a plan with the following characteristics

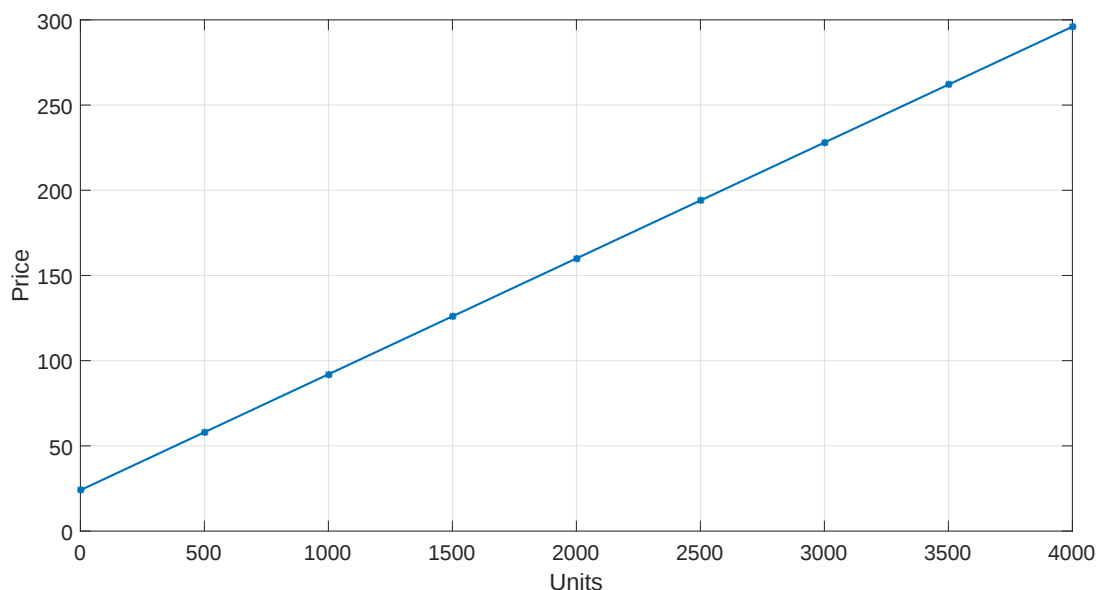
Units used	Cost in Euro
500	58
1000	92
1500	126
2000	160
2500	194
3000	228
3500	262
4000	296

- (a) Use the data above to investigate whether the pattern in the table above is linear, quadratic or exponential. Explain your conclusion

Data is linear (the first difference is constant)

Marks: 0, 3, 5

- (b) Plot the data on the graph below



Marks: 0, 3, 5

- (c) Find the equation of the curve $y=f(x)$ where y is the price in € and x is the number of units.

$$y = \frac{34}{500}(x - 500) + 58 = .0068x + 24$$

Marks: 0, 3, 5, 8, 10

- (d) How much would Eimar pay if she did not use any unit?

$$y(0) = 24$$

Marks: 0, 3, 5

- (e) The prices above do not include VAT. Eimar used 2000 units and her total bill was €181.6. Find the VAT rate on gas, correct to one decimal place

$$R = \frac{181.6}{160} = 1.135$$

VAT is 13.5%. Marks: 0, 3, 5, 8

- (f) The VAT rates will change in the near future. The new rates will work as follows:

- 15% on the first €125
- 20% on the balance

If Eimar uses 1000 units, what will be her total bill?

$$P = 95 \times 1.15 = 105.8$$

Marks: 0, 3, 5, 7

- (g) If Eimar uses 2000 units, what will be her total bill with the new VAT rates?

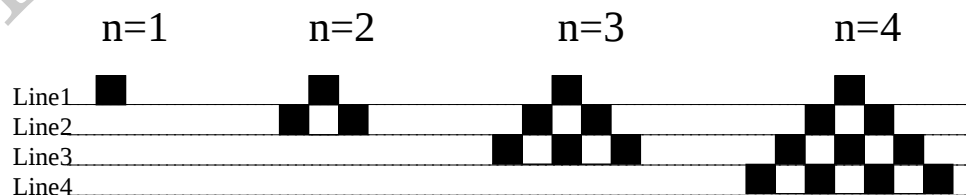
$$P = 125 \times 1.15 + 35 \times 1.2 = 185.75$$

Marks: 0, 3, 5, 8, 10

Question 8

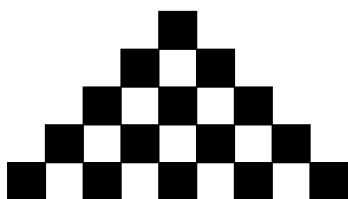
(65 Marks)

Ciara wants to draw some patterns with tiles. She uses the design shown on the picture below:



- (a) Draw the next pattern corresponding to $n=5$.

Marks: 0, 3, 5



(b) Fill in the table below

Line	1	2	3	4	5
No. of tiles	1	3	5	7	9
No. of black tiles	1	2	3	4	5
No. of white tiles	0	1	2	3	4

Marks: 0, 3, 5, 8, 10

(c) Assuming the pattern continues, the number of white tiles in the n -th line of the the n -th pattern is given by the formula

$$B_n = pn + q \quad \text{where } p, q \in \mathbb{N}$$

Find the value of p and q .

$$p + q = 0 \quad 2p + q = 1 \implies p = 1, \quad q = -1 \implies W = n - 1$$

Marks: 0, 3, 5, 8, 10

(d) How many white tiles are in the 15-th line

$$W_{15} = 15 - 1 = 14$$

Marks: 0, 3, 5, 7

(e) Show that the number of white tiles required to build the n -th patter is

$$S_n = \frac{n^2 - n}{2}$$

Sum of linear series

Marks: 0, 3, 5, 8, 10, 13, 15

(f) How many white tiles do you need to build the 25-th pattern

$$S_{25} = \frac{625 - 25}{2} = 300$$

Marks: 0, 3, 5, 8

(g) What pattern can you build if you have 190 tiles

$$n^2 - n = 380 \implies n = 20$$

Marks: 0, 3, 5, 8, 10

Question 9

(35 Marks)

A tablet manufacturer wants to maximise its profit. The daily profit (in thousands of Euros) can be calculated with the function

$$\pi(x) = -x^2 + 12x - 20$$

where x is the number of tablets produced everyday in thousands of units.

(i) Calculate the profit when the industrial produces 5000 tablets.

$$\pi(5) = -25 + 60 - 20 = 15$$

Profit is €15,000

Marks: 0, 3, 5

(ii) Calculate the derivative of the profit.

$$\pi'(x) = -2x + 12$$

Marks: 0, 3, 5

(iii) What number of tablets maximises the profit?

$$\pi'(x) = 0 \implies -2x + 12 = 0 \implies x = 6$$

The profit is maximum for 6,000 tablets

Marks: 0, 3, 5, 8

(iv) Calculate the maximum profit.

$$\pi(6) = -36 + 72 - 20 = 16$$

The profit is €16,000.

Marks: 0, 3, 5, 7

(v) What are the break even points for this profit? These points are defined as $\pi(x) = 0$.

$$-x^2 + 12x - 20 = 0 \implies -(x - 10)(x - 2) = 0$$

The break even points are 2,000 and 10,000 tablets.

Marks: 0, 3, 5, 8, 10